1986

A Counterexample to Perception of Structure from Motions

Chia-Hoang Lee

Report Number:

86-610

A COUNTEREXAMPLE TO PERCEPTION OF STRUCTURE FROM MOTION

Chia-Hoang Lee

CSD-TR-610
June 1986

A Counterexample to Perception of Structure from Motion

Chia-Hoang Lee

Department of Computer Sciences

Purdue University

West Lafayette, IN 47907

ABSTRACT

The paper of "Perception of Structure from Motion" [1][2] studied the lower bound issues of Structure from Motion. The key result is that the authors showed that two orthographic projections of four noncoplanar points admit only four interpretations of structure. This forms the basis for an algorithm (see abstract [1][2]). Unfortunately, the result is flawed. In this note, we present a counterexample which has at least five solutions.

August 14, 1986

1. Introduction

Recently, a paper entitled "Perception of Structure from Motion" [1][2] discusses lower bounds in relation to the structure from motion problem. This problem was first treated by S. Ullman [3] where three views of four noncoplanar points can uniquely determine the structure (relative depth) of these four points. In [1][2], the authors go one step further to investigate the lower bounds issue. The following are two quoted paragraphs.

> We prove that two orthographic projections of four noncoplanar points admit only four interpretations (up to a reflection) of structure. This forms the basis for an algorithm to recover structure from motion ...[1]

> **Theorem 2:** Two orthographic projections of four rigidly linked noncoplanar points are compatible with at most four interpretations[2]

Here, we would like to point out that the result (unfortunately) is wrong. In the following, a counterexample with five solutions (the reflection is not counted) is presented. Other solutions in fact could be given, but five solutions are sufficient to invalidate their result.

2. Example

The tilt and the slant of rotational axis are chosen to be 30 degrees and the rotational angle is also 30 degrees. The coordinates of points before and after transformation are given below. The left hand side represents those before motion and the right hand side represents those after motion. Note that the translation is adjusted to be zero and one of the points is chosen as reference and fixed point.

$$O = (0.0\ \ 0.0\ \ 0.0),\quad O = (0.00\ \ 0.00\ \ 0.00)$$

$$B_1 = (4.0\ \ 2.0\ \ 3.0),\quad A_1 = (3.253\ \ 2.976\ \ 3.091).$$

$$B_2 = (2.0\ \ 3.0\ \ 5.0),\quad A_2 = (1.402\ \ 2.580\ \ 5.419)$$

$$B_3 = (6.0\ \ 5.0\ \ 3.0),\quad A_3 = (3.780\ \ 6.494\ \ 3.678)$$

The rotational matrix can be computed according to [4] as follows:

[1]see Abstract of [1] and first section of [2]

[2]see [1][2], section 4

$$\begin{bmatrix} 0.8911 & -0.4185 & 0.1752 \\ 0.4475 & 0.8743 & -0.1875 \\ -0.0747 & 0.2455 & 0.9665 \end{bmatrix}$$

The following are four solutions where column vectors of the matrix on the right hand side represent space coordinates in the first scene; and column vectors of the matrix on the left hand side represent space coordinates in the second scene due to some motion.

$$\begin{bmatrix} 3.253 & 1.402 & 3.780 \\ 2.976 & 2.580 & 6.494 \\ 0.748 & 2.351 & 3.177 \end{bmatrix} = \boldsymbol{R} \begin{bmatrix} 4.0 & 2.0 & 6.0 \\ 2.0 & 3.0 & 5.0 \\ 0.0 & 1.072 & 2.357 \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} 3.253 & 1.402 & 3.780 \\ 2.976 & 2.580 & 6.494 \\ 1.248 & 5.365 & 8.471 \end{bmatrix} = \boldsymbol{R} \begin{bmatrix} 4.0 & 2.0 & 6.0 \\ 2.0 & 3.0 & 5.0 \\ -1.0 & -4.941 & -8.199 \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} 3.253 & 1.402 & 3.780 \\ 2.976 & 2.580 & 6.494 \\ 5.055 & 8.732 & 5.735 \end{bmatrix} = \boldsymbol{R} \begin{bmatrix} 4.0 & 2.0 & 6.0 \\ 2.0 & 3.0 & 5.0 \\ -5.0 & -8.478 & -5.32 \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} 3.253 & 1.402 & 3.780 \\ 2.976 & 2.580 & 6.494 \\ 0.8405 & 2.09 & 2.356 \end{bmatrix} = \boldsymbol{R} \begin{bmatrix} 4.0 & 2.0 & 6.0 \\ 2.0 & 3.0 & 5.0 \\ 0.383 & 0.0 & -1.020 \end{bmatrix} \quad (4)$$

The above four solutions and the original one which we started this example already make five solutions. To check these solutions, the readers are advised to examine the invariant of the length of each vector, and the invariant of the inner product of each two vectors. Write $[A_1\ A_2\ A_3]$ as A and $[B_1\ B_2\ B_3]$ as B. If the lengths and the inner products are invariant, then one has $A^t A = B_t B$. This gives $(A\ B^{-1})^t\ (A\ B^{-1}) = I$ where I is the identity matrix. [5] ensures that there exist a rotation $\boldsymbol{R}$ such that $A = \boldsymbol{R}\ B$.

3. Conclusion

In fact, the number of interpretation compatible with two views of four noncoplanar points depends on the each case. Most of the time, there will be an infinite number of solutions. The theory for this requires a lengthy discussion which are currently in preparation [6].

References

1. Aloimonos J. and Amit B. (1985), *Perception of Structure from Motion: Lower Bounds Issues*, Technical Report 158, Department of Computer Science, The

University of Rochester.

2. Aloimonos J. and C.M. Brown (1986), *Perception of Structure from Motion: I: Optical Flow vs.* Discrete Displacements II: Lower Bound Results", Conference of Computer Vision and Pattern Recognition, IEEE Computer Society, June 25-27, Miami, Florida.

3. Ullman S. (1979), *The Interpretation of Visual Motion*, MIT Press, Cambridge, Masschusetts.

4. Rogers D.F. and J.A. Adams (1976), *Mathematical Elements for Computer Graphics*, McGraw-Hill, New York.

5. Bellman R. (1960), *Introduction to Matrix Analysis*, McGraw-Hill, New York. p. 25.

6. Lee C.H. (1986), *Matching and Motion of Four Points in Two Views*, TR-611 Department of Computer Science, Purdue University.