A Study of Resonant State of Pressure Pulsation in a Simple Suction Pipeline of Reciprocating Compressor

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A STUDY ON RESONANT STATE OF PRESSURE PULSATION IN A SIMPLE SUCTION PIPELINE OF RECIPROCATING COMPRESSOR

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ABSTRACT

Analysis and experiment show that to supercharge the suction volume of a cylinder by means of pressure pulsation is not certain to result in delay of the valve closing. To arrange a proper suction pipeline and valve cavity, we can not only get supercharge of the cylinder, but also get economical specific power consumption, while valve working condition is good too. In this paper the methods of calculating pipe length and amplitude of pressure pulsation as well as loss of pressure at resonant state are recommended. Experiment of verification was done on a double-acting air compressor type L 2-10/7.

INTRODUCTION

It is the nature that a reciprocating compressor sucks and discharges intermittently, thereby it generates pressure pulsation in the piping system.
Generally, the pressure pulsation results in vibration of the piping system and must be limited within a permitted value.
But sometimes it can also be made use of supercharging in the suction volume of the cylinder or reducing specific power consumption, while it doesn't make the valve working condition worsen. In the reference [1], [6], [8], [9], [10], these cases were studied. In this paper we studied some rules of resonance and its influence on movement of the valve plate.

ANALYSIS

The pressure pulsation comes from intermission of the suction and variable motion of the piston during suction process.
Excitment to be stirred up by the piston varies with time as following

Single-acting (cycle 2π)

$$\left.\begin{array}{ll} u(t) = 0, & \theta \geq \omega_0 t \geq 0 \\ u(t) = r\,\omega_0\,(\sin\omega_0 t + \frac{\varepsilon}{2}\sin 2\omega_0 t), & \pi \geq \omega_0 t > \theta \\ u(t) = 0, & 2\pi \geq \omega_0 t \geq \pi \end{array}\right\} \quad (1)$$

double-acting (cycle π)

$$\left.\begin{array}{ll} u(t) = 0, & \theta \geq \omega_0 t \geq 0 \\ u(t) = r\,\omega_0(\sin\omega_0 t + \frac{\varepsilon}{2}\sin 2\omega_0 t), & \pi \geq \omega_0 t > \theta \end{array}\right\} \quad (2)$$

The excitation function can be expanded in the Fourier series

$$u(t) = \frac{\bar{U}}{2} + \sum U_m \sin(nm\,\omega_0 t + \varphi_m), \quad m = 1,2,3,\text{---}$$

for single-acting $n = 1$
for double-acting $n = 2$

Excitation source of each order harmonic component should be regarded at the top of the piston, so that both single and double acting cylinders become ordinary volume which is separated by valve, and there is only a concentrated pressure loss like orifice at the place of valve. The volume of cylinder should be replaced by a equivalent volume. The equivalent volume of the cylinder equals the steady component of harmonic analysis formula as follows

$$\left.\begin{array}{l} V(t)=A_0r\left\{1-\cos\omega_o t+\frac{e}{4}(1-\cos 2\omega_o t)\right\}+V_r, \\ \qquad\qquad\qquad\qquad \theta\le\omega_o t\le\pi \\ V(t)=0, \quad \text{single-acting}\begin{cases}0<\omega_o t<\theta\\ \pi<\omega_o t<2\pi\end{cases} \\ \qquad\quad \text{double-acting } 0<\omega_o t<\theta \end{array}\right\} \quad (3)$$

When the chink of suction valve is regarded as a pipe with zero length, cylinder and pipeline are considered as a Helmholtz Resonator.The schematic diagram of the simple suction system without strainer is shown by fig.1. Diagram u-ωt of the excitation function is shown by fig.2.

