2000

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DYNAMIC SIMULATION OF HFC-134a FLOW THROUGH ADIABATIC AND NON-ADIABATIC CAPILLARY TUBES

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ABSTRACT

A theoretical model for predicting adiabatic and non-adiabatic capillary tube performance with HFC-134a is presented. The model is based on the mass, momentum and energy conservation equations written in one-dimensional and transient differential form. The resulting set of partial differential equations is integrated by the finite-volume technique and solved by successive substitution. The transient behavior as well as the spatial distribution of the mass flow rate, specific enthalpy, pressure and vapor quality are presented and discussed. Comparisons between the transient and quasi-steady modeling approaches and between the transient behavior of adiabatic and non-adiabatic capillary tubes are also presented.

INTRODUCTION

The capillary tube is a constant area expansion device widely used in vapor-compression refrigeration systems (e.g. household refrigerators and freezers, dehumidifiers and room air conditioners). Capillaries can be of two kinds namely, the adiabatic and the non-adiabatic. In the former the tube is fully insulated while in the latter it forms a counter-flow heat exchanger with the suction line.

Although physically simple the capillary tube is a very complex expansion device. The flow in such a device offers several challenges for a phenomenological description: turbulence, heat transfer, phase-change, sonic and metastability effects all occur in the flow through capillary tubes.

During the last few years the behavior of adiabatic and non-adiabatic capillary tubes has been studied extensively, both experimentally and theoretically, at the Federal University of Santa Catarina - Brazil (Boabaid Neto, 1994, Gonçalves, 1994, Mezavila, 1995, Seixlack, 1996, Mendonça, 1996 and Zangari, 1998). The problem has always been treated as steady flow, although steady-state conditions are rarely attained in on-off controlled refrigeration systems.

The objective of this study is to present a mathematical model for transient and steady simulations of the thermal and fluid-dynamic behavior of the refrigerant flow through adiabatic and non-adiabatic capillary tubes. The model employs HFC-134a as the working fluid but it can be easily extended to other refrigerants of interest.

MATHEMATICAL MODEL

Simplifying Assumptions

The model was developed with the following key assumptions:

- The capillary is a straight, horizontal and constant cross-sectional area tube
- The viscous compressible two-phase flow is one-dimensional in the axial direction
- The heat diffusion and the viscous dissipation are neglected
- The pressure drop along the length of the suction line is neglected

- The suction line is well insulated from the surrounding air
- The metastable flow effects are neglected

Governing Equations

The fundamental equations governing the non-adiabatic capillary tube flow are derived from the mass, momentum and energy conservation laws:

$$\frac{\partial \rho}{\partial t}+\frac{\partial G}{\partial z}=0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho u)+\frac{\partial}{\partial z}(Gu)=-\frac{\partial p}{\partial z}-\tau_w\frac{P_i}{A} \tag{2}$$

$$\frac{\partial}{\partial t}(\rho h_\circ)+\frac{\partial}{\partial z}(Gh_\circ)=\frac{\partial p}{\partial t}-q_i''\frac{P_i}{A} \tag{3}$$

$$\left(\rho c_p\right)_w\frac{\partial T_w}{\partial t}=q_i''\frac{P_i}{A_w}-q_e''\frac{P_e}{A_w} \tag{4}$$

where,

$$G=\rho u \quad ; \quad \tau_w=f\,Gu/8 \quad ; \quad q_i''=\lambda_i(T-T_w) \quad ; \quad q_e''=\lambda_e(T_w-T_s) \quad ; \quad h_\circ=h+\tfrac{1}{2}u^2$$

and G is the refrigerant mass flux [kg/s.m^2], u is the mean flow velocity [m/s], ρ is the refrigerant density [kg/m^3], p is the refrigerant pressure [kPa], T is the refrigerant temperature [K], T_w is the wall temperature [K], T_s is the suction line fluid temperature [K], h is the refrigerant specific enthalpy [kJ/kg], h_o is the refrigerant stagnation enthalpy [kJ/kg], τ_w is the shear stress at the tube wall [kPa], f is the friction factor, q" is the heat flux [W/m^2] at the internal (i) and external (e) surfaces, λ is convective the heat transfer coefficient [W/m^2.K], P is the tube perimeter [m], A is the tube cross-sectional area [m^2], and A_w is the wall cross-sectional area [m^2].

