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ABSTRACT

The vortex tube is a structurally simple device with no moving parts that is capable of separating a high-pressure flow into two lower pressure flows with different energies, usually manifested as a difference in temperatures. The vortex tube is relatively inefficient as a stand-alone cooling device but it may become an important component of a refrigeration system when employed as an alternative to the conventional throttling valve. A cycle augmented with a vortex tube can offer several advantages including: efficient operation above the Joule-Thomson inversion curve, relative insensitivity to heat exchanger size, and the ability to operate with a lower pressure ratio. The extent to which these advantages are realized depends on the cycle configuration, the fluid properties, the operating conditions, and the behavior of the vortex tube.

In this paper, a semi-empirical model of the vortex tube is presented using one hypothesis regarding its underlying physics. Predictions made using this model are found to compare favorably with the limited experimental data that are available. The vortex tube model is integrated into a model of a vapor compression refrigerator and a Joule-Thomson cryogenic refrigerator. The potential for increasing the performance of a refrigeration cycle using the vortex tube is found to be extremely limited for the vapor compression cycle. However, it is shown that the vortex tube may present a significant opportunity to improve the performance of refrigeration systems using the Joule-Thomson cycle and may allow efficient operation at lower pressure ratios, with smaller recuperative heat exchangers, and with less expensive working fluids than are currently used. More complex cycles are discussed in which the vortex tube can be used to simultaneously perform phase and energy separation.

INTRODUCTION

Figure 1 illustrates a counter-flow vortex tube. High-pressure gas enters the tube through one or more nearly tangential nozzles. Colder, low-pressure gas leaves via an orifice near the centerline adjacent to the plane of the nozzles and warmer, low-pressure gas leaves near the periphery at the end of the tube opposite the nozzles. This device was described by Ranque (1933) and examined experimentally in the 1940's by Hilsch (1947). The vortex tube requires no work or heat interaction with the environment to operate. Consequently the energy separation effect must be attributable to an energy interaction that is internal to the tube, occurring between the two exiting flows. The exact mechanism of this interaction is not well understood due to the lack of reliable measurements of the temperature, pressure, and velocity distributions. Because of the strong recirculating nature of the flow field, the presence of invasive instruments strongly affects the entire flow field (Reynolds, 1962) and the high velocity, swirling flow within a vortex tube presents a significant

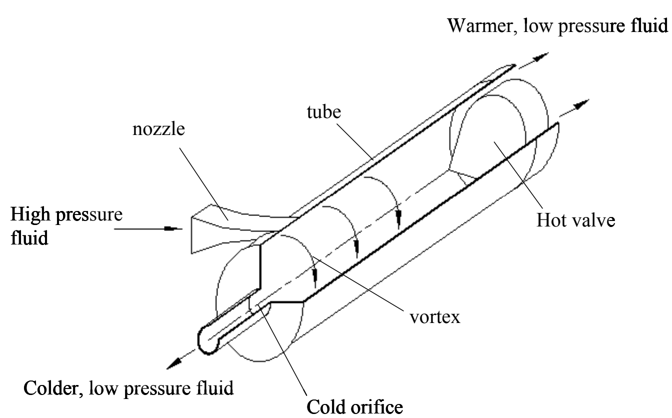

Figure 1. Counter-flow Vortex Tube

challenge to traditional, non-invasive flow visualization techniques as the large radial pressure gradients prevent any seeding particle from faithfully following the flow; instead these markers are quickly centrifuged to the tube outer diameter (Cockerill, 1998).

Despite the simplicity of vortex tube's geometry, the energy separation phenomenon is quite complex and this has lead to the publication of several, semi-conflicting theories regarding its operation. Some researchers have suggested that energy is transferred as work due to viscous shear between a fast moving, inner core and a slower moving outer annulus that is characteristic of a free vortex (e.g. Lewins et al., 1999). Other researchers have described internal, refrigeration cycles associated with fluid motion due to turbulent eddies (Hartnett et al., 1957), sound waves (Kurosaka, 1982), Göertler vortices (Stephan et al., 1983), and secondary circulation (Ahlborn et al., 2000). In this paper the behavior of the vortex tube is modeled more simply – as a nozzle in series with a counter-flow heat exchanger, an idea originally attributed to Sheper (1951). A semi-empirical model of the vortex tube is described based on this concept and predictions made using this model are shown to agree well with some experimental data. Recently the vortex tube has received some interest as a potential component in a cryogenic Joule-Thomson (Nash, 1991) or traditional vapor compression refrigeration cycle (Li et al., 2000). This paper uses the semi-empirical model of the vortex tube to evaluate the potential cycle efficiency improvement that can be attained when a conventional expansion valve is replaced with a counter-flow vortex tube in these refrigeration cycles.

