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STABILITY PROBLEMS IN LOCAL AREA NETWORKS:
A QUALITATIVE APPROACH

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**STABILITY PROBLEMS IN LOCAL AREA NETWORKS:
A QUALITATIVE APPROACH**

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The focus of this research is on the development of rigorous theoretical foundation for qualitative analysis of large-scale, complex systems. Primary motivation for the work is the need for firmly based methods applicable to study complex phenomena (properties) which occur in local computer networks, e.g., congestions and deadlocks, bistability, hysteresis, sudden changes in network behavior, inaccessibility of some regions, divergence, fairness and so forth. Emphasis is on the stability problems since only stable systems may work in practice. Stability definition assumed in this project is broad enough to cover such problems as ergodicity and nonergodicity, partial ergodicity, finiteness of some quantities of interests, practical stability, shape of steady-state distributions (bistability property) and fairness. The variability inherent in most local area networks is such that to make meaningful performance prediction it is necessary to study the evolution of a stochastic system. Such a stochastic approach is assumed throughout the project. The results are applied to study stability of some important local area networks, e.g., asymmetric buffered contention broadcast system, exponential backoff algorithm in a multiaccess network, conflict resolution algorithms in packet radio networks and token-passing networks.

1. INTRODUCTION

The evaluation of computer performance is needed during the entire life of a computer network. Performance is one of the factors that must be taken into account in the design, development, configuration and tuning of a computer network. However, as computer networks become more and more sophisticated new complications arise which pose highly non-trivial design and analysis problems. The most difficult to treat analytically are pathological behaviors of a system, e.g. congestion, deadlocks, bistability, fairness, hysteresis and so on, all of which are almost inevitably associated with all types of computer networks. Analyzing these types of behavior we focus our attention on phenomena in a network. The quantitative analysis (e.g. numerical algorithms of queueing models) that researchers have been doing up to the present seems to be inappropriate, or at least quite restricted, for these types of investigations. This comes from the fact that the models are too complex to be treated analytically, and - what is probably most important - quantitative analysis deals with numbers, values of functions, etc. while we must study structural properties, behavior and phenomena which may not possess a traditional description. In such a situation an extension of objectives of quantitative analysis is needed. This leads to a new approach called qualitative analysis which studies phenomena (properties) and mutual relationships among them. An advantage of such an analysis follows from the fact that often a gross behavior of a system is largely independent of quantitative values of system variables, and therefore, it does not need detailed quantitative analysis.

Stability in a network is a well-recognized example of the qualitative approach. It has great practical and theoretical importance since stability is a property required by all operating systems. Let us point out that instability in a computer network leads to rapid degradation in throughput and delay, sudden jumps between "good" states and "bad" states, infinite queue lengths and so forth. In a broad sense stability deals with a *required property* of a system in the presence of *perturbations*. For example, we ask if delay of a packet is smaller than a given thresh-

hold (required property) provided packet input rate is in a given region (perturbation). More generally, depending on interpretation of the *required property* and *perturbation* we obtain a number of stability definitions.

In a stochastic approach to analysis of computer networks a source of disturbances is usually the input traffic. Then stability sense depends on what one understands by required property. Existence of steady-state distribution (long-run probabilities) or convergence in a probability sense lead in a multidimensional environment to stability in the sense of ergodicity and partial ergodicity. Investigating small changes in the output distribution (e.g., queue length, delay, etc.) subject to small changes in the distribution of the input traffic (required property) we must deal with stability in the sense of robustness and continuity. Studying bistability (more generally, multistability) behavior of a system we recognize that a desired property is a particular shape of a long-run probability distribution function, which is another sense of stability. If the designer of a system requires that the delay of a packet is smaller than a given threshold he deals in fact with so called practical stability problems. Finally, if we require that all users share a resource fairly, stability in the sense of fairness is considered. We investigate these and other types of stabilities in a local area environment (multidimensional state space environment). Some of the above stabilities are well-defined (e.g., ergodicity), but need easily verifiable conditions for stability regions. Others are well-understood intuitively but do not possess precise definitions (shape of steady state distributions, fairness, etc.).

