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ANALYSIS OF COMPRESSOR BOLT FASTENERS SUBJECTED TO DYNAMIC LOADING

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ABSTRACT

Predominant cause of fractures of bolts subjected to cyclic loading is attributable to a lack of attention to proper design considerations for elastic joints.

In this paper, fundamentals of bolted elastic joints subjected to cyclic loading are reviewed. A case history involving use of elastic joint techniques on the piston bolt of a small air compressor is given. In the example, analysis of dynamic loads acting on the joint, elastic joint properties and stress analysis of the bolt are presented.

In conclusion, factors of safety are developed that enable the designer to comparatively evaluate various bolt joint options that may be available. Application of this methodology may be further extended to the design of gasketed joints.

INTRODUCTION

In the past, numerous fasteners in clamped assemblies fractured when subjected to dynamic loads because insufficient attention had been given to proper joint design. Such previous design methods usually relied solely upon the determination of maximum direct tensile stress in the bolt. Direct stress was characteristically obtained by dividing the total maximum applied load by the minimum cross sectional area of the bolt. Generous factors (usually upwards of 10.) were commonly ascribed for the ratio of bolt material tensile strength to this direct stress. Nonetheless, fatigue fractures of bolts having such misleadingly high numerical factors were not uncommon.

It has since become evident that the application of elastic joint analysis combined with dynamic stress analytical techniques have provided the compressor designer a far more reliable method. Interestingly, bolt factors of safety in the order of 1.4 when current analytical techniques are employed, afford the compressor designer far more assurance than did higher numbers associated with previous methods. On occasion, however, it remains difficult to convince some users accustomed to previous methods that such small factors of safety are adequate, particularly in critical applications.

Early investigators⊛, (1,2) reported that when dynamic external loading is applied to pre-loaded bolted assemblies such loading is divided between the bolt and clamped members in a direct relationship to respective spring constants. These spring constants are derived from Hooke's law, as illustrated in elementary equations -1- and -2-:

$$\Delta L = \frac{PL}{AE} \qquad \text{-1-}$$

Since by definition

$$K = \frac{P}{\Delta L},$$

$$K = \frac{AE}{L} \qquad \text{-2-}$$

Accordingly, spring constant for bolt is

$$K_b = \frac{A_b\ E_b}{L_b}, \qquad \text{-3-}$$

and spring constant for clamped part member is

$$K_p = \frac{A_p\ E_p}{L_p}. \qquad \text{-4-}$$

⊛ References are listed at end of paper.

Combined spring constant relationship for a bolted assembly preloaded to an initial load, F_{in} is illustrated in Fig. 1.

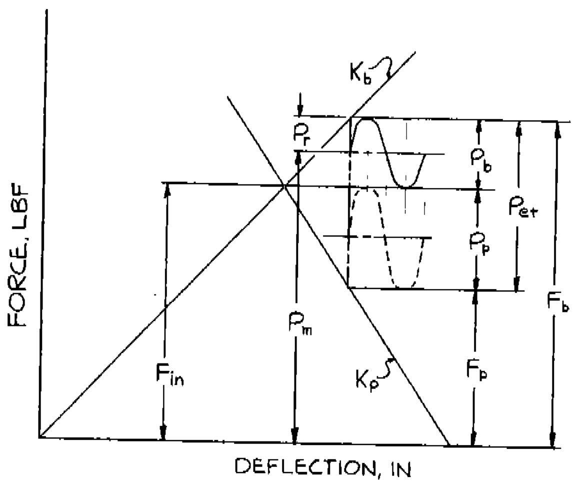

FIG. 1. CLAMPED JOINT FORCE - DEFLECTION DIAGRAM

With an external tensile double amplitude cycling load varying between 0 and P_{et} applied to a clamped joint, maximum double amplitude dynamic load felt by the bolt member is represented by P_b and maximum load felt by clamped parts is P_p. As is evident in Fig. 1, magnitude of force F_p must be greater than zero in order to prevent separation at the joint interface. Should interface separation occur, the bolt would be subjected to the full external load. Conversely, the maximum amount of pre-load is limited. F_b, total of loads F_{in} and P_b must not exceed load at yield of the bolt.