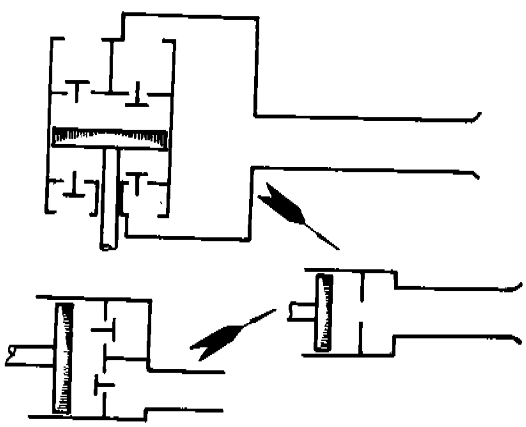


Fig.1 The simplified suction pipeline without strainer

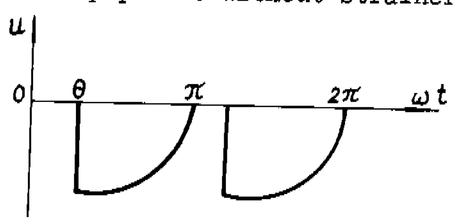


Fig.2 The excitation function

The function u(ω_ot) is smooth section by section and has a first kind of point disconnected, therefore in its harmonic analysis formula the decrease rate of harmonic coefficient of each order is not lower than 1/m.

In double-acting compressor first order harmonic takes a greater portion , The actual value of the L 2 - 10/7 compressor, expanded in Fourier Series, has proved the first order harmonic amplitude is 103.33 % of the general amplitude.

The wave composed of the first and the second order harmonics shown in fig.3 shows that the general amplitude is almost not affected by the second order harmonic.

The seoond harmonic only change the shape of the composite wave, so that the second orders can be negligible. In fig.3 θ =5 4, r = 60mm, A = 0.0593957m, ω_o=102.63 1/s, ω = 205 1/s.

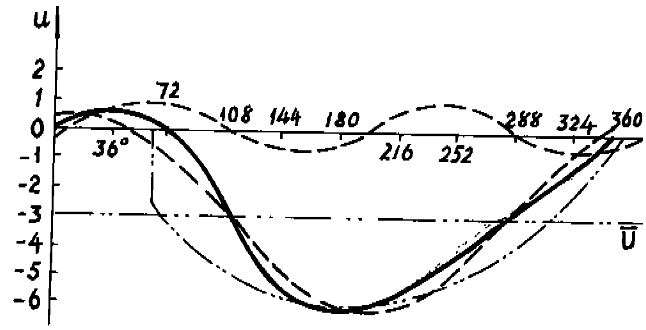


Fig.3 The excitation function of L2-10/7 Type compressor compares with the first and second order pulsation

—— the composite of first and second order

--- the first and the second order

-·-·- the excitation function

According to the reference [4], when gas column resonates in a smaller diameter pipe the input damping is in direct proportion to $(m\ \omega_o)^2$.

When suction system resonates, damping of suction system is in direct proportion to the square of velocity.

The method of harmonic analysis can be applied to measured pressure curve. The rate of first order harmonic amplitude to the general amplitude is larger than 95 %. The pressure curve measured at resonant region is shown in fig.4. These curves are similar to sine wave. It illustrates that the first order harmonic is the main part and other orders may be negligible. It also explains that the damping makes first order harmonic component smaller distortion.