EMPIRICAL MODEL OF THE VORTEX TUBE

The vortex tube's behavior can be understood using the concept of a nozzle in series with a counter-flow heat exchanger, illustrated in Figure 2. Gas enters through the nozzle at the inlet temperature ($T_{vt,in}$) and pressure ($P_{vt,in}$). As the gas passes through the nozzle its velocity is increased (to V_N) and its temperature drops accordingly (to T_N). The flow in the vortex tube is divided into two regions: an outer annulus containing the high velocity, swirling gas flowing towards the hot end and an inner core containing the more quiescent gas flowing in the opposite direction. At the hot and cold ends some of the gas is drawn off and exits the device while the remainder of the gas is recirculated. This flow pattern has been predicted by CFD analysis (Fröhlingsdorf et al., 1999) and observed experimentally (e.g. Takahama, 1965 and Ahlborn et al., 1997). The ratio between the mass flow of gas leaving the cold end and the mass flow that initially entered the vortex tube is called the cold gas mass ratio (y). The ratio between the mass flow of gas that is recirculated and that entering the vortex tube is called the recirculation factor (α).

The gas leaving the nozzle mixes with the quiescent and warmer flow that is recirculated from the core region, causing an increase in the temperature of the gas entering the outer annulus ($T_{o,0}$) and a decrease in its velocity ($V_{o,0}$). As the swirling gas near the annulus progresses axially along the length of the tube it experiences a viscous torque that acts to reduce its angular momentum. The resulting axial decay in tangential velocity has been experimentally measured (Takahama, 1981) and theoretically predicted (Sibulkin, 1962). The tangential velocity remaining at the hot end of the tube ($V_{o,L}$) depends on the length of the tube relative to the length constant of this decay process. As the swirling gas is decelerated its temperature increases because kinetic energy is converted to thermal energy. However, this swirling gas also experiences a heat transfer from the relatively warmer gas returning through the core. As a result, the temperature of the gas in the annulus at the warm end of the vortex tube ($T_{o,L}$) is greater than the inlet temperature. At the warm end of the tube the gas in the annulus sheds any remaining tangential velocity and either leaves via the hot exit or returns through the core. The static temperatures of these two streams ($T_{vt,out,h}$ and $T_{c,L}$) are increased due to this reduction in kinetic energy. The gas returning through the core is therefore at a higher temperature and has a smaller total heat capacity than the gas in the annulus, and is therefore cooled. The thermal communication between the core and annulus occurs via turbulent eddies in the absence of any boundary layer and is extremely efficient; therefore the temperature of the gas in the core can approach the nozzle exit temperature at the cold end of the vortex tube ($T_{vt,out,c}$).

This conceptual framework forms the basis of a semi-empirical model. The model consists of several simple but interrelated components. The nozzle is modeled using a nozzle efficiency (η_N) defined in the usual way as the ratio of the actual to isentropic kinetic energy. The nozzle does not see the entire pressure difference applied to the vortex tube due to the centrifugal pressure gradient that is set up at the cold end. This centrifugal pressure gradient is modeled by integrating the exit average density and a forced vortex velocity profile. An additional pressure drop is included from the flow through the cold orifice, modeled using an orifice loss coefficient. The mixing process between the recirculating gas and the gas leaving the nozzle is modeled using conservation of angular momentum and conservation of energy. The decay of kinetic energy is modeled using a friction factor correlation for turbulent flow through a rectangular duct using a Reynolds number based on the nozzle exit velocity. The interaction between the gases in the core and the annulus along the length of the tube is modeled using conventional heat exchanger equations modified to include the substantial variation in fluid kinetic energy and accounting for this effect on the internal temperature profiles. The thermal communication between these gases is modeled using a turbulent mixing length that is 10% of the tube radius.

