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ANALYTIC MODELS OF CYCLIC SERVICE SYSTEMS AND THEIR * APPLICATION TO TOKEN-PASSING LOCAL NETWORKS

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Abstract: Using the framework of cyclic-service systems with a single server, two different token-passing models are investigated. The first model is approximate, obtaining the free-token's cycle-time distribution on an asymmetric system with infinite capacity buffers and single-token operation. The second model is exact, yielding the cycle-time distribution of the free-token on an asymmetric system with unit-capacity buffers, and single-token operation. The latter result is verified using known results for symmetric, unit-capacity buffer systems. In order to demonstrate the positive effects of buffering, a small variation of the unit-capacity buffering scheme is introduced. Computational results include performance measures such as throughput, utilisation, loss probabilities, mean cycle-times, cycle-time distributions and a comparison of two buffering schemes.

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1. INTRODUCTION

Token-passing [Stal87] is currently a standard local network access method, used in ring networks and bus networks. The IEEE 802 committee standards relating to token-ring LANs and and token-bus LANs can be found in [IEEE85]. The results in this paper include the derivation of cycle-time distribution under station independence, and for small systems (i.e., $N < 10$), exact cycle-time distribution and other performance measures. The independence assumption that we refer to was used by Kuehn [Kueh79] in modelling cyclic-service systems. Our contribution is the derivation of the explicit cycle-time distribution for symmetric and asymmetric systems under station independence. More importantly, we also obtain the *exact* cycle-time distribution for an asymmetric system with unit-buffers, extending a symmetric model first introduced by Mack, Murphy and Webb [MaMW57]. It appears to be the first time that such a model has been treated, and we demonstrate that for an N station system, an exact analysis involves an $O(2^N)$ algorithm. Since token-bus networks operate in conceptually the same manner as token-rings [BuxW85], the results can also be used to analyze bus systems, as long as system parameters are given a proper interpretation.

A token-ring consists of a set of distributed stations connected in series by a transmission medium. Frames are transmitted bit by bit (i.e., sequentially) from one active station to the next. Each station regenerates each passing bit, this involving a repeater and typically a one-bit delay. A station may act as an interface between several computing devices (e.g., workstations) and the ring, so that communication between devices distributed over the ring is possible. A station gains sole right to transmit its information onto the ring when it detects a free-token (i.e., a unique sequence of bits) passing on the medium. If a station has information to transfer to another station, it must capture the passing free-token, convert it into a *busy* token (i.e., change it into a start-of-frame sequence) and then append appropriate control, status, information, and address fields, frame check sequence, and end-of-frame sequence. Any station detecting its address in the address field of a passing frame copies the information as it passes.

In this paper, we assume *single-token* operation [AnSc82], where a station does not issue a new free-token until its own busy-token has returned and is erased from the ring. When frame lengths are greater than the bit-length of the ring, single-token operation and multiple-token operation [AnSc82] are essentially the same. While the single-token strategy's efficiency is comparable to that of the multiple-token strategy, its complexity is much less, particularly in the design of robust priority schemes and error recovery [AnSc82, Stal87].

A queueing analogue of the frame arrival and service process is shown in Fig. 1. At a given station, frames arrive independently as a Poisson process and wait in a queue at the station. The free-token cycles the ring repeatedly, visiting each station and giving each station a chance to use the ring for transmission. The free-token behaves as a server who walks from one station to another in a ring of queues. When passing by a station, the free-token is transmitted bit-by-bit from one station to its downstream active neighbour. The token-passing time or *walk-time* depends on the data rate of the ring, the distance between the particular pair of stations involved, and the kind of token used. With a view towards a general model, we assume (as in [Ferg86]) that token-delaying overhead also depends on whether the station passing the free-token did so after first transmitting a frame. If the station does not transmit a frame before a token-pass, a small time called a station *switching-time* is also involved. This extension allows for rings with station dependent overheads [Ferg86].

Define the *cycle-time* C of the free-token to be the time between two consecutive visits (called *scan-instants*) of the free-token at a given station. Using nonnegative random variables for frame-transmission times and token-passing delays, C will also be a random variable, with a limiting distribution $F_C(\cdot)$ that depends on system parameters. In [HaOh72] and [Kueh79], an assumption of station independence was used to obtain the Laplace-Stieltjes transform of $F_C(\cdot)$. In this paper, we derive the distribution explicitly (both approximately and exactly), and show

how it is computed as a function of system parameters. Throughout the paper we assume that each station transmits *at most one* frame each time it captures the free-token. In the multiqueue context, an exact solution for the steady-state queueing distributions of a finite/infinite buffer system using this service-discipline (called *one-at-a-time*) is, to the best knowledge of the authors, still unknown (see [FeAm85]). In this paper we provide an exact solution for a special case of this system. The results include:

(1). Approximate cycle-time distributions (using station independence) on asymmetric (symmetric) systems with infinite capacity buffers and single-token operation.

(2). Exact cycle-time distributions on asymmetric (symmetric) systems with unit capacity buffers and single-token operation.

(3). Two different buffering schemes for frames arriving at stations (for transmission).

(4). Measures of system performance such as mean cycle-time, distribution of cycle-time, channel utilisation, throughput, empty-buffer probabilities at scan instants, loss probabilities of arriving frames, and a comparison of the two buffering schemes.

The results in (2) an (4) solve two related open problems. In (2), the open problem is obtaining the cycle-time distribution for the specified system, and in (4) the open problem is obtaining the probability that the free-token finds a given station's buffer empty at its *scan* instant (which is the instant at which the free-token arrives at the station). Takagi [Taka85] has solved the latter problem for a *symmetric* system. We test our method using Takagi's result.

In section 2 we present a model and a brief review of of past work related specifically to this problem. In section 3 a derivation of the free-token's approximate cycle-time distribution on asymmetric and symmetric systems is presented under an assumption of station independence. In section 4 we focus on an asymmetric system with unit-capacity buffers to derive the exact-cycle time distribution. This result is potentially applicable to systems of any size, except that the method used involves a Markov chain on a state space of size 2^N. While the computation of the transition probabilities is direct, the size of the transition matrix is prohibitive for $N \geq 10$. Even so, the the result gives exact values for small systems and can be used to validate less restrictive approximations. In section 4, we apply our results to a symmetric system and demonstrate favourable performance when compared to the corresponding formula in [Taka85]. Note that the formula in [Taka85] works only for $N \leq 3$. Section 4 also includes an investigation of two different buffering schemes, typical measures of performance and numerical results. Section 5 contains a brief conclusion.

2. THE TOKEN-PASSING MODEL

The token-passing protocol is modelled by a system of N independent station buffers, chained together to form a ring by sections of varying cable lengths. Figure 2 demonstrates the model parameters for station N. Denote the buffer capacity of station j by K_j, where $K_j = 1$ for a unit-capacity buffer system, and $K_j = \infty$ for an infinite capacity buffer system, $j \in S$, where $S = \{1, 2, \ldots, N\}$ is the set of N stations on the system. Frames arrive at station j according to a Poisson process with rate λ_j, $j \in S$. The token-passing time (server's walk time) between stations j and $(j \bmod N) + 1$ is given by a random variable $Y_{(j \bmod N)+1}$, with $W_j(t) = Pr(Y_j \leq t)$, $j \in S$. If the cycling free-token finds station j's buffer nonempty at its scan instant, then station j removes the free-token from the ring and transmits a frame, taking a random time X_j, with $B_j(t) = Pr(X_j \leq t)$. If not, the free-token passes the station j interface bit-by-bit. Usually, the time for this is taken to be negligible in a model. For generality, we allow this to be a random time V_j with very small mean, where $S_j(t) = Pr(V_j \leq t)$. One motivation for this is to allow for station dependent overheads (see [Ferg86]). We assume that all distributions have finite first and second moments.

The token-ring problem can be framed in terms of the *machine repairman* problem [MaMW57], a model (with $K_j = 1$) used by Kaye [Kaye72] in analyzing data transmission loops.

Kaye's contribution is an algorithmic solution for the waiting time distribution of a frame on a symmetric system with constant frame lengths and walk times. Takagi [Taka85] extends Kaye's analysis, still on symmetric systems, to arbitrary frame lengths and walk times, and obtains mean values for response times, mean cycle-time, etc. Fuhrman [Fuhr85] presents some very useful results for a symmetric system ($K_j = \infty$), yielding mean waiting times for limited, gated, and exhaustive service. For limited service where up to k frames can be transmitted by a station on each free-token capture, these results give bounds on mean waiting times. Ferguson and Aminetzah [FeAm85] obtain exact mean waiting times for an asymmetric system utilising gated service. There has been a considerable amount of work on exhaustive service systems, the most notable of these being work by Eisenberg [Eise72], Konheim and Meister [KoMe74], and Swartz [Swar80].

A somewhat different approach was taken by Hashida and Ohara [HaOh72] and Kuehn [Kueh79], in using an assumption of station independence to examine the considerably more difficult, asymmetric, one-at-a-time service systems (with $K_j = \infty$). These authors concentrate on obtaining approximate solutions for mean queue lengths and waiting times. To the best knowledge of the authors, except for results by Kaye, and the two references just quoted, explicit expressions for distributions have not been given much attention. In fact, [HaOh72] and [Kueh79] obtain only the Laplace-Stieltjes transform for the cycle-time distribution under station independence and do not investigate the distributional form any further. Distributions of important random quantities can often yield very useful information about a system. For example, using a distribution for queue length, a sound estimate of required buffer-size with specified probability for loss can be made. On the other hand, distributions can be difficult or computationally expensive to obtain, or both.

3. APPROXIMATE CYCLE-TIME DISTRIBUTIONS ($K_j = \infty$)

In this section, an assumption of station independence ([HaOh72, Kueh79]) and exponential distributions are used to obtain approximate cycle-time distributions for asymmetric and symmetric systems with infinite waiting room (i.e., $K_j = \infty$) for arriving frames at each station. In section 3.1 is obtained the probability that the free-token finds station j's buffer empty at any station-j scan instant, for each $j \in S$. In section 3.2 these empty buffer probabilities are used to obtain the approximate cycle-time distribution of the free-token. The approximation is useful since it is known to work well under high and low station loads [Kueh79].

A Cycle-Time Property

Assume that all queues possess stationary queueing distributions. For any fixed station k in S, let $C(k)$ denote the random time between consecutive free-token scans at station k. Since the free-token's cycle-time on the network is independent of station index k (see [Kueh79]), we have $C(k) = C$ for all $k \in S$. Henceforth we use C to denote the cycle-time random variable for any station in S.

■

Assumption of Station Independence

Let L_k be a Bernoulli random variable defined on $\{0, 1\}$, with $L_k = 0$ if station k has an empty buffer, and $L_k = 1$ otherwise, at any scan instant at station k. The assumption that random variables in the set $\{L_k \mid k \in S\}$ are mutually independent is called a *station independence* assumption. This assumption was used by Hashida and Ohara [HaOh72], and later by [Kueh79] in analyzing cyclic-service systems. In [Kueh79], each queue is modelled as an M/GI/1 queue. The service-time distribution a customer is taken to be the free-token's cycle-time distribution. Using the mean and variance of the approximate cycle-time distribution (obtained as a Laplace Stieltjes transform) Kuehn demonstrated that station independence performs extremely well (compared to simulated results) under high and low loads.