$V_s = 4V_c$
d =150mm
l =1240mm

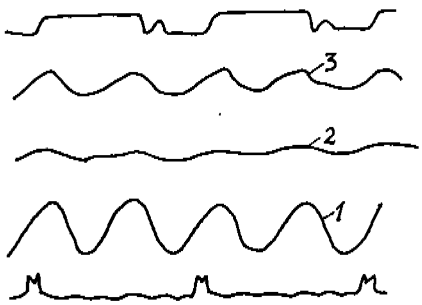


Fig.4a The case of largest amplitude
1,2,3 — measured points shown by fig.10

$V_s = 4V_c$
d =150mm
l =1750mm

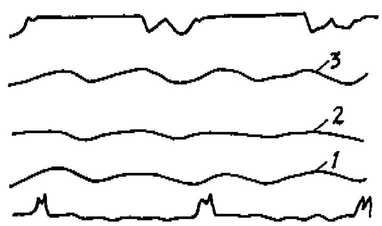


Fig.4b The case of valve closing is moved up

$V_c = 4V_c$
d =150mm
l =810mm

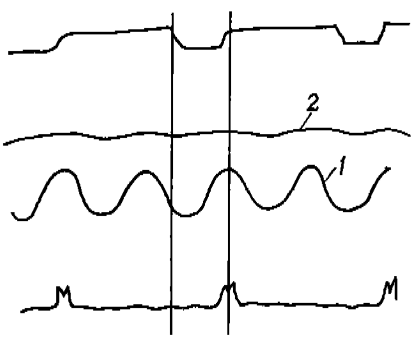


Fig.4c The case of valve opened delay

Due to the reason above, it is exact enough for calculating in engineering that the general wave is simplified as the sum of a first order harmonic and a steady flow, so that the following formula can express excitation function jast as the solid line of the fig.5.

$$u_0 = \tilde{u}_0 + \bar{U}_0 = U_0 \sin(\omega t + \varphi_0) + \bar{U}_0 \qquad (4)$$

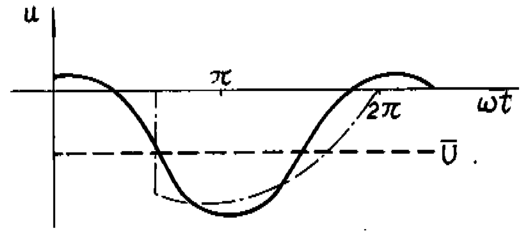


Fig.5 The diagram of simplified excitation function

CALCULATION OF RESONANT LENGTH OF THE SUCTION PIPELINE

In the reference [2], [3] the calculation of natrural frequency of gas column in complicated piping system is recommended, but how to deal with the condition that valve opens intermittently and cylinder volume change continuously, isn't mentioned. From the experiment by Sodel et., even if the amplitude of pressure pulsation is close to 30 % of the nominal pressure, acoustics equation is still applicable. Here the amplitude of pressure pulsation at resonant state in compressor isn't larger than 30 %, so that the equation can be applied at resonance. In actual compressor system the steady flow velocity isn't zero, in order to investigate the influence of steady flow velocity, the following equations are solved.

$$\rho_0 \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} - a\rho_0 u \qquad (5)$$

$$\frac{\partial p}{\partial t} = -c^2 \rho_0 \frac{\partial u}{\partial x} - \bar{U}_i \frac{\partial p}{\partial x} \qquad (6)$$

$$u = \bar{U} \frac{A_0}{A_i} - \tilde{u} = \bar{U}_i + \tilde{u}$$

ρ_0 , $\bar{U}_i$ average value at any section of suction pipeline. The solution of above equation is

$$\begin{pmatrix} A_1 \\ U_1 \end{pmatrix} = e^{-j\varphi M_0} \begin{pmatrix} \cos\varphi & j\sin\varphi \\ j\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} A_2 \\ U_2 \end{pmatrix} \qquad (7)$$

$$M_0 = \frac{U}{c}, \quad \varphi = \frac{\omega}{c} l, \quad A = \frac{p}{c\rho}$$

The difference between the formula above and the equation without steady flow velocity is only the term $e^{-j\varphi M_0}$. It illustrates that steady flow velocity makes the phase between pressure and velocity turn round an angle along the pipe, in general, value M_0 is very small, so that the amplitude and velocity almost doesn't change. Thus acoustics equation can be applied.

How to deal with valve and volume of cylinder while calculating resonant pipeline length is a question which is worth studying. It is evident that place of valve was regarded as a closed end doesn't accord with the real condition. Because the section of valve in comparison with the section of pipeline isn't very small and, for a double-acting compressor the valve is closed merely in 30 % of a cycle period, that is to say, linking intermittently of two volumes and changing continuousely of the cylinder volume must be accounted. The changing cylinder volume should be replaced by an equavalent volume, jast as shown by formula (3) and, the top of the piston is regarded as a closed end.