In this proposal we establish precise definitions of the relevant stabilities, provide easily computable conditions for stability regions in a multidimensional environment, and consider mutual relationships between different types of stabilities. Finally, we apply these results to study the stability of some important local area networks, e.g. asymmetric buffered contention (ALOHA type) broadcast systems (one hop and many hop network), exponential backoff algorithms in multiaccess networks, conflict resolution algorithms in packet radio networks and

token-passing computer networks.

2. PRELIMINARIES AND MOTIVATION FOR RESEARCH

A system works in stable fashion if it possesses *required properties* in the presence of some *perturbations* (disturbances). Only stable systems may operate in a real life. A non-trivial problem is to design stable systems and to recognize whether a system is stable or not. Moreover, a system may be stable in one sense and unstable in another sense. A sense of stability depends on what one understands by required properties and perturbations. Before we proceed to general definitions of different stabilities, let us consider an example.

Example 2.1. Packet broadcast contention (ALOHA-type) system.

Let us study a set of M geographically distributed users competing for access to a broadcast channel e.g. radio channel, cable channel or satellite channel. If no central coordination is provided, packet collision is inevitable: simultaneously transmitted packets collide and destroy each other. Behavior of such a system depends on many issues such as:

- the multiaccess protocol used, that is, how users share the common channel. There are contention protocols (e.g. ALOHA protocol [ABR] [TOB]), reservation protocols ([ROB], [SZP1], [TOB], conflict resolution protocols [MAS], [CAP], [GAL], [MAT] exponential backoff protocol [KEL] [GOD], and many others [BUX])
- whether the users are buffered [SAA] or unbuffered. In the former case we assume that each user has a buffer of infinite capacity; the latter case assumes that a user is capable of storing only one packet at a time
- if the users are indistinguishable (symmetric case) or distinguishable (asymmetric case)
- the properties of the communication channel

- mobile users versus fixed-position users
- one-hop network versus multihop network. In a one hop network, each user is in the range of all the other users.

To identify some stability problems in such a system, we must define the *required property* and *disturbances*. In stochastic systems, as the ones considered here, it is assumed that the source of the disturbance is the input traffic . On the other hand, *required property* depends on the particular system description, and an analyst's point of view. Let us investigate some properties which are of great interest to us and lead to better understanding of the system behavior. First of all, let us assume buffered users . Then the system is described by a multidimensional stochastic (hopefully Markovian) process. This process has infinite state space, so the primary question is *whether it possesses a steady-state distribution or not*. Roughly speaking, we ask if after a long period of time since system initialization the queue lengths at each buffer are finite. This leads to a notion of *ergodicity* of the system (process) and it may be considered as our first type of stability. But we are also interested in whether the average queue lengths are finite; this produces a new sense of stability. Moreover, in this type of system it might happen that some queues are infinite while others behaves quite nicely, that is, there are finite queues at some stations. In this case we say that the system is *partially ergodic*.

Consider now a single user. Assume that you are a greedy user, and you are only interested in what happens with you. Then, the problem is whether starting from an empty queue you ever return to the empty queue after a finite period of time. This problem may be formulated in terms of the *first return time*, that is, starting from a given state how much time you need to hit a given subset of the state space. It might be proved that there is relationship between ergodicity (partial ergodicity) and finiteness of first return time.

Assume now unbuffered symmetric users in the ALOHA system. Then, the system is described by a random variable which represents the number of active users (a finite, one-

dimensional Markov chain) [KLE] [TOB] [SZP1]. Since the appropriate Markov chain is finite and irreducible, it always possesses a steady-state solution, so the ergodicity problem disappears. But, it is also well known that in some circumstances the system is bistable, that is, there are two local equilibrium states, and the system oscillates between these states. How do we identify such behavior? It turns out that this phenomenon is a consequence of the fact that steady-state probabilities π_k , as a function of k , $k=0,1,\dots,M$ (these probabilities are a solution of a system of linear equations), represent a bimodal function, i.e., there are two states with high probabilities (more probable states) and the system triggers between these states giving as a result a picture of bistability. Generalizing this we may ask whether a steady-state distribution of a system is a multimodal function or not. From the practical point of view it is important to identify this property *without solving a system of linear equations* (a qualitative approach). Hence, we must study a *type (shape) of steady-state distribution*.