■

Under the assumptions of stationary queues and station independence, the cycle-time random variable can be written as $C = \sum_{j=1}^{N} \{X_j' + Y_j\}$, where X_j' is distributed as $Pr[L_j = 1] B_j(\cdot) + Pr[L_j = 0] S_j(\cdot)$ (i.e., a mixture of frame transmission time and station switching time distributions), and Y_j is distributed as $S_j(\cdot)$ (i.e., walk time distribution). That is, station independence yields limiting cycle-times that are i.i.d sums of random variables. Our interest is in the limiting density $f_C(\cdot)$ of the random variable C. In order to obtain $f_C(\cdot)$ we must first obtain the probability $Pr[L_j = 0]$ that station j's buffer is empty at its scan instants.

3.1 Empty Buffer Probabilities

Let $r_j(K_j)$ denote the probability that station j's buffer (which is of size K_j) is found empty on any scan instant, $j \in$ S. Though queue j is not an M/GI/1 queue (since service-times, which are modelled via cycle-times, are *not* independent), the probability $r_j(\infty) = Pr[L_j = 0]$ can be computed as if it were an M/GI/1 queue. A lucid explanation of this in the multiqueue context can be found in [Kueh79]. Using λ_j to denote the Poisson arrival rate at queue j, and $E(C)$ for the mean cycle-time, it follows that

$$r_j(\infty) = 1 - \lambda_j E(C) \tag{1}$$

for all $j \in$ S. Equation (1) is also true for GI/GI/1 systems (see [Kueh79]) and can be obtained via the reasoning that for queueing systems in equilibrium, each queue's arrival rate must equal its departure rate.

The expectation $E(C)$ in (1) can be obtained by investigating the flow balance of the system. At steady-state, the mean number of frames arriving at any station per cycle is equal to the mean number of frames served at that station per cycle. The mean number of frames served at station j during a cycle is identical to the probability that the free-token encounters at least one frame at queue j at this station's scan instants [Kueh79]. From this we obtain

$$E(C) = \sum_{j \in S} \{E(Y_j) + (\lambda_j E(C))E(X_j) + (1 - \lambda_j E(C))E(V_j)\} \qquad (2)$$

and consequently the mean cycle time as

$$E(C) = \frac{\sum_{j \in S} [E(Y_j) + E(V_j)]}{(1 - \sum_{i \in S} \lambda_i E(X_i) + \sum_{i \in S} \lambda_i E(V_i))} \qquad (3)$$

3.2 Cycle-Time Distributions

In this section, an expression for f_C is first developed for an asymmetric model, i.e., one with all distributions having different parameters. The arrival processes are assumed to be Poisson (λ_j), and the B_j's, S_j's, and U_j's are assumed to be exponential, with means $1/\mu_{j0}$, $1/\mu_{j1}$, and $1/\alpha_j$, respectively. The random variable C can be decomposed in terms of its various sojourn times as

$$C = \sum_{j \in S} X_j' + \sum_{j \in S} Y_j = X^* + Y^* \qquad (4)$$

where the starred terms denote the respective sums. The random variable Y_j has density $\alpha_j e^{-\alpha_j t}$, and the random variable X_j' has a density that is a mixture of the densities of X_j and V_j, given by $p_j \mu_{j0} e^{-\mu_{j0} t} + q_j \mu_{j1} e^{-\mu_{j1} t}$, with $p_j = 1 - q_j$, $j \in S$. Recall that Y_j is the time spent by the free-token to move from station j's predecessor in the ring to station j. The mixed variable X_j' is a consequence of station independence, explained thus. If station j's buffer is not empty at its scan instants (the probability of this is p_j) then a transmission of random length X_j follows, *before* the free-token can leave station j. If station j's buffer is found empty at its scan instants (the probability of this is $q_j = 1 - p_j$) then the free-token takes a small random time V_j to switch past the station. So, the time spent by the free-token at station j is a random variable X_j', distributed as a mixture of these two distinct event times. Thus, the random cycle-time of the free-token is distributed as the sum of N hyperexponentials (i.e., times spent at the N stations) and a generalised Erlangian random variable (i.e., the sum of N different walk times).

Asymmetric Systems

Let the Laplace-Stieltjes transforms of the densities of the random variables X^* and Y^* be given by $L[f_{X^*}]$ and $L[f_{Y^*}]$ respectively. For all $j \in S$, we have

$$L[f_{X_j'}] = \frac{a_{j0}}{(s + \mu_{j0})} + \frac{a_{j1}}{(s + \mu_{j1})} \tag{5}$$

with $a_{j0} = p_j \, \mu_{j0}$, $a_{j1} = q_j \, \mu_{j1}$, and

$$L[f_{Y_j}] = \frac{\alpha_j}{(s + \alpha_j)} \tag{6}$$

The transform of f_C is obtained as

$$L[f_C] = L[f_{X^*}] \cdot L[f_{Y^*}] = \prod_{j \in S} L[f_{X_j'}] \cdot L[f_{Y_j}] \tag{7}$$

Let Θ be the set of all N digit binary numbers representing the non-negative integers in the range $[0, 2^N - 1]$. An element $\mathrm{k} \in \Theta$ is an N-bit binary vector of the form $[k_1, k_2, \cdots, k_N]$. In terms of our new notation,

$$L[f_{X^*}] = \sum_{\mathrm{k} \in \Theta} \prod_{i \in S} \frac{a_{i\,k_i}}{(s + \mu_{i\,k_i})} \tag{8}$$

and

$$L[f_{Y^*}] = \sum_{j \in S} \frac{\beta_j}{(s + \alpha_j)} \tag{9}$$

where $\beta_j = \left(\prod_{\substack{i \in S \\ i \neq j}} \frac{\alpha_i}{(\alpha_i - \alpha_j)} \right) \alpha_j$.

$L[f_C]$ can now be obtained from Eqs. (7), (8) and (9). Note that $L[f_C]$ contains $2^N \cdot N$ terms, where each term has the form

$$D_{\mathrm{k}j}^*(s) = \prod_{i \in S} \frac{a_{i\,k_i} \beta_j}{(s + \mu_{i\,k_i})(s + \alpha_j)} \quad , \text{ for } j \in S \text{ and } \mathrm{k} \in \Theta. \tag{10}$$

Using partial fraction expansion, the resulting expression consists of terms $(s + \mu)$ and $(s + a)$, that are convergent for $\mathrm{Re}(s) > -\mu$ and $\mathrm{Re}(s) > -\alpha$, respectively. Upon inverting the transform in Eq. (10), we obtain

$$D_{\mathbf{k}j}(t) = \prod_{i \in S} a_{i\,k_i} \beta_j \left\{ \frac{e^{-\alpha_j t}}{\prod_{m \in S} (\mu_{m\,k_m} - \alpha_j)} + \sum_{n \in S} \frac{e^{-\mu_{n\,k_n} t}}{\prod_{\substack{m \in S \\ m \neq n}} (\mu_{m\,k_m} - \mu_{n\,k_n})(\alpha_j - \mu_{n\,k_n})} \right\}. \quad (11)$$

Finally, the cycle-time density is given as

$$f_C(c) = \sum_{j \in S} \sum_{\mathbf{k} \in \Theta} D_{\mathbf{k}j}(c). \quad (12)$$

The complexity of computing the asymmetric density can be obtained as follows. Let the time taken for each addition be t_s, and the time taken for each multiplication be t_p. Consider the expression for $D_{\mathbf{k}j}(c)$ given in Eq. (12). The second (summation) term in the sum requires a time of $N^2 t_p + (N-1)t_s$, and the first term in the sum requires a time of Nt_p. The sum itself requires a time of t_s, and the product term involving the $a_{i\,k_i}$'s and the β_j's requires an effort of $(N+1)t_p$. The time required for any $D_{\mathbf{k}j}$ is $N^2[t_p + 2N + 1] + Nt_s$. For a given value of $c \in R^+$, the effort required to compute $f_C(c)$ is $2^N N\{N^2[t_p + 2N + 1] + Nt_s\} + 2^N(N-1)t_s$. This requires an algorithm of exponential complexity and is inefficient for large N. In fact, due to the presence of the summation over the set Θ in Eq.(12), any algorithm for $f_C(\cdot)$ will always be an exponential algorithm. A comparison of analytic (continuous) and simulated (dotted) density for two, five and eight station systems is shown in Fig. 3. This demonstrates that in the infinite buffer case, the station independence assumption works well at very high loads. This can also be shown true for very low loads. Kuehn demonstrates this by computing the approximate cycle-time variance [Kueh79] and comparing it to simulated variance.

Symmetric Systems

If $\alpha_j = \alpha$, $\mu_{j0} = \mu_0$, $\mu_{j1} = \mu_1$, and $\lambda_j = \lambda$, for all $j \in S$, the required computation is reduced. In this case, $a_0 = p\,\mu_0$ and $a_1 = q\,\mu_1$. The transforms for f_{X^*} and f_{Y^*} become

$$L[f_{X^*}] = \sum_{j=0}^{N} \binom{N}{j} \left[\frac{a_0}{s+\mu_0} \right]^{N-j} \left[\frac{a_1}{s+\mu_1} \right]^{j} \tag{13}$$

$$L[f_{Y^*}] = \left[\frac{\alpha}{s+\alpha} \right]^{N} \tag{14}$$

The transform $L[f_C]$ can be obtained from equations (13) and (14). A direct inversion by partial fraction expansion will involve repeated differentiation in the computation of the coefficients of the fractions. Each term in the inverse will require the computation of $2N$ coefficients, where the N^{th} coefficient involves the N^{th} derivative of an expression of the form $(s+\zeta)^{-m} \cdot (s+\kappa)^{-n}$, where $m \le N$, and $n \le N$, this requiring an overall effort of $O(2^N)$. An apparent simplification is to write the transform of the density of X^* as

$$L[f_{X^*}] = \sum_{j \in S} \frac{d_j}{(s+\mu_0)^j} + \sum_{j \in S} \frac{e_j}{(s+\mu_1)^j} \tag{15}$$

where

$$d_j = \frac{1}{(\mu_0-\mu_1)^{N-j}} \sum_{n=1}^{N-j} \binom{N}{n} \binom{N-j-1}{n-1} (-1)^n (a_0)^{N-n} (a_1)^n \tag{16}$$

$$e_j = \frac{1}{(\mu_1-\mu_0)^{N-j}} \sum_{n=1}^{N-j} \binom{N}{n} \binom{N-j-1}{n-1} (-1)^n (a_1)^{N-n} (a_0)^n \tag{17}$$

for $j = 1, 2, 3, \cdots, N-1$, with $d_N = a_0^N$ and $e_N = a_1^N$.