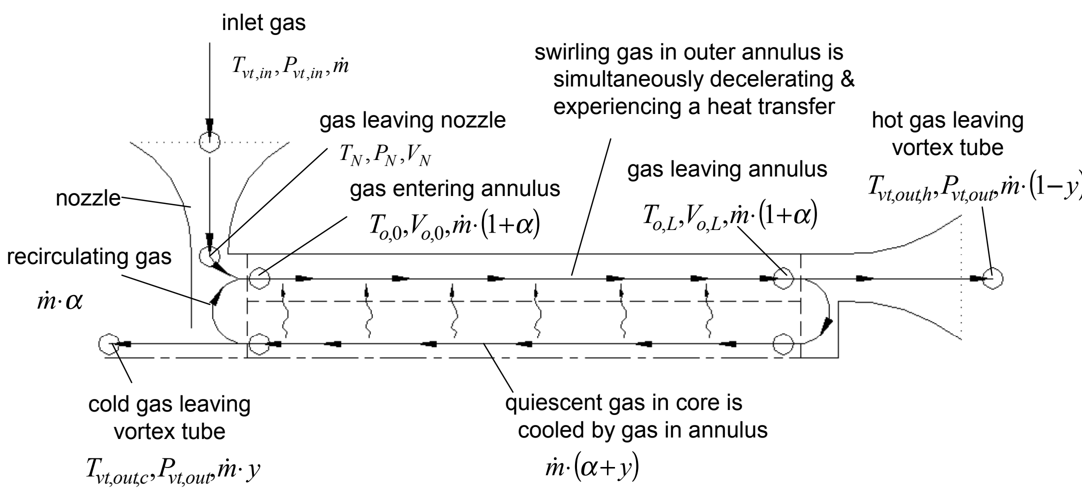

Figure 2. Conceptual Model of Counter-Flow Vortex Tube

This model is intended to capture the major physical mechanisms occurring within the vortex tube. The model is semi-empirical in the sense that the nozzle efficiency and recirculation factor are adjusted to achieve reasonable agreement between the model and the available data. Figure 3 illustrates data reported by Stephan et al. (1984) for a vortex tube operated with room temperature air at various inlet pressures as a function of the cold gas mass fraction. Also shown in Figure 3 is the performance predicted by the vortex tube model described above. Figure 4 illustrates the measured and predicted performance for the same vortex tube using helium gas. The model qualitatively reproduces the performance trends and also predicts absolute performance that is representative of the experimental measurements.