Finally, consider the situation from a users point of view. Each user wants to have the same rights to access the channel (required property). It means that the system should be *fair*.

□

Generalizing the above example we define below a number of stabilities we plan to investigate. Let a system be described by an M -dimensional stochastic process $Z^t = (Z_1^t, Z_2^t, \dots, Z_M^t)$, $Z^t \subseteq R_+^M$ where $R_+^M = \{(r_1, r_2, \dots, r_M) : r_i \geq 0, i = 1, 2, \dots, M\}$. For example, Z_i^t may represent queue length in the i -th buffer in a local area network, waiting times, delays and so forth. For simplicity of further considerations we assume that Z^t is defined on the M -tuples of nonnegative numbers, that is, $Z^t \subset C$ where $C \stackrel{def}{=} I_+^M = \{(i_1, i_2, \dots, i_M) : i_k \text{-nonnegative integers, } k = 1, 2, \dots, M\}$. We call C the state space for Z^t . We adopt the following definitions:

Ergodicity. Let steady-state probabilities π_k , $k \in I_+^M$, be defined as $\pi_k = \lim_{t \rightarrow \infty} Pr\{Z^t = k\}$. A system is *ergodic* if and only if $\pi_k > 0$ and $\sum_{k \in C} \pi_k = 1$.

Finite moments. Let $E Z_i^l$, $i = 1, 2, \dots, M$ denote the l -th moment of the i -th component of Z^t as $t \rightarrow \infty$. A system is stable if for all $i = 1, 2, \dots, M$ and given l , the moments $E Z_1^l$, $E Z_2^l, \dots, E Z_M^l$ exist and are finite.

Partial ergodicity. In some systems, steady-state distribution $\pi_k, k \in C$, may not exist for Z^t , but marginal distributions of some components of Z^t are still well-defined. Consider an example.

Let (N_1^t, N_2^t) be queue lengths in two buffers of an ALOHA-type system (see Ex.2.1). Let π_{k_1, k_2} , π_{k_1} , π_{k_2} denote $\pi_{k_1, k_2} = \lim_{t \rightarrow \infty} Pr\{N_1^t = k_1, N_2^t = k_2\}$, $\pi_{k_1} = \lim_{t \rightarrow \infty} Pr\{N_1^t = k_1\}$, $\pi_{k_2} = \lim_{t \rightarrow \infty} Pr\{N_2^t = k_2\}$.

Assume (N_1^t, N_2^t) is a two-dimensional irreducible Markov chain. If (N_1^t, N_2^t) is not ergodic then [CHU] $\pi_{k_1, k_2} = 0$ for all $(k_1, k_2) \in C = I_+^2$. But, by Fatou's lemma [RUD]

$$\begin{aligned} \pi_{k_1} &= \lim_{t \rightarrow \infty} Pr\{N_1^t = k_1\} = \lim_{t \rightarrow \infty} \sum_{k_2=0}^{\infty} Pr\{N_1^t = k_1, N_2^t = k_2\} \geq \\ & \sum_{k_2=0}^{\infty} \lim_{t \rightarrow \infty} Pr\{N_1^t = k_1, N_2^t = k_2\} = 0 \end{aligned}$$

hence π_{k_1} or π_{k_2} might be positive, and marginal distribution may exist. To generalize it, let us define a set of indices $l_1, l_2, \dots, l_n \in \{1, 2, \dots, M\}$, $l_i \neq l_j$ if $i \neq j$ and $1 \leq n < M$. Denote $I = (l_1, \dots, l_n)$ and for $k_I \in I_+^n$.

$$\pi_{k_I} = \lim_{t \rightarrow \infty} Pr\{N_{l_1}^t = k_{l_1}, N_{l_2}^t = k_{l_2}, \dots, N_{l_n}^t = k_{l_n}\}$$

Then, a system is *partially ergodic* if exists an n -tuple $I = (l_1, \dots, l_n)$ such that $\pi_{k_I} > 0$, $k_I \in I_+^n$ and $\sum_{k_I \in I_+^n} \pi_{k_I} = 1$.