The transform of the density of C can then be expressed as

$$L[f_C] = \sum_{j \in S} \frac{d_j\, \alpha^N}{(s+\mu_0)^j (s+\alpha)^N} + \sum_{j \in S} \frac{e_j\, \alpha^N}{(s+\mu_1)^j (s+\alpha)^N} \tag{18}$$

where the computation of the coefficients of the partial fraction expansion is standard [VaRa82]. An arbitrary term from the first summation in Eq. (18), say

$$D_j^*(s) = \frac{d_j\, \alpha^N}{(s+\mu_0)^j (s+\alpha)^N} \tag{19}$$

can be seen to invert to

$$D_j(t) = d_j\, \alpha^N \left\{ \sum_{k=0}^{j-1} \frac{\xi_k\, t^{j-k-1} e^{-\mu_0 t}}{\Gamma(j-k)} + \sum_{k=0}^{N-1} \frac{\zeta_k\, t^{N-k-1} e^{-\alpha t}}{\Gamma(N-k)} \right\} \tag{20}$$

and $E_j(t)$ can be obtained in the same manner, with coefficients ξ_k' and ζ_k', from the term in the second summation. Finally, the density for C in the symmetric case is given by

$$f_C(c) = \sum_{j \in S} D_j(c) \;+\; \sum_{j \in S} E_j(c). \tag{21}$$

The complexity of an algorithm using (21) can be determined by investigation of the function D_j. Consider the quotient immediately after the first summation in D_j. The partial fraction coefficient ξ_k requires an effort of $t_p(N + 3k - 1)$, the power of t requires $t_p(j - k - 2)$, the gamma function requires $t_p(j - k - 1)$, and obtaining the quotient with the final products requires $3t_p$. For a fixed value of k, the quotient term requires a time of $t_p(N + 2j + k - 1)$. Varying k from 0 to $(j - 1)$ to obtain the first summation requires an overall effort of $t_p[5j^2/2 + j(N - 3/2]$, for each $j \in S$. In like fashion, the second summation can be seen to require an overall effort of $t_p[7N^2/2 - 3N/2]$, for each $j \in S$. All products involved in the computation of a single D_j require a total effort of $t_p[5j^2 + j(N - 3/2) + 7N^2/2 - N/2 + 1]$. All sums involved in the same computation require a total effort of $t_s(N + j - 1)$. The total time required to compute the first summation in (21) is given by $t_p[5(N + 1)^2N^2/4 + N(N + 1)(N - 3/2)/2 + 7N^3/2 - N^2/2 + N] + t_s[N(N - 1)/2]$. By symmetry, E_j can be shown to require the same effort. Thus, the symmetric density requires an algorithm of polynomial complexity.

4. EXACT CYCLE-TIME DISTRIBUTIONS ($K_j = 1$)

We now restrict our attention to the case of unit-capacity buffers (i.e., through the rest of the paper $K_j = 1$, for all $j \in$ S). Our objective is to compute the exact cycle-time distribution of the free-token. This is done via an embedded Markov chain on a state space of size 2^N. So a disadvantage of our approach is that it is feasible only for small systems, i.e., $N \leq 9$. However, we stress that this is an exact result and can be used as a guide to approximations. This general technique has led to new insights and corresponding results for infinite and finite capacity buffer systems [ReNi86, ReSz87].

In a system with unit-capacity buffers, at most one frame is allowed to queue at each station at any given time. Ordinarily, an arriving frame may not enter a station's buffer if a frame is already queued there. A station's buffer is considered empty at the *end* of the station's transmission. Thus, frames arriving to find the buffer occupied (even while the occupant's transmission is in progress) are lost. Let us call this a *restricted-buffering* (RB) scheme. Next, suppose that this restriction is relaxed in the following manner. A frame arriving at a station to find a buffer occupied *and* the occupant's transmission in progress is allowed to enter the station's buffer, even while the previous occupant is leaving the buffer, much like a spooling mechanism. In systems that are large, or those that work with *long* frames, it may be of some interest to differentiate between these two possibilities. Let us call this a *relaxed-buffering* (XB) scheme. Both buffering schemes can be treated in a unified manner.

Let t_j^k denote the k^{th} scan-instant of station j, $j \in S$, $k \geq 1$. Assuming steady-state operation (i.e., we are examining the stationary versions of interesting stochastic processes), let time $t=0$ be the time at which the free-token makes an *exit* from station N to begin the walk to station 1. Let the term *cycle* denote the sequence of station visits occurring between consecutive scans at station N. The length of a cycle is a *cycle-time*, and the k^{th} cycle-time C_k is given by $C_k = C_N^k = | t_N^k - t_N^{k-1} |$, for $k > 1$. Thus, t_j^k is station j's scan-instant, occurring during the the k^{th} cycle, $j \in S$, $k \geq 1$.

Let Z_j^k be a random variable whose value is 1 if station j's buffer is found nonempty at time t_j^k, and $Z_j^k = 0$ otherwise, for each $j \in S$. Because each station's buffer is of unit capacity, it is clear that Z_j^{k+1} depends on

(i). the length $C_j^{k+1} = | t_j^{k+1} - t_j^k |$ of the time interval $(t_j^k, t_j^{k+1}]$ between the k^{th} and the $(k+1)^{th}$ scan of station j, and

(ii). the set of random variables $\{Z_j^k, Z_{(j \bmod N)+1}^k, \cdots, Z_N^k, Z_1^{k+1}, \cdots, Z_{j-1}^{k+1}\}$, since C_j^{k+1} is a function of this set of random variables, for each j, $j \in S$, and $k \geq 1$. We now state a theorem that we shall require for our computations. The proof can be found in the Appendix.

THEOREM

Let $Z(t) = \langle Z_1(t), \ldots, Z_N(t) \rangle$ be a continuous time stochastic process with components defined by $Z_j(t) = Z_j^k$, for $t \in (t_N^k, t_N^{k+1}]$. Also, let $Z_k = \langle Z_1^k, \cdots, Z_N^k \rangle$. Then,

(a) $\{Z(t) ; t > 0\}$ is a semi-Markov process, and

(b) $\{Z_n ; n > 0\}$ is a Markov chain,

each with state space given by the set of all N-bit binary vectors Θ_N.

∎

In order to obtain the free-token's cycle-time distribution, the probability transition matrix of the chain $\{Z_n\}$ must be obtained. We wish to compute the probability $Pr[Z_{k+1} = z' \mid Z_k = z]$ for an arbitrarily chosen k, $k > 1$, and all N-bit binary vectors $z = \langle z_1, \cdots, z_N \rangle$, $z' = \langle z_1', \cdots, z_N' \rangle$, with $z, z' \in \Theta_N$. Let $T_j = z_j X_j + (1 - z_j) V_j$ be the amount of time spent by the free-token at station j during the k^{th} cycle (i.e., this time is either the amount of time it takes station j to transmit its frame, or it is the amount of time it takes the free-token to switch past an empty station j buffer). Similarly, let $T_j' = z_j' X_j + (1 - z_j') V_j$ denote the time spent by the free-token at station j during the $(k+1)^{th}$ cycle. If a random variable appears primed (e.g., Y_j', or T_j') then the random variable corresponds to some random time in the $(k+1)^{th}$ cycle. If a random variable appears unprimed (e.g., Y_j, or T_j), then the random variable corresponds to some random time in the k^{th} cycle.

Relaxed-Buffering Scheme (XB)

Ordinarily, a frame which arrives to find a station's buffer occupied is lost by the system. If we relax this condition just a little so that a newly arriving frame is allowed to enter a station's buffer as long as the buffer is either empty *or* the frame currently occupying the buffer is currently undergoing transmission, a station will lose fewer frames.

Let $C_j^{k+1} = t_j^{k+1} - t_j^k$ denote the time between the k^{th} and the $(k+1)^{th}$ scans of station j, $j \in S$, $k > 1$. The random variable C_j^{k+1} can be explicitly written as

$$\begin{aligned} C_j^{k+1} &= t_j^{k+1} - t_j^k \\ &= \sum_{k=j}^{N} T_k + \sum_{k=j}^{N-1} Y_{(k \bmod N)+1} + Y_1' \\ &+ \sum_{k=1}^{j-1} T_j' + \sum_{k=1}^{j-1} Y'_{(k \bmod N)+1} \end{aligned} \tag{22}$$

for each station $j \in S$. Let $\xi_j(Z_j^{k+1} \mid Z_j^k, C_j^{k+1})$ denote the marginal probability that station j has Z_j^{k+1} frames (i.e., either 0 or 1) at the $(k+1)^{th}$ scan-instant, given that Z_j^k frames (i.e., either 0 or 1) were found here at the k^{th} scan-instant, *and* the time between these two scans is C_j^{k+1}. Then,

$$\xi_j(Z_j^{k+1} = z_j' \mid Z_j^k = z_j, C_j^{k+1} = c_j) = z_j'(1 - e^{-\lambda_j c_j}) + (1 - z_j')e^{-\lambda_j c_j} \tag{23}$$

for $z_j, z_j' \in \{0,1\}$ and each j, $j \in S$. Note that the probability does not explicitly involve z_j (i.e., see right-hand side of Eq.(23)). In fact, z_j affects the probability only via its prescence in the definition of C_j^{k+1} (i.e., T_j in Eq.(22)). Henceforth, we denote this probability by $\xi_j(Z_j^{k+1} \mid C_j^{k+1})$ with the understanding that z_j is involved through C_j^{k+1}, for each $j \in S$.

*Restricted-Buffering Scheme (*RB*)*

This is the conventional interpretation of a unit-capacity buffer. If a frame arrives at a station to find the buffer occupied (i.e., the frame occupying the buffer has not been transmitted over the medium), the arriving frame is lost by the system. Let the random variable D_j^{k+1} be defined as

$$\begin{aligned} D_j^{k+1} &= t_j^{k+1} - t_j^k \\ &= (1 - z_j)V_j + \sum_{k=j}^{N} T_{(k \bmod N)+1} + \sum_{k=j}^{N-1} Y_{(k \bmod N)+1} + Y_1' \\ &+ \sum_{k=1}^{j-1} T_j' + \sum_{k=1}^{j-1} Y'_{(k \bmod N)+1} \end{aligned} \tag{24}$$

for each station $j \in S$. Note that the only difference between Eqs.(22) and (24) is that the T_j in (22) is replaced by $(1-z_j)V_j$ in (24). If $z_j=0$, then D_j is the time between the k^{th} and the $(k+1)^{th}$ scan-instants at station j. If $z_j=1$, then D_j is the time between the k^{th} *exit* of the free-token from station j and the $(k+1)^{th}$ scan-instant at station j, $j \in S$. Let $\delta_j(Z_j^{k+1} \mid Z_j^k, C_j^{k+1})$ be the marginal probability that station j has Z_j^{k+1} frames (i.e., either 0 or 1) on the $(k+1)^{th}$ scan, given that Z_j^k frames (i.e., either 0 or 1) were found queued at the k^{th} scan, and the time between these two scans is D_j^{k+1}. Then,

$$\delta_j(Z_j^{k+1}=z_j' \mid Z_j^k=z_j, D_j^k=d_j) = z_j'(1-e^{-\lambda_j d_j}) + (1-z_j')e^{-\lambda_j d_j} \tag{25}$$

for $z_j, z_j' \in \{0,1\}$ and each j, $j \in S$. Note that just as in the XB situation, the probability on the right hand size involves z_j only via D_j^{k+1}. Henceforth, we write the expression on the left-hand side of (25) as $\delta_j(Z_j^{k+1} \mid D_j^{k+1})$, with the understanding that z_j is involved in the definition of D_j^{k+1} (see Eq.(24)) for each $j \in S$.