The equation to find out resonant length of pipeline shown by fig.6a is

$$tg\frac{\omega l}{c} = \frac{cA}{\omega V} \tag{8}$$

Shown by fig.6b is

$$tg\frac{\omega l}{c} = \frac{A_1\cos kl_2 - \frac{A_1}{A_2}\sin kl_2\frac{\omega V}{c}}{A_2\sin kl_2 + \frac{\omega V}{c}\cos kl_2} \tag{9}$$

where $V = V_s + V_o$

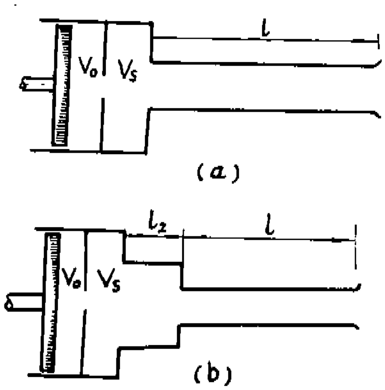


Fig.6 sketch of the simplified cylinder and suction pipeline system

$$V_o = \frac{Ar}{\pi}\int_{\theta}^{\pi}\left\{1-\cos\omega_o t + \frac{e}{4}(1-\cos 2\omega_o t) + V_r\right\} d\omega_o t$$

With the assumption of opened end p = o, the calculating method mentioned above is performed. In fact, pressure pulsation could travel out in spherical surface wave at the opened and, thus damping of the inlet of pipeline doesn't eqal zero. Frome reference [4] damping of opened end as follows

$$Z \simeq \rho c\left(1-\frac{2J_1(W)}{W}\right) - J\rho c M(w) \tag{10}$$

here

$W = \frac{\omega D}{c} = \frac{2\pi D}{\lambda}$

J(w) is the Basal Function

$M(w) = \frac{\pi}{4}\int_0^{\pi/2}\sin(w\cos\alpha)\sin^2\alpha\, d\alpha$

By and large in compressor $w < 0.5$, then limit value of impedance of acoustics at opened end is the following value

$$Z \simeq \frac{\rho D^2\omega^2}{8c} + \frac{i\omega 4\rho D}{3\pi} \tag{11}$$

When the stimulating frequency resulted from the compressor is lower, damping is smaller, thus impedance to the pipe inlet is $\frac{4\rho D}{3\pi}\omega$ shown by fig.7. Namely radiation effect of inlet is equal to adding a section pipe as long as $4D/3\pi$

$$l = l_o + \frac{4D}{3\pi} \tag{12}$$

When the width of hinder in inlet shown by fig.7. is far less than wavelength of pressure pulsation, impedence is 0.3

$$l_o = l - 0.3D \tag{13}$$

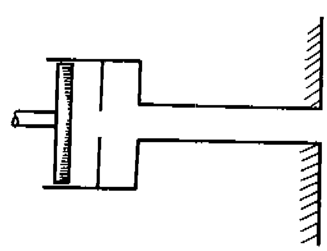


Fig.7 Construction of inlet

From formula (13), the larger the pipe diameter, the greater the radiation effect. calculated values in comparison with experemental values is in table 1.

table 1.

ω_0	V	D	l_0	l_0'	error %
205.25	0.04362	100	425	405	4.6
205.25	0.04362	150	991	980	1.1
205.25	0.02752	100	660	645	2.3
205.25	0.02752	192	1014	990	2.5
205.25	0.02752	150	1260	1240	1.5

* l_0 —calculated, l_0' —measured

CALCULATION OF RESONANT AMPLITUDE OF PLESSURE PULSATION OF SUCTION SYSTEM OF ACTUAL COMPRESSOR

In pipeline the vibration mode is standing wave, then velocity can be written as follows at the first order resonance

$$u = U_0\, U(x)\, Q \sin(\omega t - \frac{\pi}{2}) + \frac{A_0}{A}\bar{U} \qquad (14)$$

The U(x) is ribration mode of the velocity with given boundary condition and without damping, it can be got from wave equation at $U_l = 1$, $p_l = 0$.
The Q is a constant depending on piping system and frequency. It expresses what degree a system at resonant state amplicfies pulsation to. Q can be got frome energy equation. as a result

$$Q = \sqrt{\frac{3\pi A_0 \rho C p(x_0)}{8 B_1 U_0}} \qquad (15)$$

where

$$B_1 = \int_l \frac{\rho A a}{2D} U^3(x)\,dx + \Sigma \frac{\rho A_i \xi_i}{2} U^3(x_i)$$