A similar set of equations was also derived for the suction line flow (Hermes, 2000). The solution of the complete set of equations provides the temporal and spatial distribution of the refrigerant mass flow rate, pressure, and specific enthalpy as well as the wall temperature distribution. The refrigerant pressure and specific enthalpy at the inlet of the capillary tube and the refrigerant pressure at the outlet of the capillary tube are the model's boundary conditions. The continuity equation's boundary condition is obtained by means of a iterative procedure based on the evaporating or sonic pressure, depending on the refrigerant status at the outlet of the capillary tube. Fauske's choked flow criterion (Fauske, 1962) is used to evaluate whether the flow is critical or not. The initial conditions are inferred from the system equalization pressure (4,9 bar) and from the surrounding air temperature (43°C).

Friction Factor

The single-phase flow friction factor was calculated using Churchil's equation (Churchill, 1977). The two-phase flow friction factor was evaluated using a variation of Erth's equation (Erth, 1970) where the capillary tube inlet properties (Reynolds number and quality) are replaced by local properties at the entrance of each control volume (Mezavila, 1995).

Heat Transfer Coefficients

Gnielisnki's equation (Gnielisnki, 1976) was adopted to calculate the single phase flow convective heat transfer coefficient in the capillary tube. This equation was also applied to the single phase flow in the suction line, but using Blasius' friction factor and an equivalent diameter for annular flow (Kakaç, 1987). Pate's correlation (Pate, 1982) was applied to the two phase flow region of the capillary tube and suction line. The heat transfer coefficient between the capillary tube and the surrounding air was calculated from Churchil & Chu's equation (Churchil & Chu, 1975).

Thermodynamic and Thermophysical Properties

Thermodynamics properties values of refrigerant HFC-134a required by the model in solving the governing equations were internally evaluated by the model as a function of refrigerant pressure and specific enthalpy. Four basic equations were employed (Wilson & Basu, 1988): equation of state, i.e. the relation between the pressure, temperature and specific volume (or density); saturation vapor pressure, specific heat capacity at constant pressure, and density of the saturated liquid as functions of temperature. The thermophysical properties of the refrigerant were obtained from a polynomial fit of data from a commercially refrigerant property data base (Gallagher et al., 1993).

NUMERICAL SOLUTION

The numerical solution was carried out by discretization of the one-dimensional continuity, energy and momentum equations based on a control volume formulation The numerical method employed in this work is similar in many aspects to that developed by Escanes et al. (1995), with the discretization nodes located at the inlet and outlet sections of the control volumes.

In order to assure numerical stability, a completely implicit scheme was employed to integrate the transient terms. The whole set of equations can be generically represented by equation (5). The integration of this equation over a control volume of length Δz (see Figure 1) provides,

$$\left(\overline{\rho\phi} - \overline{\rho\phi}^{\circ}\right)\frac{\Delta z}{\Delta t} + (G\phi)_k - (G\phi)_{k-1} = \bar{S}\Delta z \tag{5}$$

where, the subscript (°) denotes the previous time-step value of a specific dependent variable (ϕ) and the superscript ($^{-}$) represents the average value of this variable over the control volume. The average value was determined from the trapezoidal rule:

$$\overline{\rho\phi} = \frac{1}{\Delta z}\int_{z_{k-1}}^{z_k} \rho\phi dz = \frac{\rho_k\phi_k + \rho_{k-1}\phi_{k-1}}{2} \tag{6}$$

The same approach was also employed for the source term S. The trapezoidal rule was employed in this work as the integration method because it can better characterize the high pressure and enthalpy gradients at the outlet of the capillary tube than lower order methods. The suction line and the capillary tube computational meshes were coupled accordingly to the scheme shown in Figure 2. The proposed numerical solution requires less CPU time and is easier to be implemented than the traditional finite volume method (Patankar, 1980).