■

Under the assumption of stationary transitions, C_j^k and D_j^k do not depend on the cycle index k. For any k, given that $Z_k=z$ and $Z'_{k+1}=z'$, $z, z' \in \Theta_N$, the random variables $C_j=C_j^{k+1}$ and $D_j=D_j^{k+1}$ clearly depend on the states z and z′, as can be seen in (22) and (24). Observe that the set of random variable $C_1, \cdots, C_N$ is a *dependent* set of random variables because C_j overlaps with C_i in real time, $i, j \in S$, $i \neq j$. Similarly, the random variables $D_1, \cdots, D_N$ are dependent random variables. Let the joint distribution of $C_1, \cdots, C_N$ be denoted by $F_{C_1, \cdots, C_N}(\cdot, \cdots, \cdot)$ and let the joint distribution of $D_1, \cdots, D_N$ be denoted by $F_{D_1, \cdots, D_N}(\cdot, \cdots, \cdot)$. In the XB case, the probability that the free-token defines a vector z′ given that the last vector defined was z is given by

$$P_{XB}(z' \mid z) = \int_0^\infty \int_0^\infty \cdots \int_0^\infty \prod_{j=1}^{N} \xi_j(z_j' \mid c_j)\, dF_{C_1, \cdots, C_N}(c_1, \ldots, c_N) \tag{26}$$

where the product of the $\xi_j(\cdot \mid \cdot)$, for $j \in S$, follows from conditional independence of events at various stations. In similar fashion, the RB case is obtained as

$$\mathrm{P}_{RB}(\mathbf{z}' \mid \mathbf{z}) = \int_0^\infty \int_0^\infty \cdots \int_0^\infty \prod_{j=1}^{N} \delta_j(z_j' \mid d_j)\, \mathrm{d}\,\mathrm{F}_{D_1,\cdots,D_N}(d_1, \ldots, d_N). \tag{27}$$

where the product of the $\delta_j(\cdot \mid \cdot)$, for $j \in S$, also follows from conditional independence. In actuality, the expressions in (26) and (27) need to be developed computationally only for the case $\mathbf{z}' = (0,\ldots,0)$, for each $\mathbf{z} \in \Theta_N$. The corresponding probabilities for every other value of $\mathbf{z}'$ can be obtained from this initial expression, for each $\mathbf{z} \in \Theta_N$. This follows from the fact that (26) and (27) can also be represented as products of joint integrals, where the products come about due to independence [ReSz87] between pieces of the overlapping cycle-times for the different stations (e.g., in the XB scheme, T_1 is unique to station 1, T_2 is common to stations 1 and 2 , Y_2 is unique to station 1, Y_3 is common to stations 1 and 2, etc.).

Explicit form of the Cycle-Time Distribution

Let the random time between two consecutive free-token scans at station N be denoted by the random variable C, and its distribution by $F_C(\cdot)$. Let P_{XB} and P_{RB} be the probability transition matrices for the 2^N state Markov chain $\{Z_n\}$, as given by Eqs. (26) and (27), respectively. Correspondingly, let $\{\Phi_{\mathbf{z}}(XB)\ ;\ \mathbf{z} \in \Theta_N\}$ and $\{\Phi_{\mathbf{z}}(RB)\ ;\ \mathbf{z} \in \Theta_N\}$ denote the invariant vectors of P_{XB} and P_{RB}, respectively. The limiting cycle-time distribution $F_C(\cdot)$ of the free-token is given by

$$F_C(t) = \sum_{\mathbf{z} \in \Theta_N} \Phi_{\mathbf{z}}(RB)\, [U_1 * \cdots * U_N * \{z_1 B_1 + (1 - z_z) S_1\} * \cdots * \{z_N B_N + (1 - z_N) S_N\}](t) \tag{28}$$

where the "*" is used to denote the convolution operation. The cycle-time distribution for the XB counterpart is obtained by using the distribution $\{\Phi_{\mathbf{z}}(XB)\}$ in place of $\{\Phi_{\mathbf{z}}(RB)\}$in (28).

As an illustrative example, suppose that the distributions $B_j(\cdot)$, $S_j(\cdot)$ and $U_j(\cdot)$ are all exponential, with means $1/\mu_j$, $1/\beta_j$, and $1/\alpha_j$, respectively, for $1 \le j \le N$. Further, let $a_i = \alpha_i$ and $a_{N+i} = z_i\, \mu_i + (1 - z_i)\, \beta_i$ for $1 \le i \le N$. In this case, (28) can explicitly be written as

$$F_C(t) \;=\; \sum_{z \in \Theta_N} \Phi_z(RB) \left[\sum_{j=1}^{2N} \xi_j(z)\, a_j\, e^{-a_j t} \right] \tag{29}$$

where $\xi_j(z) = \prod_{\substack{k=1 \\ k \neq j}}^{N} \frac{a_k}{a_k - a_j}$, $1 \le j \le 2N$. This distribution is essentially a linear combination of exponentials.

5. NUMERICAL RESULTS AND APPLICATIONS

In this section we obtain some performance measurements using the results obtained from the previous section. In addition, we give some graphical results meant to illustrate the nature of cycle-times and other interesting performance issues, the computations again done with the aid of the results from the last section. The following experiments focus on a token ring configuration, with an appropriate choice of parameter values. The method is also applicable to token-bus systems, with proper interpretation of parameters.

As an example, consider a 10 Mbps token-ring system with $N=3$ stations such that the distance between stations 1 and 2 is 300 m, between stations 2 and 3 is 200 m, and between stations 3 and 1 is 100 m. Assume that the delay introduced at each station is a one-bit time. Using the standard propagation velocity of 2×10^8 m/s, the bit-length of each link between two-stations (on the average) is 10 bits, thus yielding a ring bit-length of 33 bits. Assume that each station transmits frames of exponentially distributed lengths. Frames from stations 1, 2 and 3 have mean lengths of 200, 600 and 400 bits, respectively. In using the single-token rule, a station that transmits a frame must wait for the busy-token to return before it can issue a new free-token. Since frame lengths are longer than the bit-length of the ring, a new free-token cannot be issued until the end of the sender's own transmission. Thus, the station transmission-times are the times for which each station monopolises a free-token, with $E(X_1)=2\times10^{-5}\ s$, $E(X_2)=6\times10^{-5}\ s$, and $E(X_3)=4\times10^{-5}\ s$. Token-passing times between stations are taken to be exponentially distributed, with $E(Y_1)=10^{-7}\ s$, $E(Y_2)=3\times10^{-7}\ s$, and $E(Y_3)=2\times10^{-7}\ s$. Assume also that switching times are exponentially distributed, with means $E(V_j)=E(Y_j)/100\ s$, $1 \le j \le 3$.

In Figures 4.1a and 4.2a can be seen the cycle-time density of the free-token on an XB system, for low channel traffic. Figure 4.1a displays small cycle-times (i.e., time less than $20\mu s$), and Figure 4.2a displays larger cycle-times. The reason for the two-part display is to emphasise the long-tailed nature of these densities. Figures 4.1b and 4.2b graph the moderate traffic situation (again in two parts), and Figure 4c graphs the entire cycle-time density for high traffic. Note that the cycle-time density can be bimodal, though not markedly so. Also, for high traffic, cycle-times tend to take on a greater range of large values with higher probability.

Exact Probabilities for Buffer Status at scan instants

The probability $r_j(1)$ that station j's buffer is found empty (for the RB scheme) on any scan of station j is given by

$$r_j(1) \quad = \sum_{z \in \{z \in \Theta_N \mid z_j = 0\}} \Phi_z(RB) \tag{30}$$

(where $\{\Phi_z(XB)\}$ is used for the XB scheme), for each $j \in S$. In treating a *symmetric* system, Takagi [Taka85] obtains this probability explicitly for up to $N=3$. The Markov chain used for this has two major differences from the Markov chain that we have used. First, it relies heavily on symmetry. Second, the nature of the embedding is different. That is, in [Taka85] a state is taken to be the status of the different buffers *at the instant when station 1 is polled* (i.e., scanned) by the free-token. In our formulation, we ignore symmetry. Additionally, embedding is done at scan instants at station N, *but* the state of the system this instant is a *record* of the status of at stations at *their respective scan instants*. In Table 1 below we demonstrate results for $N=3$ stations, with the first column showing numbers resulting from the formula (see Eq. (31b) in [Taka85]), and the second column showing numbers resulting from specialising our asymmetric results to the symmetric case. The third column displays arbitrarily chosen values of λ_j, $1 \le j \le 3$. For this computation, all mean walk times are $10^{-7}\ s$, mean switching times are zero (since the analysis in

[Taka85] utilises switching times of zero), and mean frame transmission times are $10^{-5}\ s$ for each station. All times are exponentially distributed random variables. The numbers in both columns can be seen to agree closely, with small differences resulting from round-off error in our computations.

Formula	*Algorithm*	λ_j
0.982987137	0.982933061	0.0031
0.978101621	0.977993843	0.0037
0.972541497	0.972348627	0.0042
0.966276227	0.965957982	0.0048
0.959280956	0.958788073	0.0053
0.951536921	0.950811442	0.0059
0.943031779	0.942007698	0.0064
0.933759850	0.932364091	0.0070
0.923722269	0.921875948	0.0075
0.912927019	0.910546954	0.0081
0.901388860	0.899389268	0.0086
0.889129139	0.887423459	0.0092

TABLE 1.

Mean Cycle-time

The mean cycle-time on this system is

$$E(C) = \sum_{i \in S} \left[E(Y_i) + r_i(1)E(V_i) + (1 - r_i(1))E(X_i) \right] \tag{31}$$

In Figures 5 through 9, the experiments are done with all parameters fixed except station 3's arrival rate (i.e., λ_3), and parameters otherwise indicated on the graphs. The arrival rates are fixed at $\lambda_1 = 0.009$, and $\lambda_2 = 0.006$ for stations 1 and 2. In Figure 5 can be seen the effect of station 3's frame length on the mean cycle-time of the system for varying values of λ_3. In Figures 6 through 9, $E(X_3) = 10^{-4}\ s$, and this is the only parameter change. In other words, station 3 has longer frames (on the average) than stations 1 and 2. Figures 6a, 6b and 6c display the limiting vector distributions $\{\Phi_z(XB)\ ;\ z \in \Theta_3\}$ and $\{\Phi_z(RB)\ ;\ z \in \Theta_3\}$ for the XB and RB cases, respectively, for low, moderate and high channel traffic. The difference appears to be considerable for moderate to high traffic.