Partial finite moments. Define a function $f: I_+^M \rightarrow R$, R is a set of real numbers. Consider $Ef(Z^t)$. For example, if $f(\cdot)$ is a projection on the i -th axis, that $f(Z) = Z_i^t$, and $Ef(Z) = EZ_i$ is an average of Z_i ; if $f(Z) = Z_{l_1} + Z_{l_2}$, $l_1, l_2 \in \{1, 2, \dots, M\}$, then $Ef(Z) = EZ_{l_1} + EZ_{l_2}$ is the sum of average values of Z_{l_1} and Z_{l_2} . Then, we say that a system is *stable with respect to a function* $f(\cdot)$ if there exists a function $f(\cdot)$ such that $Ef(Z) < \infty$.

Practical Stability. Let D be an average delay for a packet in a local area network with total input rate λ packets per unit of time. We want to know how big an input traffic should be provided $D \leq D_{\max}$, where D_{\max} is a given number. Generalizing it, let λ represent an input parameter and Λ is a set of admissible values of λ . Let also $c(\lambda)$ be a criterion function for a system, e.g. delay, average queue length or a probability of loss. Define a set of required properties as $C = \{c : c(\lambda) \leq c_{\max}\}$. Then, we say a system is *stable with respect to* (Λ, C) if the following holds $\lambda \in \Lambda$ implies $c \in C$.

Shape of Steady-State Distribution. Assume for simplicity Z^t is a one-dimensional Markov chain with finite state space $C = \{k : 0 \leq k \leq M\}$. Then, steady-state probability vector $\pi = [\pi_0, \pi_1, \dots, \pi_M]$ is a solution of a system of linear equation $\pi P = \pi$, where $P = \{p_{ij}\}_{i,j=0}^M$ is a transition matrix. Consider the probabilities $\pi_k, k = 0, 1, \dots, M$ as a function of k . We denote it as $\pi(k)$. Some properties of a system (e.g., bistability) depend on the type (shape) of the function $\pi(k)$, as we have seen in Example 2.1. It is important to know if $\pi(k), k \in C$ is a unimodal function (only one maximum), bimodal (two maxima) or n -modal (n maxima of $\pi(k)$) function. Bimodal distributions of $\pi(k)$ may produce a bistable behavior, which is obviously an undesirable phenomena. We say that *a system is stable in the sense of shape of steady-state distribution distribution if $\pi(k)$ is unimodal function for $k \in C$* . The problem is that we want to identify this stability without solving the system of linear equations, i.e., knowing only transition matrix P we investigate $\pi(k)$ as a function of $k \in C$.

Fairness. From the user point of view it is important if all users have the same rights to access a resource (e.g. channel). However, though it is easy to understand intuitively whether a system is fair or not, there is no widely accepted definition of fairness. In fact, most systems in the real world are unfair, so a "smart" definition of the unfairness coefficient, F , should be found. Then, we say that a system is *stable with respect to fairness if $F \geq F_{\max}$* , where F_{\max} is a given thresh-

hold. The main problem here is to precisely define a class of fair systems, and introduce a "good" definition of fairness coefficient (see for example [SZP1] [GER2] [REG]). One application of fairness is in the study of load balancing problems in distributed systems [NI].

□

There are number of questions associated with stability problems. First of all, we want to know if a system is stable or not with respect to a given property. For stable systems we may further ask what amount of the property it possesses, what does it mean for one system to be more stable than another, and so forth. For example: for partial ergodic system we say the system S_1 is more stable than the system S_2 if the number of ergodic components in S_1 is greater than the number of ergodic components in S_2 . For fair systems we establish ordering relationships with respect to the fairness coefficient F . In general, we seek to establish ordering relationships with respect to a given property for stable systems.