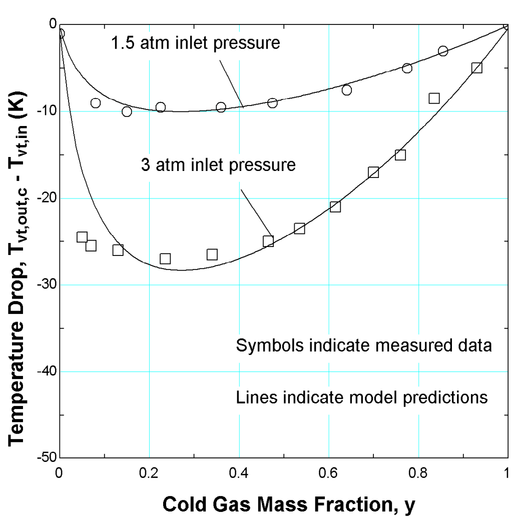

Figure 3. Measured vs Predicted Performance with Room Temperature Air

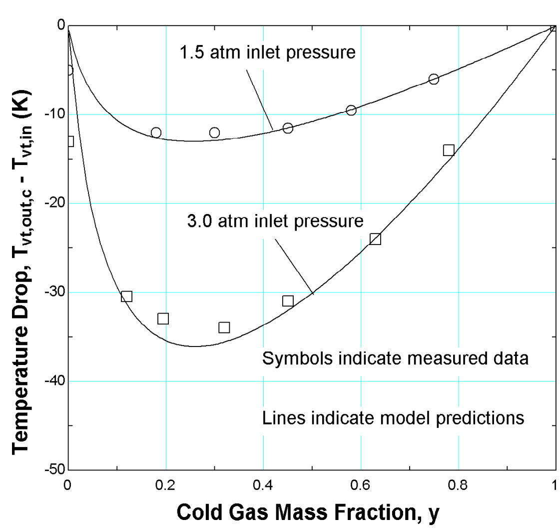

Figure 4. Measured vs Predicted Performance with Room Temperature Helium

This physical explanation of a vortex tube's operation is based on a heat transfer between the cold, high velocity flow leaving the nozzle and the low velocity flow in the core. This explanation implies that the fluid leaving the cold end of a "perfect" vortex tube can be no colder than the fluid leaving an isentropic nozzle subjected to the same inlet and exit pressures. The next section shows that this limit is rather restrictive and it has some important implications relative to the utility of the vortex in refrigeration systems.

APPLICATION OF VORTEX TUBE TO REFRIGERATION CYCLES

The relative inefficiency of the vortex tube as a stand-alone cooling device has thus far limited its use. The coefficient of performance (COP) of the vortex tube as a refrigerator, cooling provided per work required to compress the fluid, is very low (less than 0.1 near room temperature) relative to a domestic refrigeration cycle. However, its simplicity makes it an extremely compact, reliable, affordable, and flexible alternative in some special applications. A more exciting and potentially much broader use for the vortex tube exists if it can be integrated into a refrigeration system as an alternative to the conventional throttling valve in order to accomplish the required expansion process in a less irreversible manner. The vortex tube model developed in the previous section will be employed to evaluate the utility of the vortex tube within vapor compression and Joule-Thomson refrigeration systems. The results of these analyses must be qualified by the statement that the vortex tube model has been developed for and verified against data taken at room temperature under single-phase operating conditions. The performance of a vortex tube may be substantially different in the two-phase and non-ideal conditions encountered within a refrigeration system but the data available for these operating conditions are scarce and often conflicting. The analysis remains a valuable tool provided that the underlying physical model is accurate and it clearly points out those situations where the potential exists for significant improvements in cycle performance.

Vapor Compression Cycle

Figure 5 illustrates a conventional vapor compression refrigeration cycle with a throttling valve and Figure 6 illustrates the corresponding ideal thermodynamic cycle using a typical synthetic refrigerant, R134a. The process of expanding the two-phase fluid through the valve, from state (3) to state (4), follows a line of constant enthalpy. In the situation where the valve is replaced by a vortex tube, the corresponding expansion for the fluid extracted from the cold side of the tube would be limited to an isentropic process, for reasons described earlier. The limiting exit states would therefore be (4c) and (4h), as indicated in Figure 6. Clearly, under these assumptions the vortex tube is not capable of producing a temperature separation effect under the vapor dome – the hot and cold fluids leave at the same temperature and at most we might expect a difference in quality based on centrifugal forces. This fact has been experimentally observed in those few instances where a vortex tube has been operated in the saturated region, for example by Takahama et al. (1979) using steam. Therefore, there is no benefit associated with the use of a vortex tube within a pure refrigerant, vapor compression refrigeration cycle.

The isenthalpic and isentropic temperature change between two isobars are not identical when the refrigeration cycle extends into the super-critical region or utilizes non-azeotropic mixtures and in these situations it is possible that the application of a vortex tube may be beneficial. Carbon dioxide is a naturally occurring substance that has recently received renewed attention as a potential alternative to synthetic refrigerants (Lorentzen, 1994) because it is non-flammable, non-toxic, readily available, and compatible with most materials. However the theoretical COP of a carbon dioxide refrigeration system is not as high as that of most synthetic refrigerants, in large part due to throttling losses (these inefficiencies explain more than 40% of the total loss according to Robinson et al., 1996). The potential for increasing the performance of a carbon dioxide cycle via modification of the expansion mechanism is therefore high. Li et al. (2000) theoretically investigated a carbon dioxide cycle with an expansion valve, a work extracting expansion device, and a vortex tube and reported that use of a vortex tube could improve the performance relative to the valve. This analysis made no attempt to model the vortex tube beyond invoking the 1st and 2nd law of thermodynamics. If the physical model described above is used to model the vortex tube in this situation then the predicted benefit in cycle performance disappears, again because the temperature separation effect goes rapidly to zero as the vapor dome is approached.