System Utilisation

Given that the free-token defines a vector $\mathbf{z} \in \Theta_N$, the mean channel utilisation for that cycle is given by

$$E(U \mid \mathbf{z}) = \frac{\sum_{j=1}^{N} z_j E(X_j)}{\sum_{j=1}^{N} z_j E(X_j) + (1-z_j)E(V_j) + E(Y_j)} \tag{32}$$

and mean system channel utilisation is

$$E(U) = \sum_{\mathbf{z} \in \Theta_N} E(U \mid \mathbf{z}) Pr(\mathbf{z}) \tag{33}$$

where the appropriate limiting vector distribution is substituted for the XB and RB cases, respectively. Figure 7 graphs mean system utilisation (called U) versus arrival rate λ_3. It is clear that the XB scheme makes a significant difference. Note that the mean throughput of the system is the same as that given by Eq.(32), but with the substitution $E(X_j)=1$ for all $j \in S$ (i.e., number of successful transmissions per unit time). In Figure 8 is plotted the throughput S of station 3 alone versus the arrival rate λ_3. The probability of frame loss for any station is computed via the relation $S_j = \lambda_j \ (1-L_j)$, where S_j is station j's throughput, and L_j is the probability that a station-j frame is lost. Using $S = S_3$, and $L = L_3$, Figure 9 compares the frame-loss probability for the XB and RB cases.

6. Conclusion

We have shown that an asymmetric token ring with unit buffers can be analyzed exactly using an $O(2^N)$ algorithm. This is done by obtaining the distribution of cycle-time, and then deriving various performance measures from this distribution. For such systems, there is much useful information contained in the cycle-time distribution. It is unfortunate that for $N \geq 10$, the computations are impractical, though workable in principle. In such cases, practioners usually resort to an assumption of symmetry, effectively reducing complexity from $O(2^N)$ to $O(N)$. It is important to note that drawing conclusions about system performance from symmetric models can be hazardous, especially since it is often the very asymmetry of the observed system that causes it to behave in a manner that arouses our interest. It is fairly easy, though tedious, to show that for $N < 10$, symmetric and asymmetric systems can behave very differently.

In addition to analytic evaluation of small systems for various parameter ranges, our results can be used to compare systems with different buffering policies. A comparison of a variation (XB) of the normal buffering scheme (RB) when unit-capacity buffers are used is an immediate application of our methods. Though XB can intuitively be expected to perform better than RB, a rigorous comparison of the two requires an exact model of the kind developed here. Current work includes applications to multiple-token operation and single-frame operation for rings with bit-lengths smaller/greater than frame-lengths, and the formidable asymmetric system with finite/infinite capacity buffers and one-at-a-time service.

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APPENDIX

THEOREM

Let $Z(t) = <Z_1(t), ..., Z_N(t)>$ be a continuous time stochastic process with components defined by $Z_j(t) = Z_j^k$, for $t \in (t_N^k, t_N^{k+1}]$. Also, let $Z_k = <Z_1^k, \cdots, Z_N^k>$. Then,

(a) $\{Z(t); t > 0\}$ is a semi-Markov process, and

(b) $\{Z_n; n > 0\}$ is a Markov chain,

each with state space given by the set of all N-bit binary vectors Θ_N.

PROOF :

We are interested in cycles of the free-token defined at steady-state scan-instants of station N. With $t=0$ as the instant at which the free-token makes an *exit* from station N (to begin the walk to station 1), let t_j^k denote station j's scan-instant in the k^{th} cycle, for $k \geq 1$. For each k, $k \geq 1$, $<Z_1^k, \cdots, Z_N^k>$ represents a record of station events possessed by the free-token at time $t = t_N^k$. Each such record (an N-bit string of binary digits) is taken to be a state of the system. Without loss of generality, assume that the free-token resides in state $<0, \cdots, 0>$ at time $t=0$. The free-token is seen to make state transitions at the time instants $t_N^1, t_N^2, ..., t_N^k, ...$ Given that $Z_j^k = z_k$ and $Z_j^{k+1} = z_j'$, for some integer k, $k \geq 1$, let $T_j = z_j X_j + (1-z_j) V_j$, and $T_j' = z_j' X_j + (1-z_j') V_j$, for $z_j, z_j' \in \{0,1\}$, $j \in S$. That is, given that z and z′ are two consecutive states in Θ_N, the conditional random variables T_j and T_j' describe the amount of time spent by the free-token at station j while in states z and z′, respectively.

We now show that the discrete time process $\{Z_k\}$ is an N-dimensional Markov chain on the 2^N state-space Θ_3, embedded at station-N scan instants. Assuming Poisson arrivals of rate λ_j at queue j and the RB scheme, let $A(C_j^{k+1})$ represent the number of customers who arrive at queue j during the time C_j, for each $j \in S$. For the XB scheme this number is denoted by $A(D_j^{k+1})$. Since each buffer is of unit capacity, it follows that

$$<Z_1^{k+1}, Z_2^{k+1}, ..., Z_N^{k+1}> = <[Z_1^k - 1]^+, [Z_2^k - 1]^+, ..., [Z_N{}^k - 1]^+> \tag{34}$$

$$+ <I_1^{k+1}, I_2^{k+1}, ..., I_N^{k+1}>$$

where $[X-1]^+ = \max(X-1, 0)$, and

$$I_j^{k+1} = \begin{cases} 1 & A(C_j^{k+1}) > 0 \\ 0 & A(C_j^{k+1}) = 0 \end{cases}$$

for the RB scheme, for each $j \in S$. The XB scheme is identical, except that I_j is defined in terms of $A(D_j)$. From the stochastic equation in (34), we conclude that the sequence $\{Z_k\}$ is an N-dimensional Markov chain defined on Θ_N. The amount of time spent by the process in each state $<z_1, ..., z_N> \in \Theta_N$ is a random time depending on the next state $<z_1', ..., z_N'>$ to be entered. Thus, the continuous time process $\{Z(t)\}$ is a semi-Markov process (see [Cinl75]).

■

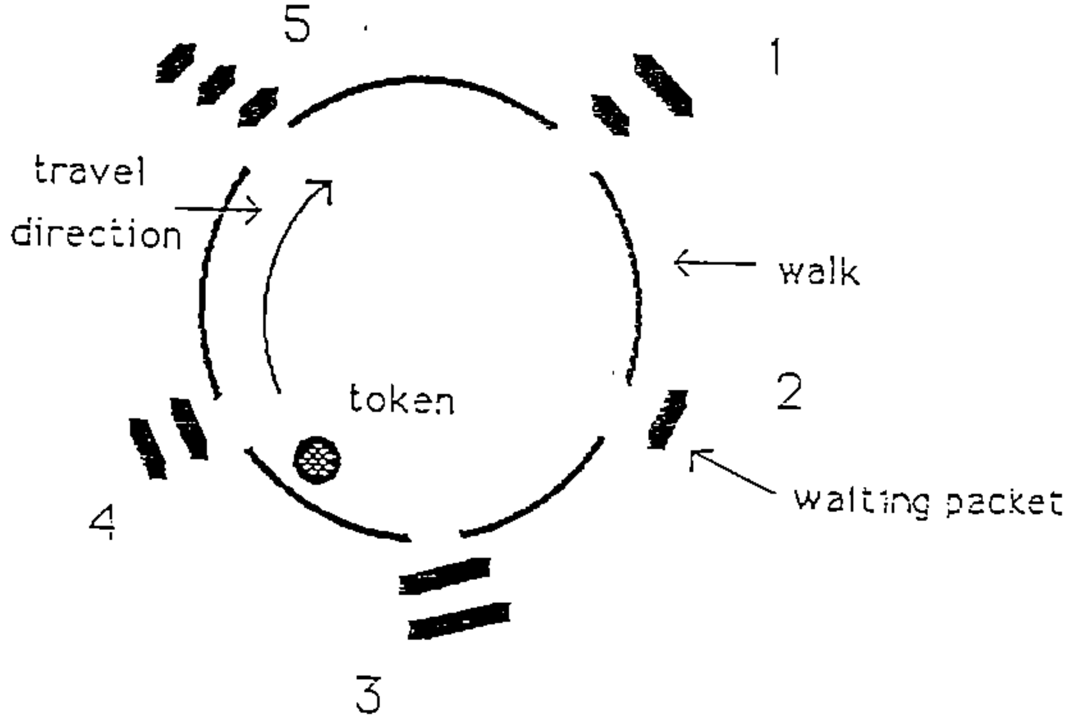

Fig. 1 Conceptual view of Token-passing on bus and ring

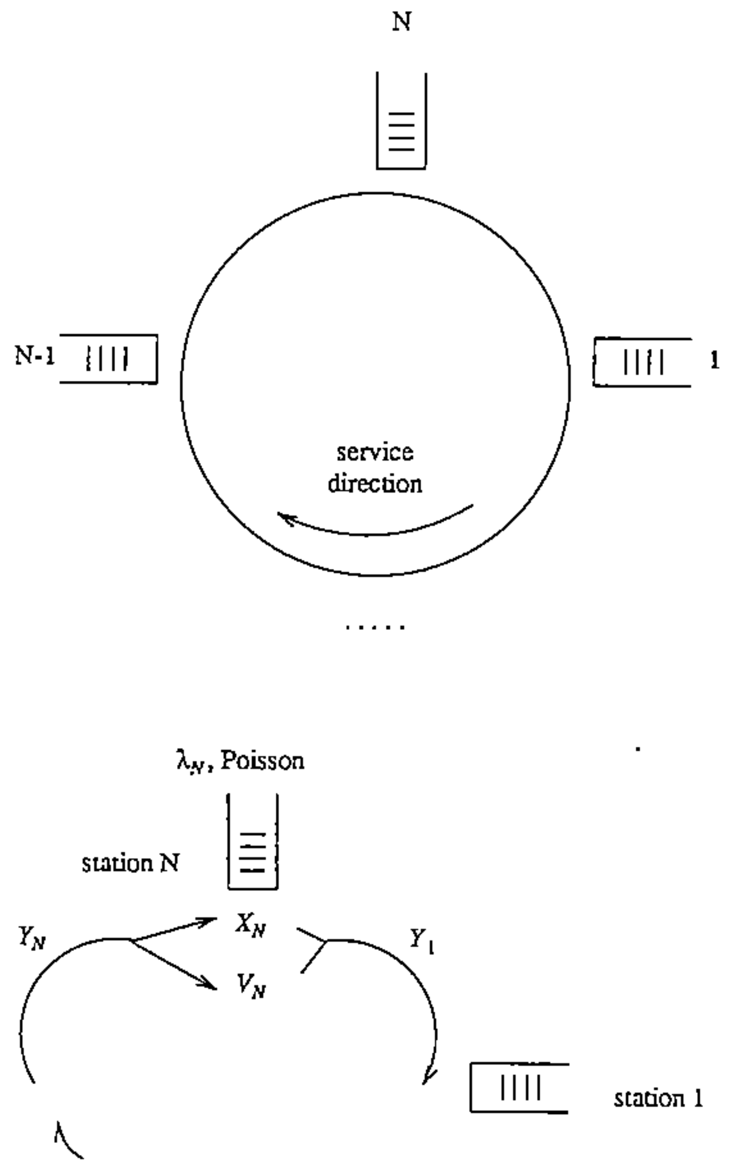

Figure 2. Multiqueue and cyclic server, infinite buffers.