3. Quantitative analysis versus qualitative analysis

In this proposal we study a qualitative property of computer networks namely, stability problems. In fact, stability investigation is an example of the qualitative analysis of a stochastic model. In a long-term plan we wish to explore other qualitative aspects in analyses of computer networks. Therefore, in this section we give a brief description of such an analysis, and point out of some differences between these and quantitative analysis.

We study pathological behavior in a large scale complex system, e.g., computer networks. As soon as pathological behavior is detected, a new system is designed which eliminates such behavior. However, a new type of phenomena may occur. For example: flow control and error control [GER] lead to deadlocks. Moreover, such an approach introduces very complicated feedback mechanisms into the system. These mechanisms are typically non-linear and pose highly non-trivial design and analysis problems. This follows from such facts as [OLD]:

- individual feedback may produce a variety of surprising phenomena.
- human intuition based on linear systems cannot be transferred to non-linear systems
- the behavior of a system is frequently not determined by the values of externally controllable variables.

In such a situation a new approach is needed. We focus our effort on studying qualitative (system) properties of (stochastic) models and phenomenological aspects of system behavior. Most of these models are sufficiently complex so that bounds and approximations for their characteristics (valid for a class of systems) are of particular importance. Qualitative properties of stochastic models constitute an important theoretical basis for approximate methods. These also characterize the influence of quantities describing the behavior of the constituent elements or components of the system, on parameters describing the system as a whole, and provide sufficient insight into the behavior of a system. To define more precisely what qualitative analysis is we list below some of its features in comparison to properties of quantitative analysis.

Generally speaking, *quantitative analysis determines values of a function describing a system*. For example, for a given system we may calculate throughput-delay performance, queue length distribution, probability of packet loss, and even optimization of some performance characteristics [NEU], [HEI]. It should be quite obvious that in most cases approximate models are analyzed using in addition, approximate methods. Then a natural question arises: how good is the approximation and for what class of systems is the approximation valid? Let us, as an example, consider the ALOHA system [TOB] [ABR]. It is well known that there are at least two approximate methods: Poisson approximation (one assumes that input traffic to a channel is Poisson) and fluid approximation. Quite recently it was noted that the Poisson approximation “omits” a very important property of the system, namely, bistability and hysteresis properties. Fluid approximation gives a better insight into the behavior. However, it does not precisely

explain why such a behavior occurs. It appears that such an exploration needs quite advanced mathematics, namely catastrophe theory [OLD] [NEL] which is an example of typical qualitative methods.

Qualitative analysis studies abstract properties and phenomena as well as mutual relationships among them for a class of systems. For example, qualitative analysis tells us whether a system has or does not have discussed property (stability, congestion, deadlocks, etc.), it gives us a theoretical basis for approximate analysis or for finding particular bounds. Finally it informs us about pathological behavior of a system. To be more specific, let us list the most important properties of qualitative analysis:

- The analysis studies a class of systems instead of a particular system; so it may constitute basis for bounds and approximation methods.
- Qualitative analysis should precede quantitative analysis since it does not often need detailed quantitative analysis; e.g., to study ergodicity property of a Markov chain we must only determine sign of the average drift.
- Qualitative techniques in many cases can be understood, appreciated and utilized by the knowledgeable non-specialists, and the analysis may create a continuum between the essentially intuitive arguments and semi-formal proofs.
- Often a gross behavior of a system is largely independent of quantitative values of system variables and then qualitative mode of the behavior becomes dominant. Qualitative analysis determines whether there are undesirable modes of the behavior and under what conditions they might appear.