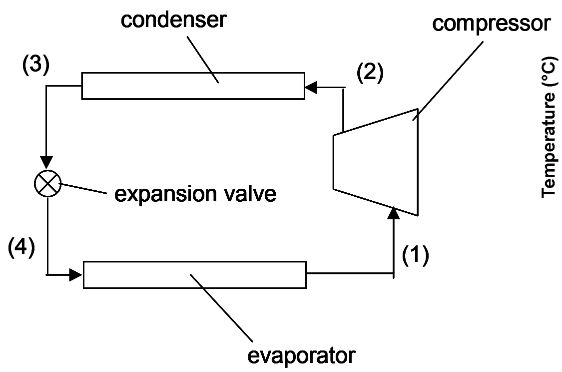

Figure 5. Vapor Compression Refrigeration Cycle

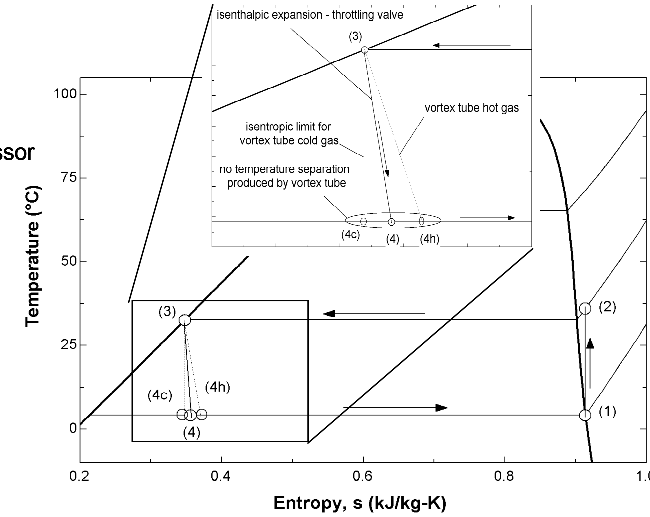

Figure 6. Ideal Vapor Compression Refrigeration Cycle

Joule-Thomson Cycle

Joule-Thomson (JT) cryogenic refrigerators rely on expansion through a throttling valve yet do not necessarily operate in the vapor dome. These refrigeration devices are used in low cost applications or in situations where reliability is of paramount importance such as tactical cryocoolers for infrared detectors, refrigeration for electronics, cryotherapy probes, and cryocoolers for space-borne detectors. Figure 7 illustrates schematically the Joule-Thomson cryogenic refrigeration cycle and Figure 8 illustrates the cycle qualitatively on a T-s diagram. Notice that the JT cycle relies on the fact that a temperature drop is produced during the isenthalpic expansion through the valve (from state 2 to state 3). This allows a small warming of the refrigerant as it accepts the refrigeration load (from state 3 to state 4) before entering the recuperative heat exchanger. In order to be viable, the JT cycle must operate in a region where the Joule-Thomson coefficient, the partial derivative of temperature with respect to pressure at constant enthalpy, is positive. This limitation explains why typical JT systems operate with very high pressure ratio and require large, high effectiveness recuperative heat exchangers.