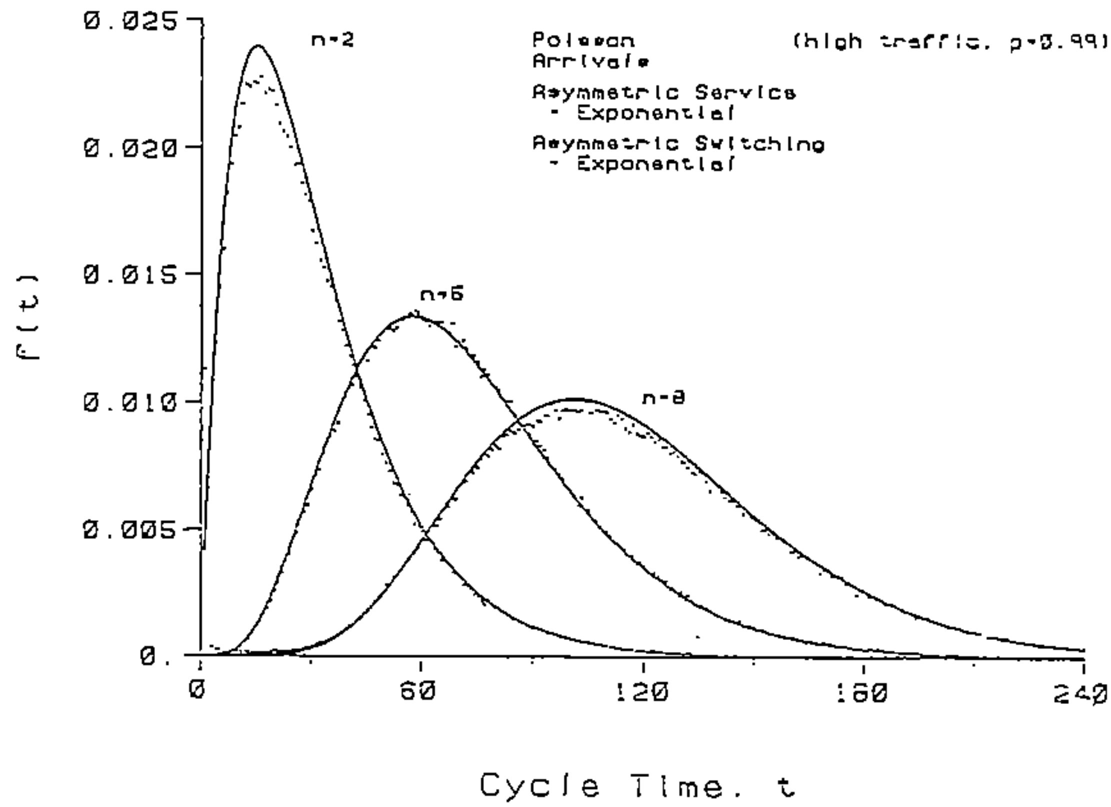

Fig. 3 Simulated versus approximate densities for high loads

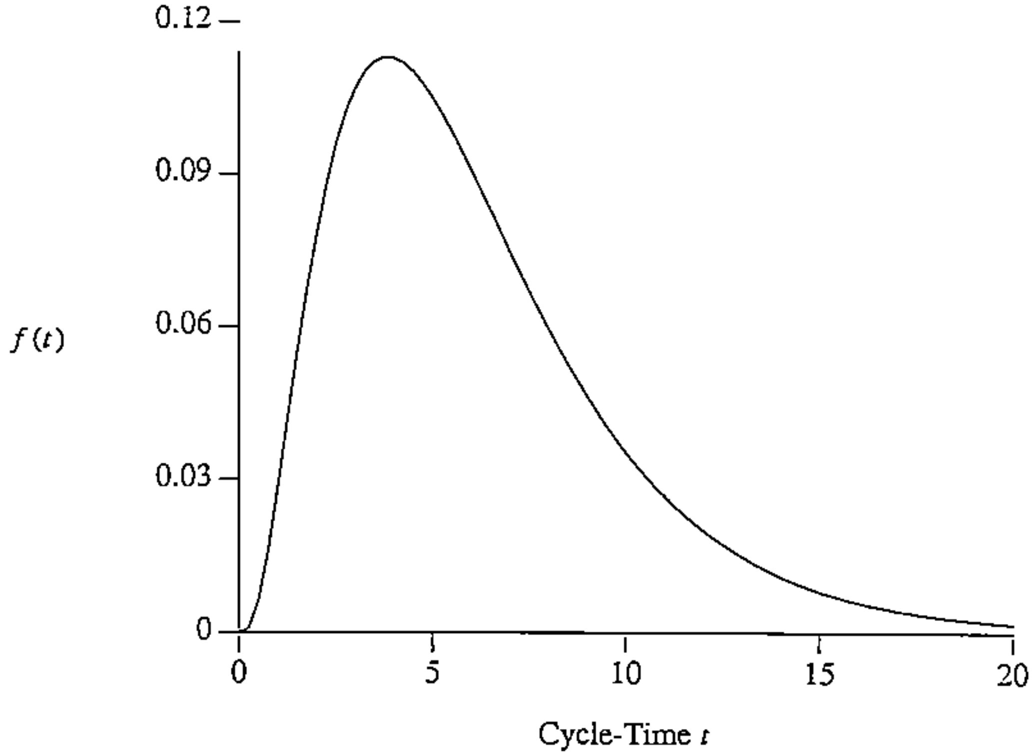

Figure 4.1a (Cycle-Time density [XB])

Low Traffic ($\lambda_1=0.001$, $\lambda_2=0.002$, $\lambda_3=0.003$)

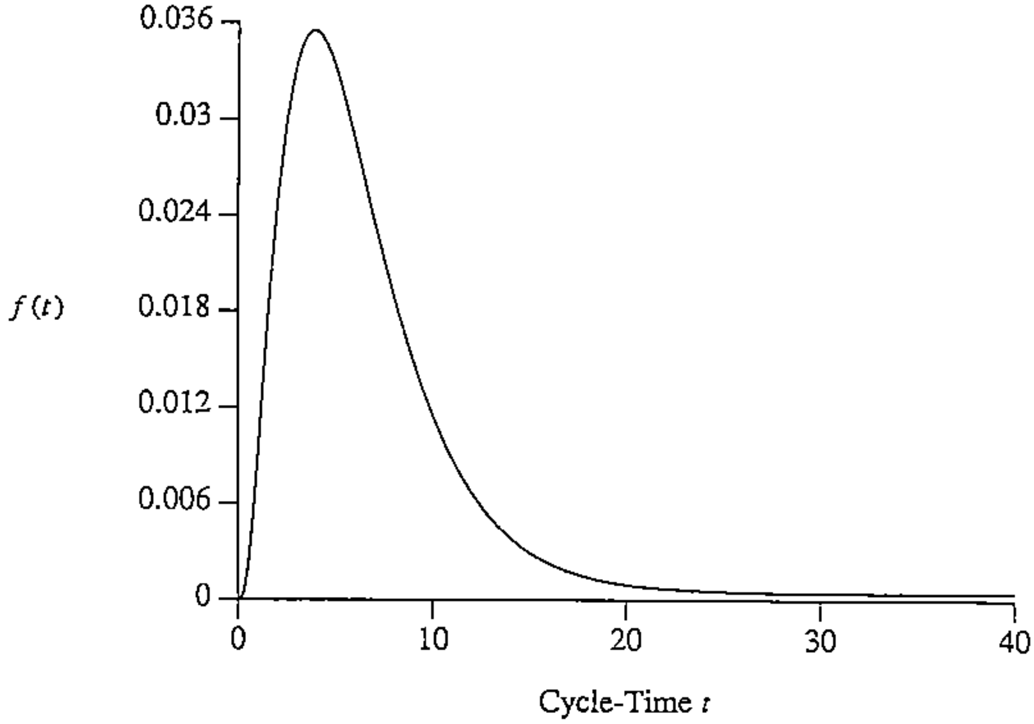

Figure 4.1b (Cycle-Time density [XB])

Moderate Traffic (λ_1=0.001 , λ_2=0.002, λ_3=0.02)

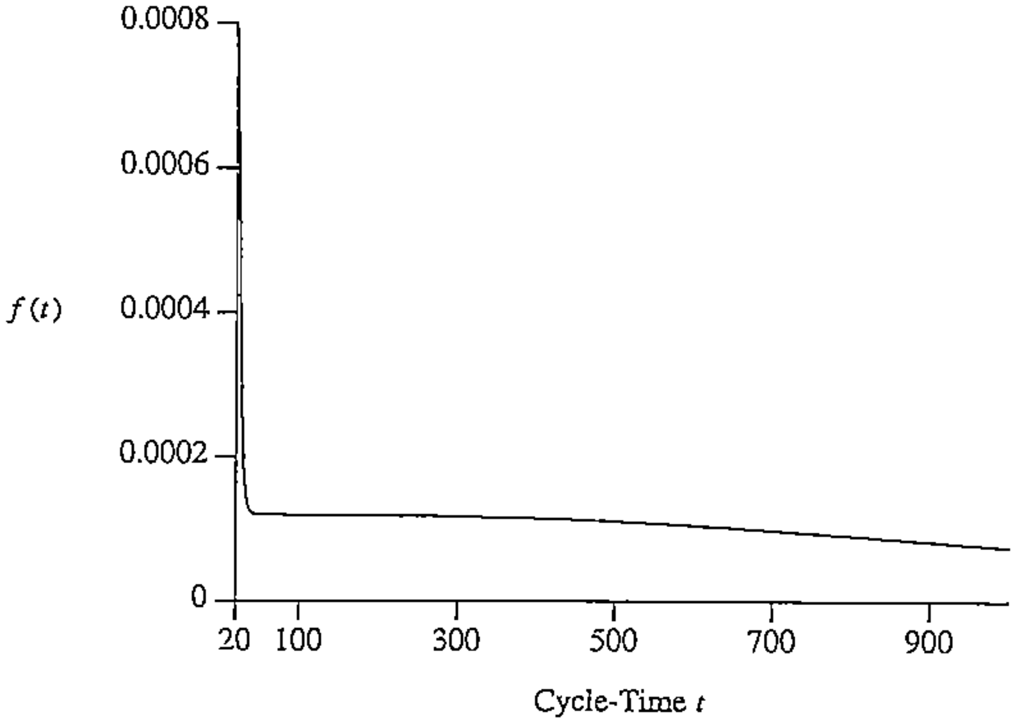

Figure 4.2a (Cycle-Time density [XB])

Low Traffic (λ_1=0.001 , λ_2=0.002, λ_3=0.003)