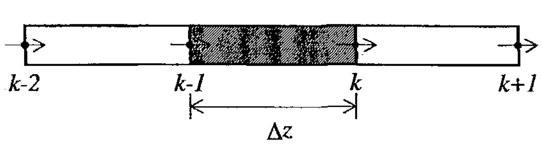

Figure 1 - Control volume

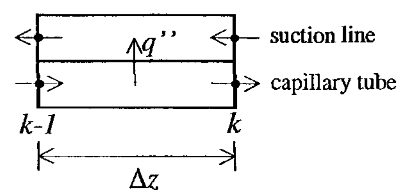

Figure 2 - Conflation of computational meshes

Computational Grid

A non-uniform computational grid was employed due to the high gradients produced at the end of the capillary tube. The computational grid was generated by the following equation (Escanes et al., 1995):

$$\Delta z_k = \frac{L}{tanh\,\xi}\left[tanh\left(\xi\frac{k}{n}\right) - tanh\left(\xi\frac{k-1}{n}\right)\right] \tag{7}$$

where ξ is the concentration factor, which has a value greater or equal to zero ($\xi = 0$ indicates a uniform grid), L is the tube length and n is the number of control volumes.

Figure 3 illustrates a grid with 50 volumes and $\xi = 2.5$, for a 4.5 m long capillary tube. Both the capillary tube and the suction line grids used in this work were generated with 400 volumes and $\xi = 5.0$.

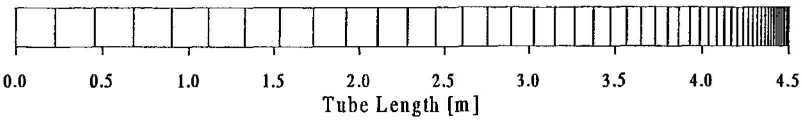

Figure 3 - Non-uniform grid (50 volumes and $\xi = 2.5$).

Numerical Algorithm

As the refrigerant flow can reach the sound speed at the capillary tube exit and the sonic pressure is previously unknown, an iterative procedure is required to determine the mass flow rate through the capillary tube. At each time step and with a guessed value for the refrigerant mass flow rate, the governing equations are integrated in the flow direction to calculate the capillary length. A new value for the mass flow rate is then calculated by the secant method, employing the difference between the calculated and actual capillary length as the error function to be minimized. The size of the smallest control volume is employed as the convergence criterion. The discretized equations are solved by successive substitution until the relative error between two consecutive iterations for the refrigerant stagnation enthalpy, mass flow rate, pressure and wall temperature are less than 10^{-6}. A 0.5 linear under-relaxation factor was employed for all variables in order to improve the convergence process. A similar procedure was employed for the suction line.

NUMERICAL RESULTS

Capillary Tube Geometry

The geometric characteristic of the concentric capillary tube- suction line heat exchanger considered in this work are presented in Table 1 and in Figure 4.

Table 1 - Geometric features of capillary tube and suction line

Dimension	Unit	Value
Capillary tube inner diameter	[mm]	0.674
Capillary tube outer diameter	[mm]	2.10
Capillary tube length	[m]	4.5
Adiabatic inlet length	[m]	2.55
Heat exchanger length	[m]	1.94
Suction line inner diameter	[mm]	4.80
Suction line outer diameter	[mm]	6.12

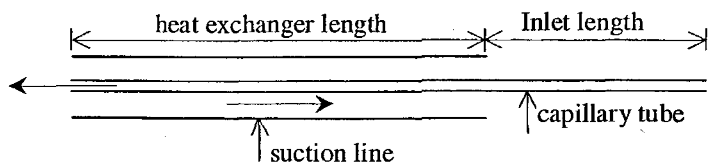

Figure 4 - Capillary tube – suction line heat exchanger geometry

Steady-State Simulation and Model Evaluation

The current model results were compared to the ones provided by the CAPHEAT computer code (Mezavila & Melo, 1996). Three capillary tube inlet conditions were examined: Case 1 – two phase flow (quality = 0.051); Case 2 – Saturated liquid flow and; Case 3 – subcooled flow (subcooling = 2.64°C). For all situations the condensing and evaporating pressures were 15.53 bar and 1.29 bar, respectively. For the adiabatic condition a good agreement between the current model and the CAPHEAT program mass flow rate predictions have been found (see Table 2). On the other hand, a small discrepancy between the sonic pressure values have been observed. This reflects the different sizes of the control volumes at the end of the capillary tube; less than 0.1mm in the present model and 1.0 mm in the CAPHEAT program. For the non-adiabatic situation a reasonable agreement between the mass flow rates predicted by both models have also been found, but the difference between the sonic pressure predictions is a little higher than the one observed for the adiabatic capillary tube (see Table 3). This again can be explained by the different sizes of the control volumes at the end of the capillary tube.