In pulsation flow, concentrated loss can be calculated from

$$p_i - p_i' = \frac{\rho}{2}\xi_i \left(\bar{U}\frac{A_0}{A_i} + \tilde{u}\right)^2 sgn\left(\frac{A_0}{A_i}\bar{U} + \tilde{u}\right) \qquad (16)$$

The sub i shows the order of the pipe sections; the P_i is instantaneous pressure in front of section i, the p_i' is behind that section. Formula (16) is expanded in Fourier series, and the steady term is $(\bar{p}_i - \bar{p}_i')$, first order term is $(\tilde{p}_i - \tilde{p}_i')$.

Let $g_i = \frac{A_i}{A_0}\frac{U(x_i)}{\bar{U}}$

when $g_i < 1$, from harmonic analysis

$$\Delta\bar{p}_i = -\frac{\rho}{2}\xi(2 + g_i)\left(\frac{A_0}{A_i}\bar{U}\right)^2$$

$$\Delta\tilde{p}_i = \frac{\rho}{2}\xi\, 2g\left(\frac{A_0}{A_i}\bar{U}\right)^2$$

when $g_i > 1$

$$\Delta\bar{p}_i = -\frac{\rho}{2}\xi\left(\frac{7.66\, g_i + 0.562\pi}{\pi}\right)\left(\frac{A_0}{A_i}\bar{U}\right)^2$$

$$\Delta\tilde{p}_i = \frac{\rho}{2}\xi\left(\frac{8}{3}g_i^2 + 6\right)\left(\frac{A_0}{A_i}\bar{U}\right)^2$$

Pressure loss calculated by means of this method is about 87 % larger than measured pressure loss, but in the same pipeline the measured pressure loss is about 4 ~ 8 times than the $\Sigma \frac{\rho}{2}\xi\left(\frac{A_0}{A_i}\bar{U}\right)^2$ as turbulent flow loss in pipeline

EXPERIMENT AND ANALYSIS

Experiments have been made on an L2-10/7 type compressor.
Pressure pulsation, valve plate movement and pressure loss are recorded. The distribution of measuring point of pressure is shown by fig.8.

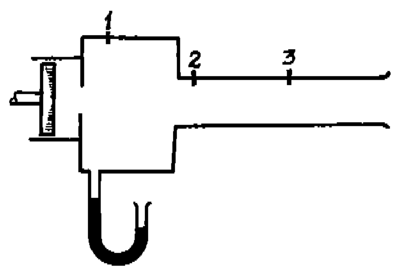


Fig.8 The place of measured points of pressure

In experiments two kinds of valve eavities $V_s = 0.04362\ m^3$, $V_s = 0.02754\ m^3$ in volume and three kinds of pipeline d=100mm, d=150 mm, d=129 mm in diameter were used.

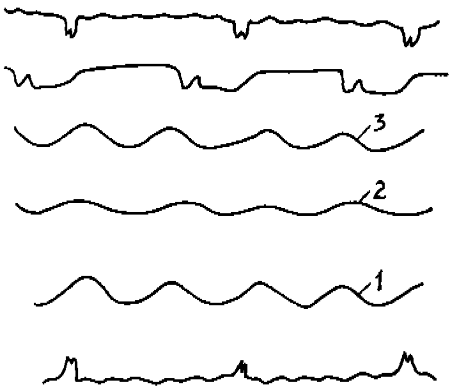
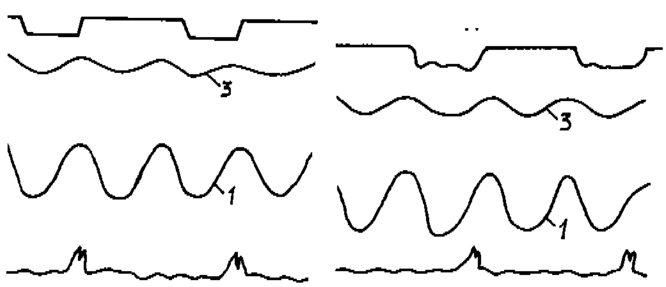