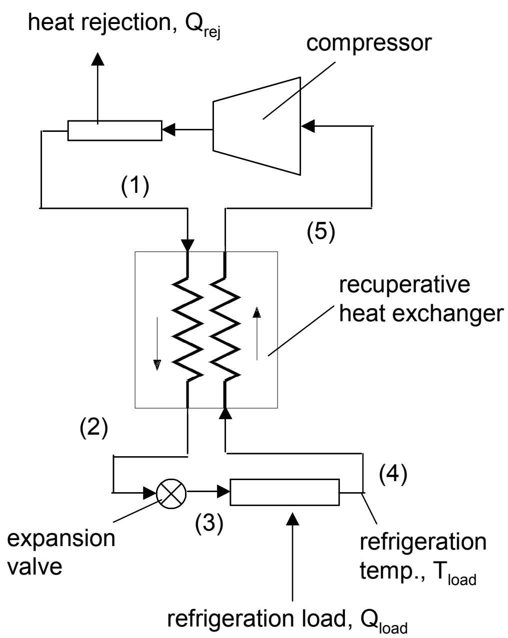

Figure 7. Joule-Thomson Refrigeration Cycle

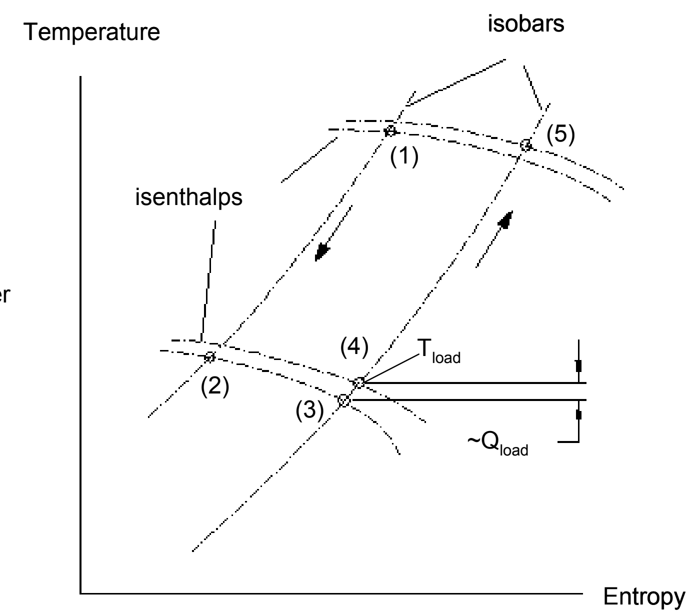

Figure 8. Joule-Thomson Refrigeration Cycle on T-s Diagram

The vortex tube produces a large temperature drop in the cold exit fluid regardless of the Joule-Thomson coefficient. For example, the isenthalpic temperature drop associated with expanding room temperature air from 3 atm to 1 atm is only 0.2 °C yet Figure 3 shows that a temperature drop of 27°C can be achieved at these conditions using a vortex tube, an increase of two orders of magnitude. Figure 9 illustrates the simplest possibility for implementing a cryogenic refrigeration cycle using a vortex tube rather than the expansion valve. The fluid leaving the cold exit is used to accept the refrigeration load and the fluid leaving the hot exit bypasses the recuperative heat exchanger. Figure 10 illustrates the temperature entropy diagram for this vortex tube cycle; the state points from the original JT cycle are retained for illustration. The temperature entering the recuperative heat exchanger, state (1), remains the same but the temperature of the leaving gas, state (2), is increased as a result of unbalancing the heat exchanger. The temperature drop associated with the fluid passing through the cold side of the vortex tube is much larger than it would be had it undergone an isenthalpic expansion (the temperature of state (3) is reduced), but some part of the flow has been sacrificed to accomplish this effect. The cold fluid is heated to the same refrigeration temperature, state (4), prior to entering the recuperative heat exchanger.

A model of the refrigeration cycle shown in Figure 9 has been developed using the vortex tube model already described. For a given mass flow rate ($\dot{m}$) and pressure ratio (PR), the refrigeration load (Q_{load}) can be predicted as a function of refrigeration temperature (T_{load}) given the characteristics of the heat exchanger (a total conductance UA), the vortex tube, and a cold gas mass fraction (y). Figure 11 illustrates the refrigeration load as a function of the cold gas mass fraction using air as the working fluid for one set of operating conditions (T_{rej} = 300 K, PR = 5, $\dot{m}$ = 10 g/s, UA = 10 W/K, and T_{load} = 250 K). When the cold gas mass fraction is equal to unity, the cycle is equivalent to a JT cycle – all of the gas passes through the vortex tube, undergoes an isenthalpic process, passes through the refrigeration load, and returns through the recuperative heat exchanger. Notice that under these operating conditions a JT cycle is not viable, the refrigeration load is negative because the pressure ratio is low and the heat exchanger small. As the cold gas mass fraction is reduced the refrigeration power increases and reaches a maximum value of 94 W at an optimal cold gas mass fraction near 0.5. As the cold gas mass fraction is reduced, three interrelated effects occur; there are two negative effects – unbalancing the heat exchanger and reducing the mass flow rate through the refrigeration load that are countered by a large positive effect - increasing the temperature drop from state (2) to state (3). The optimal value of cold gas mass fraction balances these effects and is therefore somewhat larger than the cold gas mass fraction that maximizes the temperature drop through the vortex tube, near 0.25 as shown in Figures 3 and 4.