As is further evident in Fig. 1, and dependent upon stiffness of bolt and clamped parts, only a portion of external load is directly added to preload. Loads on bolt and on clamped part are given by the relationships

-5-
$$F_b = F_{in} + \frac{K_b}{(K_b + K_p)} \times P_{et}$$

-6-
$$F_p = F_{in} - \frac{K_p}{(K_b + K_p)} \times P_{et}$$

In some dynamic load situations P_{et} is other than a non-reversing type load, in many cases varying between positive and negative values. Maximum and minimum values of F_b are calculated from equation -5-. Such boundary F_b values are the basis for dynamic analysis of the bolt.

STRESS ANALYSIS

A modified Goodman diagram for 17-4 PH, condition H1075 steel is illustrated in Fig. 2. Typical ultimate tensile strength of this material is 145000 psi,

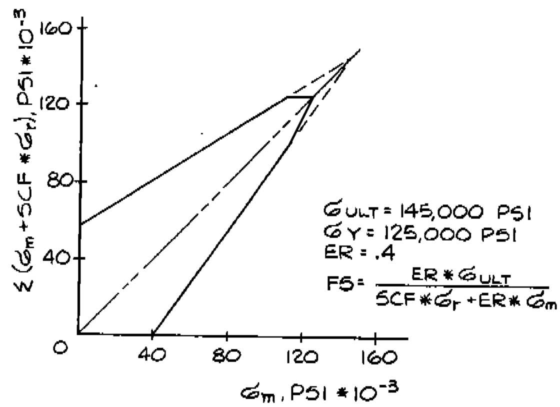

FIG. 2. MODIFIED GOODMAN DIAGRAM. 17-4 PH CONDITION H1075

(3,4) yield strength 125000 psi and endurance ratio 0.4. Factor of safety is given by

-7-
$$FS = \frac{ER \times \sigma_{ult}}{SCF \times \sigma_r + ER \times \sigma_m}$$

Stress concentration factors upwards of 4.5 are typical for cut thread having root radii. However, when manufacturing care is not taken higher stress concentration factors may exist. Factors of 2. to 2.5 are normal for well rounded roots of rolled threads. (5,6)

EXAMPLE OF DYNAMICALLY LOADED PISTON BOLT

A partial cross-sectional view of piston and piston bolt assembly of a 100 cfm air compressor is shown in Fig. 3.

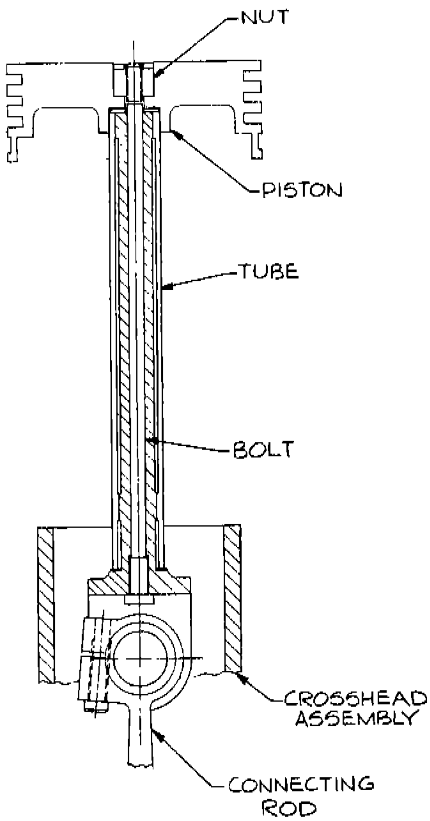

FIG. 3. CROSS-SECTION OF PISTON AND BOLT ASSEMBLY.