Table 2 – Current model versus CAPHEAT program – adiabatic capillary tube

		Case 1	Case 2	Case 3
Mass flow rate [kg/h]	Present work	1.693	2.003	2.334
	CAPHEAT program	1.690	2.017	2.313
	Difference	+0.003	-0.014	+0.021
Sonic pressure [bar]	Present work	1.38	1.59	1.82
	CAPHEAT program	1.36	1.76	1.88
	Difference	+0.02	-0.17	-0.06

Table 3 - Current model versus CAPHEAT program – non-adiabatic capillary tube

		Case 1	Case 2	Case 3
Mass flow rate [kg/h]	Present work	1.865	2.241	2.649
	CAPHEAT program	1.805	2.183	2.514
	Difference	+0.060	+0.058	+0.135
Sonic pressure [bar]	Present work	1.28	1.45	1.64
	CAPHEAT program	1.45	1.62	1.78
	Difference	-0.16	-0.17	-0.14
Suction line outlet temperature [°C]	Present work	36.7	37.4	37.8
	CAPHEAT program	37.3	37.8	37.8
	Difference	-0.6	-0.4	0.0

The refrigerant flow for the three situations under study are illustrated in Figures 5 and 6 for an adiabatic and for a non-adiabatic capillary tube, respectively. For the adiabatic capillary tube the refrigerant specific enthalpy remains constant along almost the entire length of the capillary with a small drop close to the tube exit due to the flow acceleration. For the non-adiabatic capillary tube the refrigerant specific enthalpy remains constant along the adiabatic inlet length of the capillary tube. Along the heat exchanger length the refrigerant specific enthalpy is reduced due to the heat exchanges with the suction line. Close to the capillary end the refrigerant pressure is considerably reduced but the refrigerant enthalpy remains almost constant, once the heat transfer to the suction line is not considerable. The enthalpy variation at the outlet region is again due to the rise of kinetic energy, as in the adiabatic case. As expected, the quality is reduced when compared to the adiabatic situation resulting in a higher liquid flow.

Dynamic Simulation – Quasi-Steady and Dynamic Adiabatic Models

A comparison of the current model with a quasi-steady one was conducted in order to quantify the influence of the transient terms. The quasi-steady result was obtained by solving the set of equations using a very large time-step (Δt ~ 10^{200}).

Figure 7 shows the temporal variation of the mass flow rate through an adiabatic capillary tube computed with the transient and quasi-steady models. For the transient model, both the inlet and outlet mass flow rates are shown. It can be seen that the mass flow rate predicted by both models are quite close and that they get even closer as the time goes by. It can also be observed that the inlet and outlet mass flow rates predicted by the transient flow rates are always fairly close.

Figure 7 also shows that the mass flow rate rapidly increases to a maximum value, decreases to a local minimum and then rises again, at a continuously lower rate, to a steady-state condition. The valley can be explained by the variations in the refrigerant viscosity and density at the capillary tube inlet. At the simulation start-up, the refrigerant quality is high and consequently the fluid viscosity and density are low. As liquid is forming at the tube inlet, the viscosity increases, reducing the mass flow rate. On the other hand, the density rises with the presence of liquid, increasing the mass flow rate. The minimum is then explained by the combination of these two effects. As expected, the quasi-steady mass flow rate lies between the inlet and outlet transient mass flows rates. However, at the valley, the quasi-steady values are lower than the transient counterpart, as the flow immediately reflects the boundary condition variations.

Figure 8 illustrates the spatial and temporal variations of the refrigerant quality. The quality profiles are quite similar for all times; with a minor change from the start-up to the steady-state condition. For the 5s profile, a small difference between the refrigerant qualities calculated by the two models can be observed. After 30s there is almost no distinction between the transient and quasi-steady quality profiles.