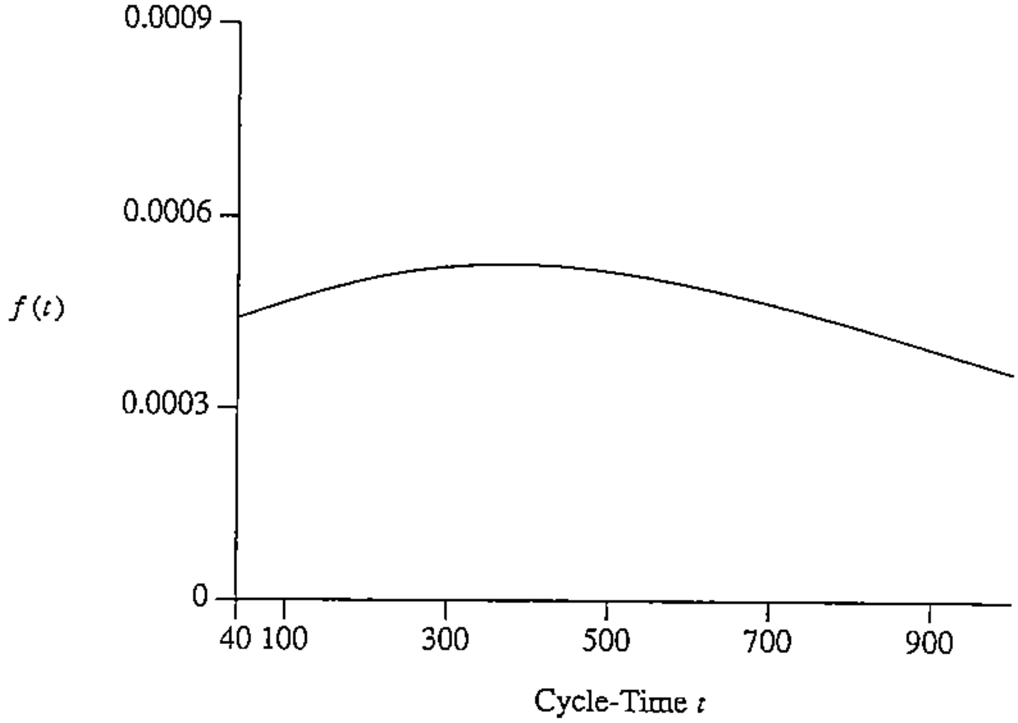

Figure 4.2b (Cycle-Time density [XB])

Moderate Traffic ($\lambda_1=0.001$, $\lambda_2=0.002$, $\lambda_3=0.02$)

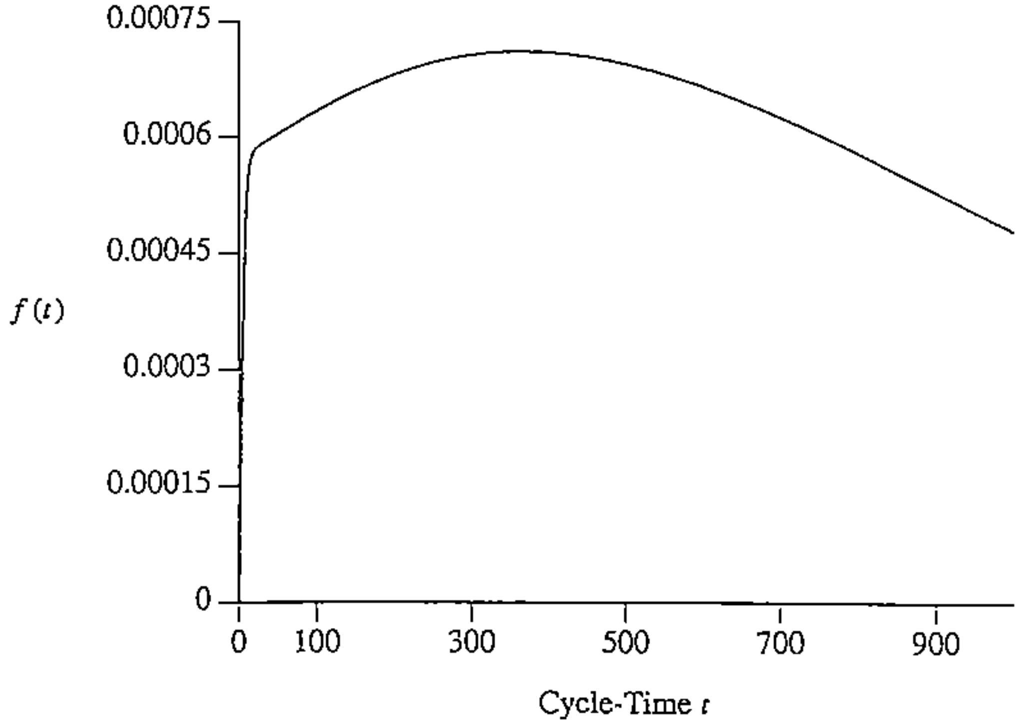

Figure 4c (Cycle-Time density [XB])

High Traffic (λ_1=0.001 , λ_2=0.002, λ_3=0.4)

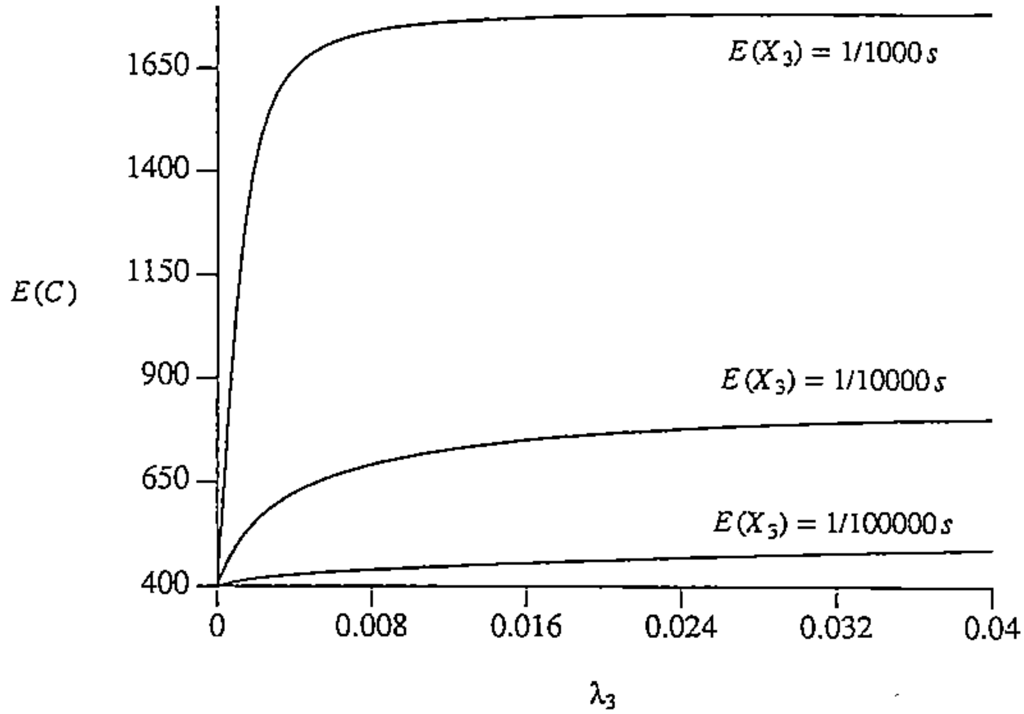

Figure 5 (Mean Cycle-Time [XB] vs. Arrival rate)

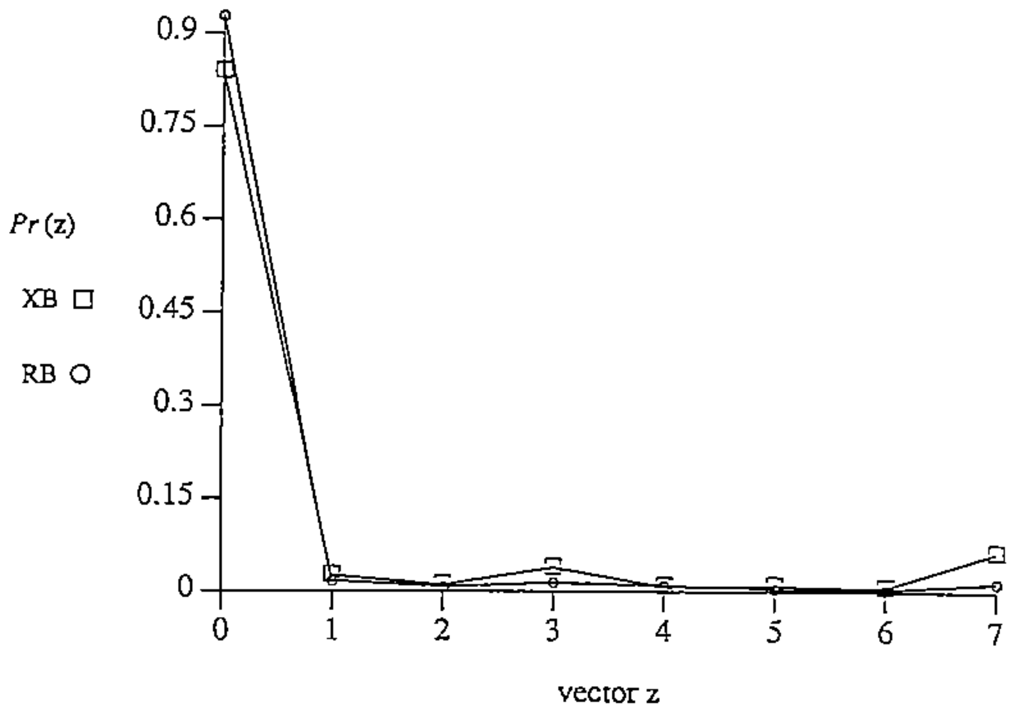

Figure 6a (Limiting vector distribution)

Low Traffic (λ_1=0.001 , λ_2=0.002, λ_3=0.003)