Function of the bolt, shown at the axial center, is to clamp the piston to the crosshead assembly through the guide tube. Present bolt material used is 17-4PH, condition H1075 steel. Specified pre-load torque on piston nut is 31 lbf-ft. Fatigue fractures had occurred through cut threads, and bolts with rolled threads had been substituted. A comparison of factors of safety between the two thread forms was required, along with recommendations for possible material substitution, if this was found to be necessary.

A number of bolts of both thread forms were examined under an optical comparator and findings were quite revealing, as illustrated in Fig. 4. Root radii of the 3/8-16 UNC2A cut threads were practically non-existent and were extremely sharp. Measured root diameter was 0.278". Rolled threads, on the other hand, were well rounded in the root area and measured root diameter was 0.288". As stated previously, a stress concentration of 4.5 was deemed appropriate for the cut threads. A stress

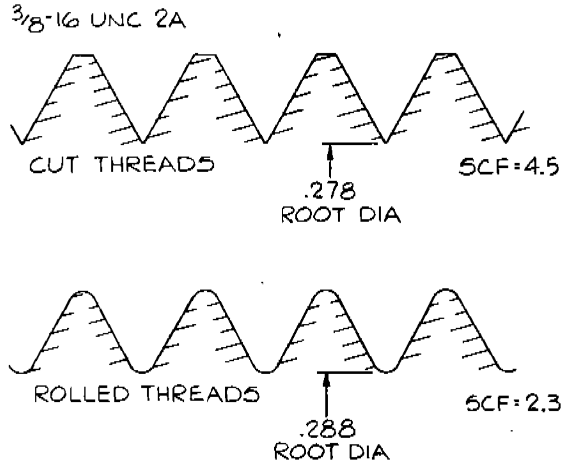

FIG. 4. DIAGRAMMATIC COMPARISON OF BOLT THREADS.

concentration factor of 2.3 was used for the rolled threads.

The 100 cfm air compressor in this example is of Y configuration with two 8.25" dia. single acting first stage cylinders and one 6.5" dia. single acting second stage cylinder. Piston stroke is 2.75" and crankshaft rotational speed is 830 rpm. All of the bolts that fractured had cut threads and were typically located at the second stage piston assembly. Applied joint force -vs- crank angle relationships were accordingly calculated for three cases of piston load: unloaded, stage I loaded and stage II loaded. These are illustrated in Figs. 5, 6 and 7, respectively. Since the piston weight for both stages is identical, Fig. 5 is common for both stages. Applied cylinder load P_{et} for piston inertia ranged from -161 lbf to 210 lbf, P_{et} for stage I loaded ranged from -1916 lbf to 184 lbf and P_{et} for stage II loaded ranged from -4192 lbf to -977 lbf.

In the example a series of washers and varying sectional geometry contributed to stiffness of overall clamped part in the relationship (7)

$$-8- \qquad K_p = \frac{1}{\sum\left(\frac{1}{K_n}\right)}$$

Numerical value of K_p was found to be 1.2301E6 lbf/in. Similarly, overall stiffness of bolt was 0.2477E6 lbf/in.