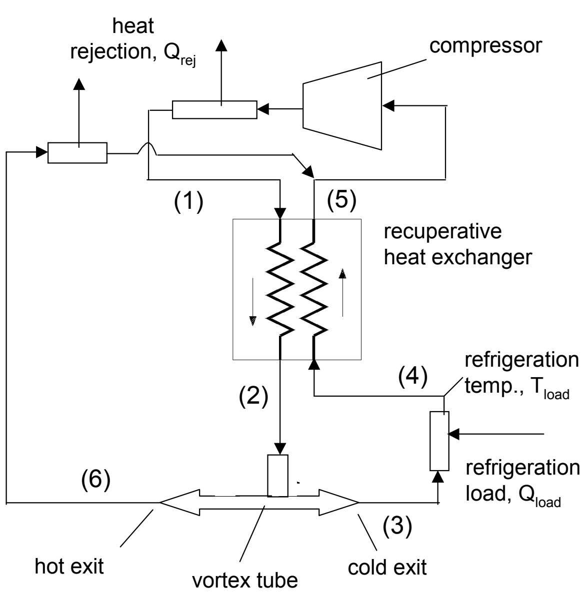

Figure 9. Vortex-Tube Refrigeration Cycle

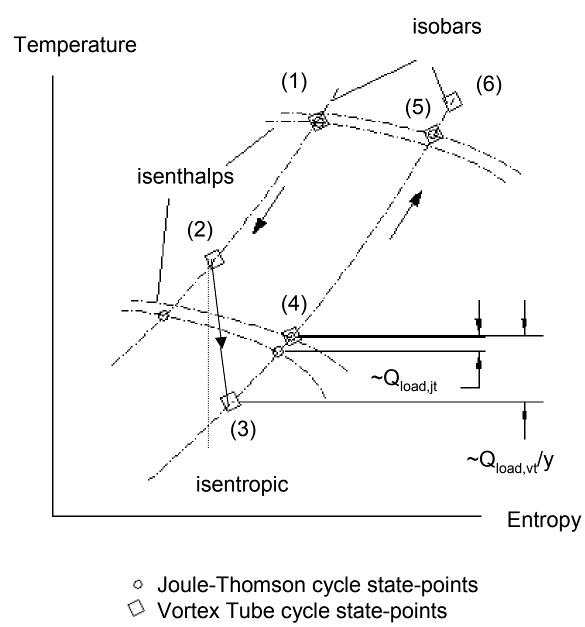

Figure 10. Vortex-Tube Refrigeration Cycle on T-s Diagram

Figure 12 illustrates the refrigeration load for the vortex tube cycle using the optimal cold gas mass ratio as a function of the load temperature and at several pressure ratios for one set of operating conditions (air, T_{rej} = 300 K, $\dot{m}$ = 10 g/s, and UA = 10 W/K). Also shown in Figure 12 is the performance of a JT cycle at identical operating conditions. Notice that the vortex tube cycle significantly outperforms the JT cycle; this is particularly true as the pressure ratio or the heat exchanger size is reduced.

This investigation clearly illustrates that the addition of a vortex tube has the potential to significantly increase the performance of a single-stage JT system and motivates further study into these devices and their use in cryogenic refrigeration systems. Over the last decade, large increases in the efficiency of JT systems have been made possible through the use of mixed gas refrigerants (Little, 1998) and this technology can be augmented by the use of phase separators as originally proposed by Kleemenko (1959). There is an obvious parallel between these phase separators and the natural separation mechanism that occurs within the vortex tube due to centrifugal effects (Marshall, 1977). There exists then the possibility of performing augmented expansion simultaneously with some phase separation within a vortex tube in a mixed gas cycle. Complex, multi-stage cycles can be considered with the potential to significantly out-perform the currently available JT technology without adding significantly to cost or detracting from reliability.