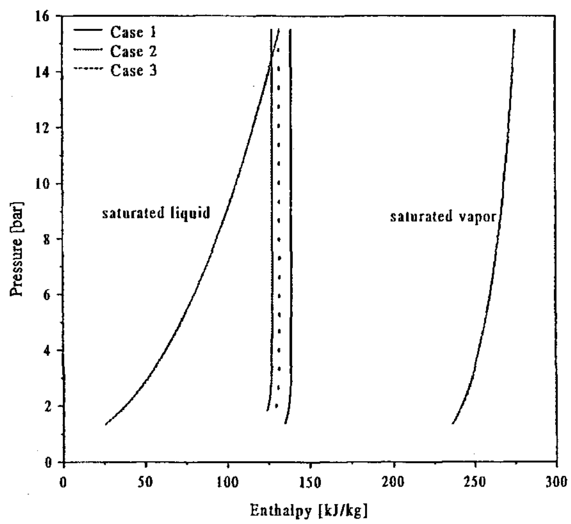

Figure 5 - Adiabatic steady-state results in a p-h diagram

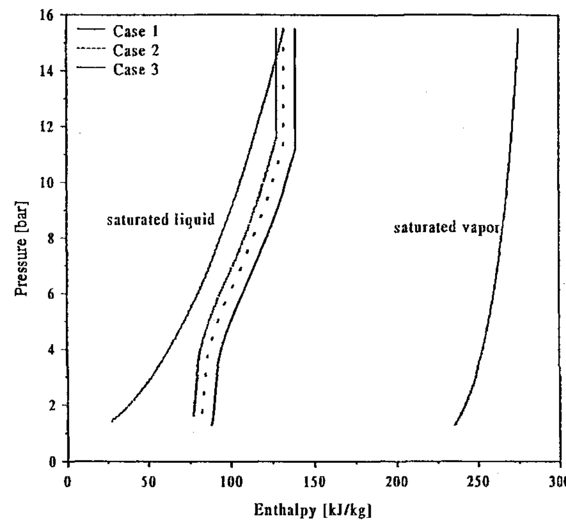

Figure 6 - Non-adiabatic steady-state results in a p-h diagram

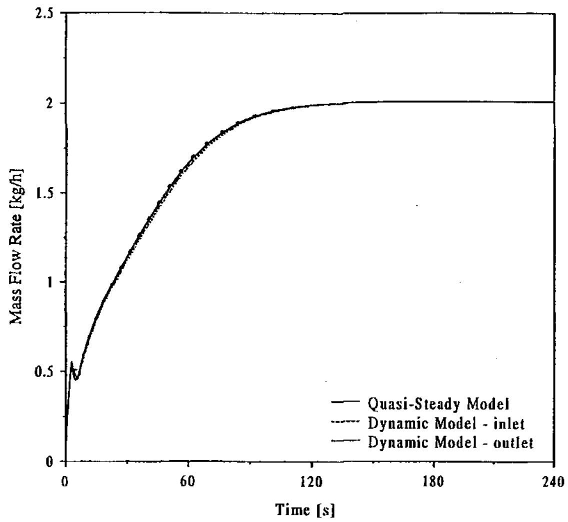

Figure 7 - Mass flow rate variation with time (adiabatic model)

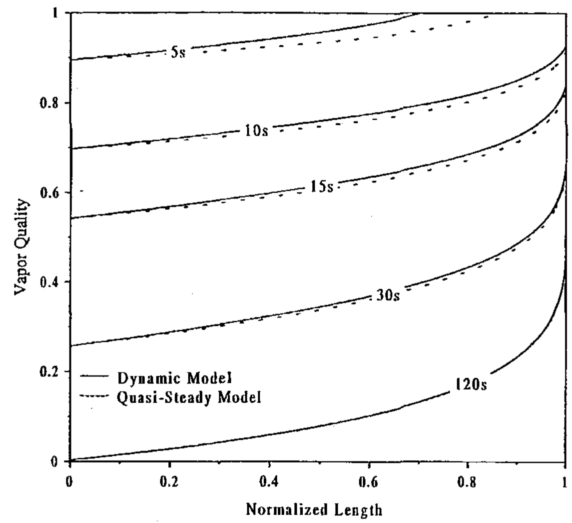

Figure 8 - Vapor quality profiles at various times (adiabatic model)

Dynamic Simulation – Quasi-Steady Non-Adiabatic Model

As the quasi-steady and transient results are fairly close, only the quasi-steady results are shown for the non-adiabatic flow. Figure 9 shows the temporal variation of the mass flow rate through the capillary tube. As in the adiabatic case, the mass flow rate increases rapidly to a maximum value, decreases to a local minimum and then gradually increases to a steady-state condition. A comparison of the adiabatic (Figure 7) and non-adiabatic flows (Figure 9) reveals that the mass flow rate variation is higher in the latter, once the heat transfer to the suction line reduces the refrigerant quality.