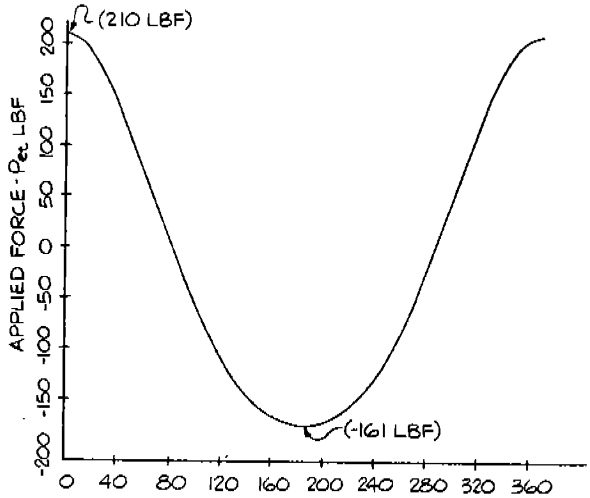

FIG. 5. P_{et} -vs- CRANK ANGLE: CYLINDER UNLOADED

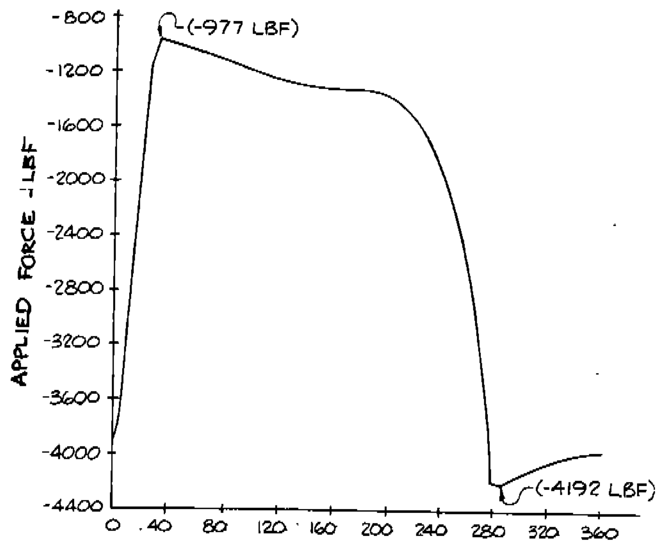

FIG. 7. P_{et} -vs- CRANK ANGLE: STAGE II CYLINDER LOADED.

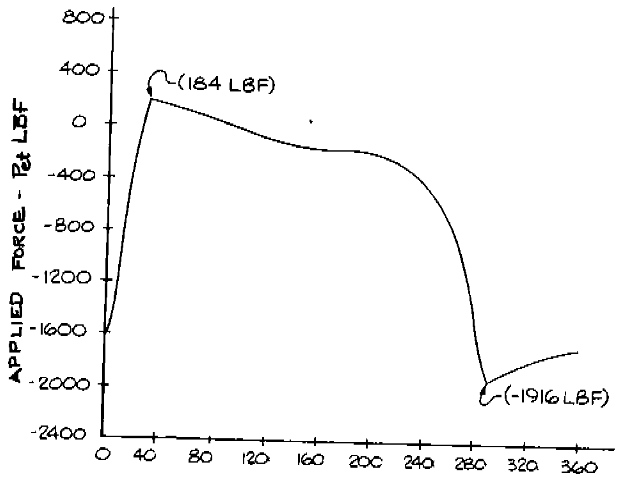

FIG. 6 P_{et} -vs- CRANK ANGLE: STAGE I CYLINDER LOADED

Bolt preload on clamped assembly, F_{in} with dry threads was obtained from the generalized relationship

$$-9- \quad F_{in} = \frac{T}{.018d}$$

With 3/8" bolt torqued to 31 lbf.ft, F_{in} was 4593 lbf.

Simplifying equations -5- and -6- for present specific preload and spring constants result in

$$-10- \quad F_b = 4593 + .16761\ P_{et}\text{, and}$$

$$-11- \quad F_p = 4593 - .83238\ P_{et}.$$

Calculated forces and stresses acting on bolt are summarized on Table 1.