Performance evaluation of computer networks is based on analysis of *stochastic models*, more precisely: queueing models. There are a number of properties for which detailed study leads to better understanding of system behavior. Some of them are: regularity [KAL],

ergodicity [PAK] [TWE] [SZP3], stability [KLE], robustness [STO], monotonicity [STO], insensitivity [HEY], boundness in some sense [KAL], types of distribution [KEI1],[KEI2],[SZP4], [NEL], modes of system behavior (congestion, deadlock, etc.), fairness [SZP1] [REG], [MRS] and so forth. Detailed description of some of these properties will be given in the next section. On the other hand, it must be pointed out that qualitative analysis of computer networks is identified also by methods which are used, and not only by the object of study. Therefore we include in such an analysis methodological studies, i.e., particularly methods which are applied to investigate properties and behavior of a class of systems. There are many qualitative techniques ranging from typical stochastic methods as stochastic inequalities, limiting distributions, asymptotic analysis, etc., and ending on very new power techniques such as geometric and topological methods, convexity methods, Lyapunov theory, topological dynamics, global analysis [HIR] and the very attractive catastrophe theory [POS].

4. RESEARCH OBJECTIVES

In this section we discuss methodology used to establish stability conditions . In particular, we outline problems and propose some preliminary results for ergodicity and nonergodicity in a multidimensional environment, we point out a relationship between partial ergodicity and attainability, and finally we propose an approach to study a shape of steady-state distribution.

4.1. Ergodicity of multidimensional Markov chains

We propose here a few methods to establish ergodicity conditions for a multidimensional Markov chain. In particular, we determine so called ergodicity regions, that is, a set of input parameter values (e.g. input rates) which assures that a system is stable in the sense of ergodicity. We discuss *Lyapunov function methods, comparison tests, unbounded random walk method and asymptotic analysis approach.*

Let $Z^t = (Z_1^t, Z_2^t, \dots, Z_M^t)$ be an M -dimensional Markov chain. We assume it is aperiodic and irreducible [CHU]. Then, *ergodicity refers to a problem of existing steady-state solutions*, that is, $\lim_{t \rightarrow \infty} Pr\{Z^t = k | Z^0 = i\} = \pi_k > 0$, where $k \in C$, $C = I_+^M$, I_+ is a set of nonnegative integers.

Lyapunov Functions - generalized drift.

Lyapunov functions have found wide applications in classifying Markov chains since the work of Foster [FOS]. Pakes [PAK], Marlin [MAR], Rosberg [ROS2] and Tweedie [TWE] extended Foster's criteria for wider class of Markov chains, but there are only a few papers which deal with the multidimensional case (see [ROS1] [SZP3], [SEN2]). Moreover, most of these criteria are sufficient for ergodicity but not necessary. To overcome it we also consider sufficient condition for nonergodicity of a Markov chain, as proposed by Kaplan [KAP] (see also [SEN1] [SZP2]).

Let us start with some examples. Assume Z^t is a one-dimensional Markov chain and let $d(k) = E\{Z^{t+1} - Z^t | Z^t = k\}$ be the average drift of Z^t at $k \in C$. Foster has proved that an *irreducible, aperiodic Markov chain is ergodic if $d(k) < -\epsilon$, $\epsilon > 0$ for all $k \in C - H$, where H is finite subset of C , and $|d(k)| < \infty$ for $k \in C$.*

Example 4.1

Consider a Markov chain N^t described by the following stochastic equation $N^{t+1} - N^t = X^t - Y^t$. Then $d(k) = E\{X^t | N^t = k\} - E\{Y^t | N^t = k\}$. If N^t represents queue length in a (discrete) $M | G | s$ queueing system, then X^t and Y^t are input process and departure process respectively. Let λ be average input rate and μ service rate for each server. Then, $d(k) = \lambda - \min\{k, s\}\mu$ and for $H = \{0, 1, \dots, s-1\}$ we find that $d(k) < 0$ for $k \in C - H$ if $\lambda < \mu s$ (see also [GRO]), which establishes sufficient condition for ergodicity of N^t .