Figure 10 shows the quality profiles for the non-adiabatic capillary flow at different times. It can be seen that the quality profiles are quite similar for all times with a continuous change from the start-up to the steady-state condition.

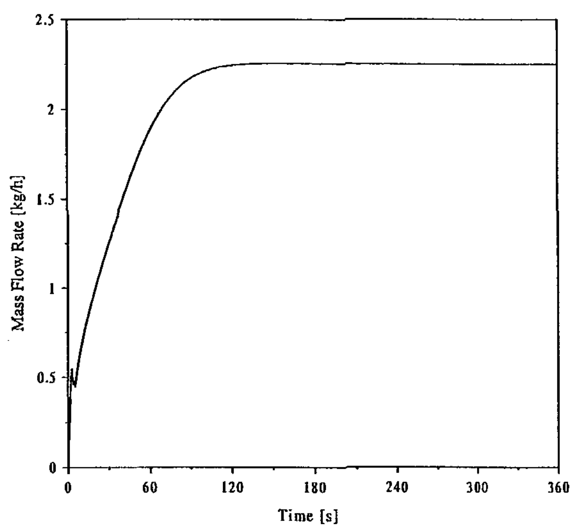

Figure 9 - Mass flow rate variation with time

(non-adiabatic model)

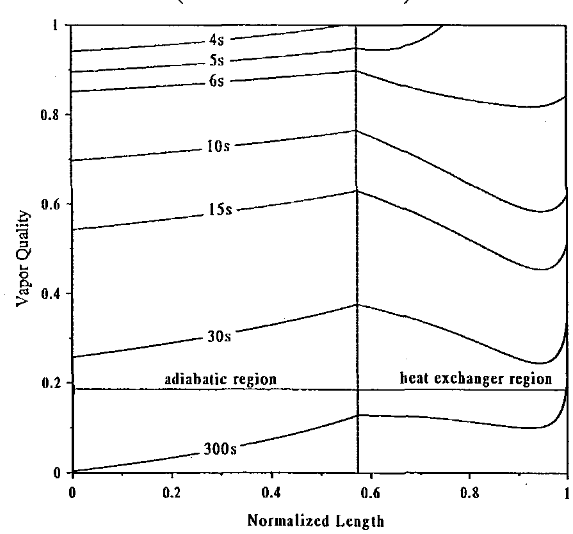

Figure 10 - Vapor quality profiles at various times (non-adiabatic model)

CONCLUDING REMARKS

A transient model for predicting adiabatic and non-adiabatic capillary tube performance was presented. The simulation was implemented on the basis of a control volume formulation of the conservation laws. The most relevant feature of this model is the computation of the spatial and temporal distributions of the refrigerant pressure, specific enthalpy, vapor quality and mass flow rate.

The transient analysis was conducted with imposed boundary conditions (Hermes, 2000). A comparison with a quasi-steady model revealed that the transient terms have a minor influence on the mass flow rate. The quasi-steady model was then employed to simulate the more complex non-adiabatic capillary tube flow. State-state comparisons with the CAPHEAT program were also performed and a reasonable level of agreement has been found.

In recent years, knowledge regarding the transient behavior of refrigerating systems has become of great importance. It is therefore necessary to predict the system performance in advance and so to optimize the system design by minimizing the energy consumption. This paper presents a transient numerical model for adiabatic and non-adiabatic capillary tubes that can be used as part of a refrigerating system simulation tool, not only to reduce the energy consumption but also to significantly reduce the number of real refrigerators tests needed.

ACKNOWLEDGMENTS

The authors are grateful to *Multibrás Eletrodomésticos S.A.* for sponsoring this research program and technical discussions, in particular, Mr. Marco E. Marques. The authors are also grateful to Mr. Roberto H. Pereira (*Embraco S.A.*) for his support during this project. The continued support for this research program from *Conselho Nacional de Desenvolvimento Cientifico* (CNP_q) is also duly acknowledged.

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