Case	Load, lbf.					Nom. Stress σ, PSI			
						Cut Thds.		Rolled Thds.	
	P_{et}	F_b	F_p	P_m	P_r	σ_m	σ_r	σ_m	σ_r
Unloaded, Max. P_{et}	210	4628	4418	4597	31	75730	510	70570	470
Unloaded, Min. P_{et}	-161	4566	4727						
Stg. I, Max. P_{et}	184	4624	4440	4448	176	73280	2900	68280	2700
Stg. I, Min. P_{et}	-1916	4272	6188						
Stg. II, Max. P_{et}	-977	4429	5406	4160	269	68530	4430	63860	4130
Stg. II Min. P_{et}	-4192	3890	8082						

TABLE 1

Summary of Bolt Forces and Stresses (F_{in} = 4593 lbf.)
17-4 PH Condition H1075

Nominal stresses of cut threads are slightly higher due to their smaller minimum cross sectional area compared to the well rounded roots of rolled threads. Mean and cyclic loads P_m and P_r are graphically illustrated on Fig. 8 for unloaded and stage II loaded cases.

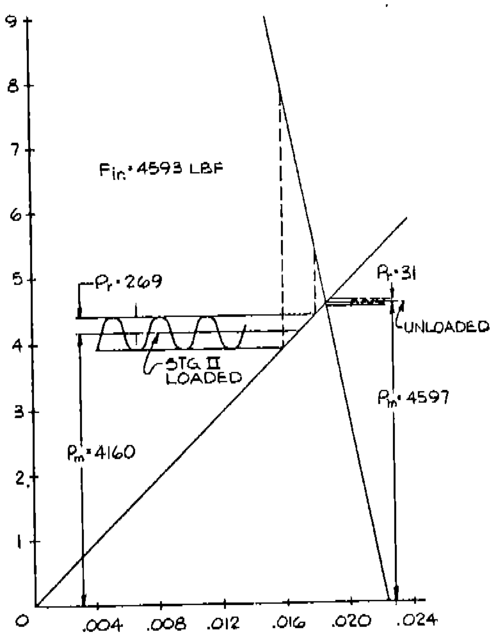

FIG. 8. ELASTIC JOINT DIAGRAM WITH DYNAMIC LOADS

Equation -7- can be simplified for 17-4PH condition H1075 bolt material to:

$$-12- \quad FS_c = \frac{58000}{4.5\ \sigma_r + .4\ \sigma_m} \quad \text{for cut threads}$$

and to

$$-13- \quad FS_r = \frac{58000}{2.3\ \sigma_r + .4\ \sigma_m} \quad \text{for rolled threads.}$$

Values of bolt cyclic stress, bolt mean stress and factors of safety are summarized on Table 2 for 17-4 PH condition H1075 bolt material. As is evident, a significant increase in factor of safety was attained by use of rolled threads compared to cut threads.

Thread Form	Load Case	σ_r, PSI	σ_m, PSI	FS
Cut	Unloaded	510	75730	1.78
Cut	Stg. I Loaded	2900	73280	1.37
Cut	Stg. II Loaded	4430	68530	1.22
Rolled	Unloaded	470	70570	1.98
Rolled	Stg. I Loaded	2700	68280	1.73
Rolled	Stg. II Loaded	4130	63860	1.66

TABLE 2

Summary of Cyclic and Mean Stresses and Factors of Safety
17 4 PH Condition H1075, F_{in} = 4593 lbf

ALTERNATE MATERIAL STUDY

Inasmuch as the application was considered to be highly critical, the customer opted for a factor of safety greater than 1.66 and questioned whether another bolt material was appropriate. Upon evaluation Multiphase Alloy MP35N* was considered as a possible alternate bolt material.

Applicable properties of MP35N material were (8)

$$\sigma_{ult}' = 260{,}000 \text{ psi}$$

$$ER' = .346$$

$$E' = 33.5 \text{ E6 psi @ } T_f = 175$$

On the basis of revised bolt modulus, K_b' was 0.2912E6 lbf/in., and

$$-10a- \quad F_b' = 4593 + .19142\ P_{et}, \text{ and}$$

$$-11a- \quad F_p' = 4593 - .80858\ P_{et}.$$

Bolt forces and stresses using MP35N bolt material are summarized on Table 3.