Fig.9 Pressure pulsation of each me measured point

Pressur curve recorded in fig. 9 shows the phases of three measured point are almost same. This illustrates that the assumption of standing wave accords with real condition. The curve is like first order harmonic. These curves shown by fig.4.
During opening of valve, pressure curve in cylinder is parallel with pressure curve in valve cavity this explains that gas pulsations in cylinder and out of cylinder are synchronous. It brings to light that gas in equivalent volume of cylinder joins in pulsation.



$V_s=6V_c$	$V_s=6V_c$
d =150mm	d =150mm
l =860mm	l = 980mm

Fig.10 The largest supercharge case compares with the largest amplitude

The largest pressure loss measured corresponds to the best supercharge case. The length of pipeline is less than the length corresponaded by the largest amplitude case. At the same time pressure amplitud is also less than the largest amplitud. In this case the curve of valve plate movement shows the closing of valve isn't delayed, and valve plate is nestled down valve seat without dashing, valve opening isn't delay too. It can be seen by fig.10.
In case of the largest amplitud closing of valve has been delayed for 12°, the surpercharge isn't the largest, because at the close point pressure isn't maximum.
The closing of valve is delayed principally by pulsation of velocity when the design of valve is pefect.
During closing of the valve the direction of velocity of pulsation is towards inside of cylinder, so that impulse of velocity delays the closing of valve, and vice versa closing of valve is moved up.
Though rising of pressure almost doesn't affect closing of valve, it affects opening of valve strongly. During opening of valve pressure in cavity is droping, then opening of valve could be delayed. The fig.4b shows the case in which closing of valve is moved up. The fig.4c shows the case of valve opened delay. If the pressure is intending to maximum during the process of valve opening, it will help valve plate opening. If the velocity of pulsation is rushing into cylinder with maximum value, it will help valve plate closing, then power consumption during suction period will be decreased.
Generaly the two cases mentioned above don't coincide each other, exept damping and clearance volume vanished.
In case of fig.4b corresponding pipe length is $l=1.35l_o$, figure shows pressure pulsation, also affectes operation of compressor; in case of fig.4c corresponding pipe length is $l=0.628\ l_0$ and pressure pulsation amplitude is 81 % of the maximum. So that only keeping away from the region of

$l=(0.8 \sim 1.2)\ l_0$ to avoid resonance isn't good enough.

CONCLUSION

Frome the resulte of experiment and analysis above, following conclusions can be got

1. To make use of the pressure pulsation supercharge, valve working condition isn't certain to become badly
2. A method of calculating pipe length of gas column resonance is reccmmended.
3. The method of pressure loss colculating coincides with reality basically.
4. The method of calculating resonant amplitude in actual compressor can meet needs of engineering basically.

NOTATION

A Section of pipe
C Adiabatic velocity of sound
D diameter of the suction of pipe
e the ratio of radius of crankshaft to length of connecting rod
l length of pipe
l_0 length of pipe at resonant state
$\bar{p}$ steady component of pressure pulsation at the excitation source
p_a amplitude of first order harmonic pressure pulsation of gas column
p instantaneous valve of pressure of gas column
$\tilde{p}$ instantaneous valve of pressure of first order pulsation of gas column
Q resonant factor of first order
r radius of crankshaft
$\bar{U}$ steady flow velocity at excitation s source
U amplitude of velocity pulsation
U_0 amplitude of first order harmonic component of velocity at excitation source
u instantaneous valve of gas column flow velocity
$\tilde{u}$ instantaneous value of gas column first order pulsation velocity
V_c cyeinder volume
V_r clearance volume
V_s volume of valve cavity
V_e equavilent volume of cylinder
a friction factor
λ wavelength of first order harmonic
ξ coefficient of concentrated pressure losses
ρ gas density
ω_0 angular velocity of the machine
ω first order harmonic angular velocity

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