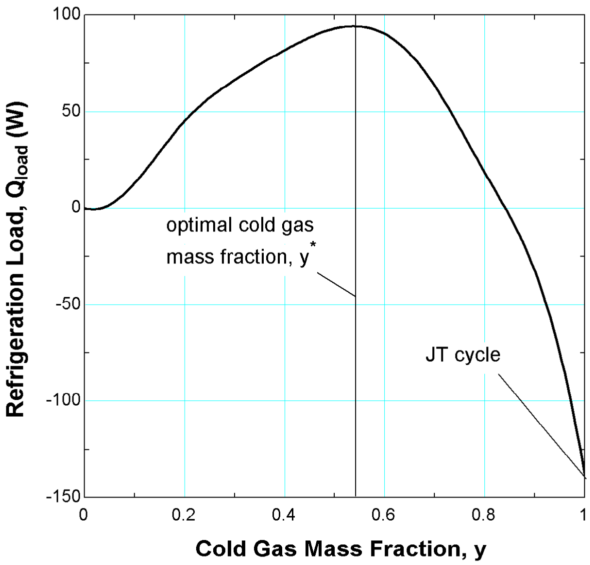

Figure 11. Refrigeration Load as a Function of Cold Gas Mass Fraction (air, T_{rej} = 300 K, T_{load} = 250 K, PR = 5.0, m = 10 g/s, UA = 10 W/K)

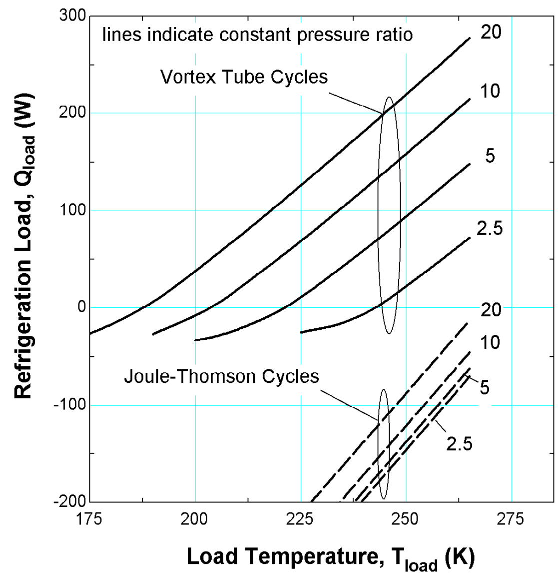

Figure 12. Refrigeration Load as a Function of Load Temperature for Various Pressure Ratios (air, T_{rej} = 300 K, m = 10 g/s, UA = 10 W/K)

CONCLUSIONS

A simple model of the vortex tube is described that captures the physics related to one possible operating mechanism. The model is shown to faithfully reproduce a limited set of data if two empirical parameters are adjusted. The semi-empirical model is subsequently used to evaluate the potential performance benefit associated with replacing the throttling valve in a refrigeration system with an appropriately optimized vortex tube. The key conclusions of this study are that:

- the performance of a conventional vapor compression refrigeration cycle cannot be augmented through the application of a vortex tube because no temperature separation can occur beneath the vapor dome,
- the performance of a vapor compression cycle operating in the near super-critical region, such as a carbon dioxide refrigeration cycle, is negligibly increased by the application of a vortex tube,
- the performance of a Joule-Thomson refrigeration cycle operating with a pure substance can be dramatically improved by application of a vortex tube, and
- the vortex tube's ability to separate 2-phase gas mixtures based on density may enable high performance, multi-stage cryogenic refrigeration systems.

In summary, the vortex tube has the potential to improve the performance of cryogenic refrigeration systems based on the Joule-Thomson cycles and its derivatives. A more intimate knowledge of the working principles of the device will allow a more accurate assessment of its potential in this regard and may allow vortex tubes to be designed specifically for the operating conditions required by these cycles. Experimental measurements of the vortex tube's capacity to provide temperature and constituent separation when operated at cryogenic conditions and with gas mixtures are required.

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