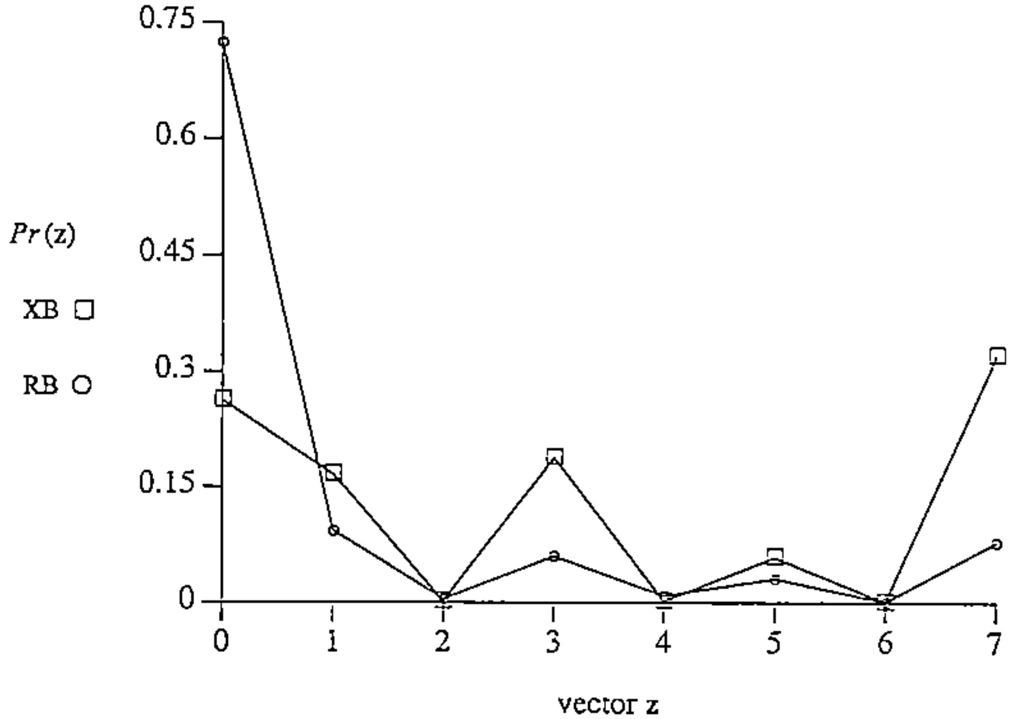

Figure 6b (Limiting vector distribution)

Moderate Traffic ($\lambda_1=0.001$, $\lambda_2=0.002$, $\lambda_3=0.02$)

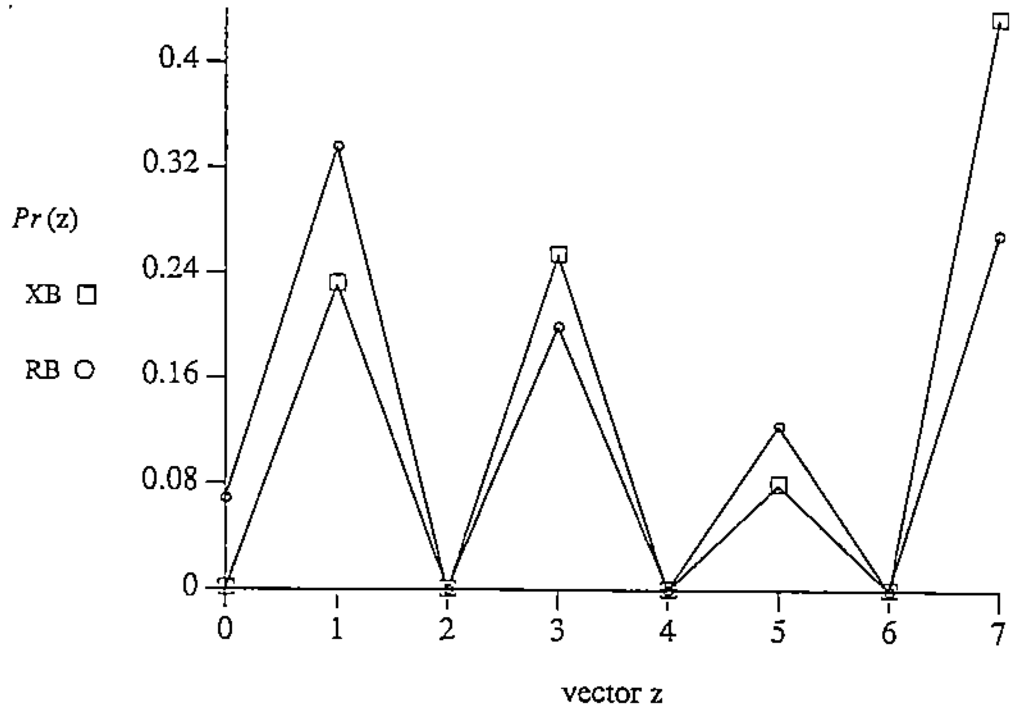

Figure 6c (Limiting vector distribution)

High Traffic ($\lambda_1=0.001$, $\lambda_2=0.002$, $\lambda_3=0.4$)

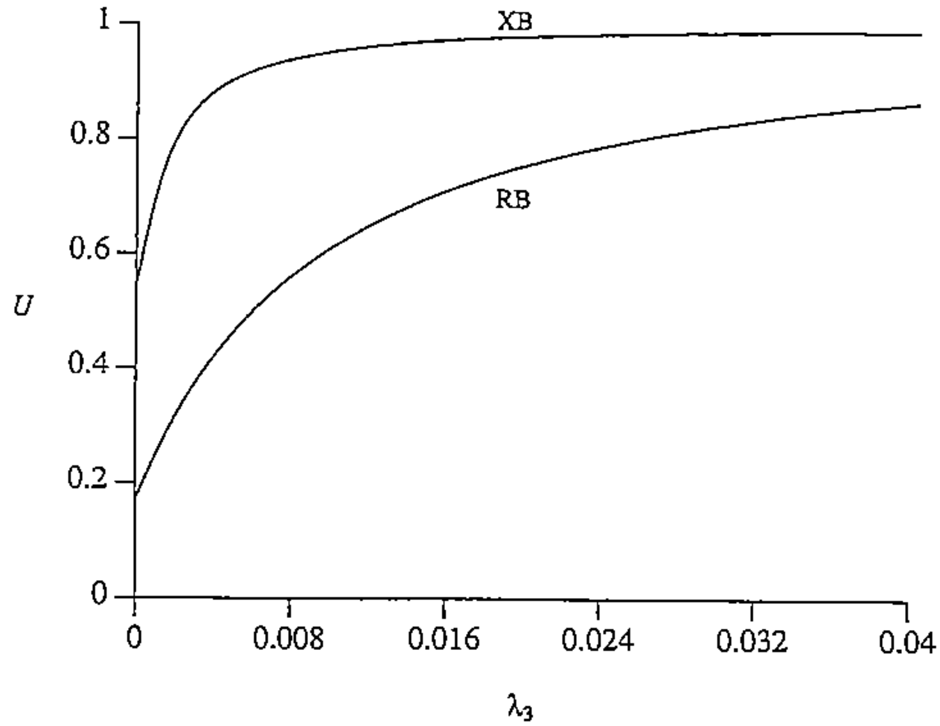

Figure 7 (Utilization vs. Arrival rate)

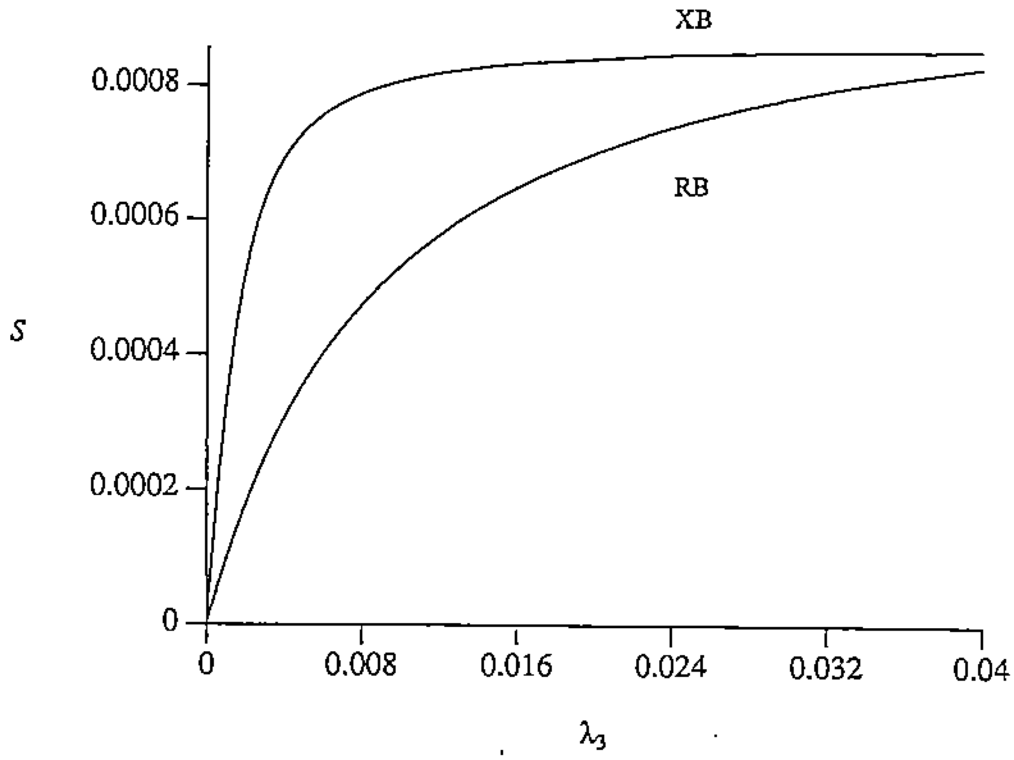

Figure 8 (Throughput vs. Arrival rate)

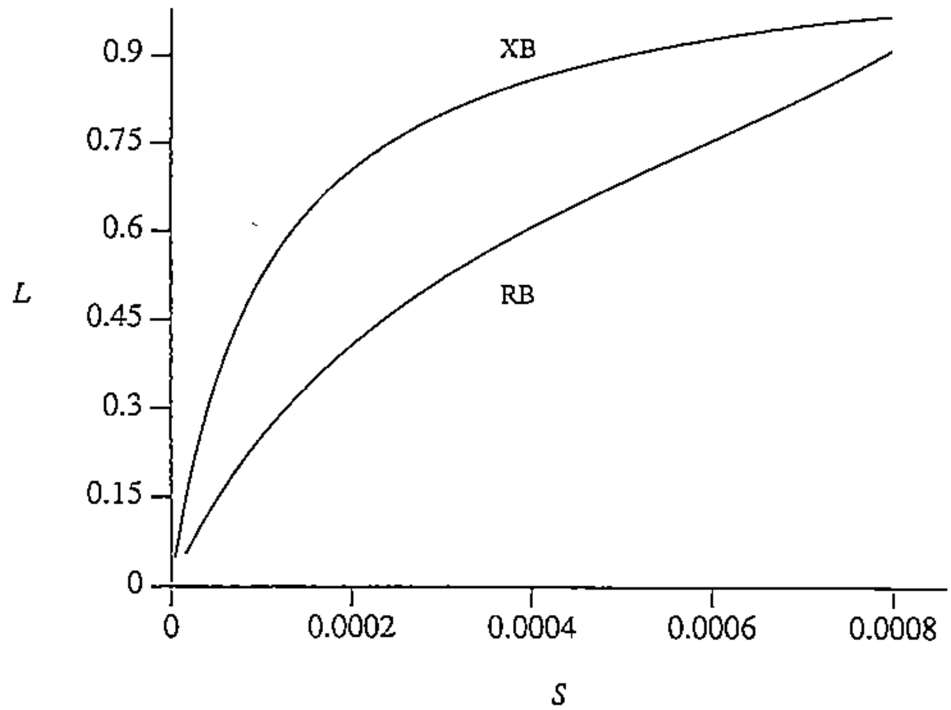

Figure 9 (Throughput vs. Loss probability)