	Load, Lbf					Nom Stress σ, PSI			
						Cut Thds		Rolled Thds	
Case	P_{et}	F_b'	F_p'	P_m'	P_r'	σ_m'	σ_r'	σ_m'	σ_r'
Unloaded Max.	210	4433	4423	4597	36	75730	590	70570	550
Unloaded Min.	-161	4562	4723						
Stg. I Max.	184	4628	4444	4427	201	72930	3310	67960	3080
Stg. I Min.	-1916	4226	6142						
Stg. II Max.	-977	4406	5383	4098	308	67510	5070	62910	4730
Stg. II Min.	-4192	3790	7982						

TABLE 3

Summary of Bolt Forces and Stresses (F_{in} = 4593 lbf)
Multi-Phase Alloy MP35N*

Rewriting equation -7- for Multiphase MP35N alloy results in:

$$-12a- \quad FS_c' = \frac{90000}{4.5\ \sigma_r' + .346\ \sigma_m'}, \text{ and}$$

$$-13a- \quad FS_r' = \frac{90000}{2.3\ \sigma_r' + .346\ \sigma_m'}.$$

* Registered trademark of Standard Pressed Steel Co.

Bolt cyclic stresses, mean stresses and factors of safety for Multiphase alloy are summarized in Table 4. As is again evident by comparing Tables 2 and 4, a significant increase in factor of safety is obtainable with Multiphase alloy and rolled threads.

Thread Form	Load Case	σ_r', psi	σ_m', psi	FS'
Cut	Unloaded	590	75730	3.12
Cut	Stg. I Loaded	3310	72930	2.24
Cut	Stg. II Loaded	5070	67510	1.95
Rolled	Unloaded	550	70570	3.50
Rolled	Stg. I Loaded	3080	67960	2.94
Rolled	Stg. II Loaded	4730	62910	2.76

TABLE 4

Summary of Cyclic and Mean Stresses and Factors of Safety Multi-Phase Alloy MP35N*. F_{in} = 4593 lbf.

CONCLUSION

Use of elastic joint analysis, in conjunction with dynamic stress analysis, affords the compressor designer a reliable method in determining fastener factor of safety. A bolt material option was offered that provided significant increase in bolt factor of safety. Elastic joint analytical techniques may be further applied in cases of gasket design. (9)

NOTATION

A	Element area, in^2
A_b	Bolt cross sectional area, in^2
A_p	Effective cross sectional area of clamped part, in^2
d	Bolt nominal diameter, in
E	Modulus of elasticity, lbf/in
E_b	Modulus of elasticity of bolt, lbf/in
E_p	Modulus of elasticity of clamped part, lbf/in
ER	Endurance ratio of material
F_b	Total load on bolt, lbf
F_{in}	Bolt pre-load on clamped assembly, lbf
F_p	Net residual load on clamped part, lbf
FS	Factor of safety
FS_c	Factor of safety for cut threads
FS_r	Factor of safety for rolled threads
K	Element spring constant, lbf/in
K_b	Bolt spring constant, lbf/in
K_r	Spring constant of clamped part element, lbf/in
K_p	Spring constant of clamped part, lbf/in
L	Element length, in
L_b	Effective length of bolt, in
L_p	Effective length of clamped part, in
P	Load, lbf
P_b	Double amplitude cyclic load on bolt, lbf
P_{et}	Applied external dynamic load, lbf
P_p	Double amplitude cyclic load on clamped part, lbf
P_m	Mean load on bolt, lbf
P_r	Cyclic load on bolt, lbf

SCF	Stress concentration factor of bolt thread
T	Set-up torque, lbf-ft
T_f	Operating temperature on bolt, ^{0}F
ΔL	Change in element length, in
σ_m	Mean stress, lbf/in^2
σ_r	Cyclic stress, lbf/in^2
σ_{ult}	Ultimate tensile strength of bolt material, lbf/in^2
'	Prime notation denote values using alternate material

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