□

The theorem remains true if we extend definition of drift. Define a function $V: C \rightarrow R_+$ called further Lyapunov function. For Z^t and $V(k)$ we introduce an operator (generalized drift) $AV(k)$, $k \in C$ as $AV(k) = E\{V(Z^{t+1}) - V(Z^t) | Z^t = k\}$. For example, assuming $V(k) = k$ we obtain $AV(k) = d(k)$. Then, it is shown that $AV(k) < -\epsilon$ for $k \in C - H$, H a finite subset of C , is sufficient for ergodicity of Z^t . In multidimensional environment two problems arise. The drift function is a vector $\mathbf{d}(\mathbf{k}) = (d_1(\mathbf{k}), d_2(\mathbf{k}), \dots, d_M(\mathbf{k}))$ where $d_i(\mathbf{k}) = \{Z_i^{t+1} - Z_i^t | Z^t = \mathbf{k} = (k_1, k_2, \dots, k_M)\}$, and any "reasonable" condition for ergodicity is violated on infinitely many states.

Example 4.2 *Two queues in the ALOHA system.*

Two queues compete for an access to a single ALOHA channel (see Ex.21). Probability of transmitting a packet from queue i is r_i ; let $\bar{r}_i = 1 - r_i$. Input rates are λ_1 and λ_2 , respectively. We denote by $N^t = (N_1^t, N_2^t)$ queue lengths in the buffers. Then for any $k_1, k_2 > 0$

$$d_1(k_1, 0) = \lambda_1 - r_1; \quad d_1(0, k_2) = \lambda_1, \quad d_1(k_1, k_2) = \lambda_1 - r_1 \bar{r}_2 \tag{4.1a}$$

$$d_2(k_1, 0) = \lambda_2; \quad d_2(0, k_2) = \lambda_2 - r_2, \quad d_2(k_1, k_2) = \lambda_2 - \bar{r}_1 r_2 \tag{4.1b}$$

As you see $d_i(k_1, k_2)$ is positive for an infinite number of states, e.g. either on N_1 or on N_2 axes.

□

By appropriate choice of the Lyapunov function we may avoid some of the above problems.

For example: define $V_i: R_+ \rightarrow R_+$, and $V(\mathbf{k}) = \sum_{i=1}^M c_i V_i(k_i)$ ($V(\mathbf{k})$ is separable function of

$\mathbf{k} = (k_1, \dots, k_M)$). Then $AV(\mathbf{k}) = \sum_{i=1}^M c_i AV_i(\mathbf{k})$ where $AV(\mathbf{k}) = E\{V_i(Z_i^{t+1}) - V_i(Z_i^t) | Z^t = \mathbf{k}\}$, and c_i

are nonnegative constants, $i = 1, 2, \dots, M$. Using the generalized drift criteria we immediately

prove that Z^t is ergodic if for a finite set $H \subset C$, $AV(\mathbf{k}) = \sum_{i=1}^M c_i AV_i(\mathbf{k}) < 0$. In particular, if

$V_i(k_i) = k_i$, then $AV(k) = \sum_{i=1}^M c_i d_i(\mathbf{k})$ where $d_i(\mathbf{k})$ is i -th component of a drift vector.

Example 4.3. *Continuation of Ex.4.2*

Let $H = \{(0,0)\}$, and $V_i(k_i) = k_i, i=1,2$. Then the system is ergodic if

$$\begin{aligned} c_1\lambda_1 + c_2\lambda_2 &< c_1r_1 \\ c_1\lambda_1 + c_2\lambda_2 &< c_2r_2 \\ c_1\lambda_1 + c_2\lambda_2 &< c_1\bar{r}_1\bar{r}_2 + c_2\bar{r}_1\bar{r}_2 \end{aligned} \quad (4.2)$$

i.e., the above three inequalities must be satisfied simultaneously. Moreover, a stability subregion (set of such values $\lambda = (\lambda_1, \lambda_2)$ that satisfy (4.2)) strongly depends on the constants c_1, c_2 . A proper choice of c_1 and c_2 is essential to determine the largest stability subregion.

□

Example 4.4. *Exponential Back-off Algorithms in Ethernet [GOD] [KEL]*

Let us consider the two users as in Ex.4.2, however, now each user additionally determines how many times the transmitted packet was involved in a collision. We denote this number by B_i^t for i -th user and assume that the probability of transmitting a packet is equal to $2^{-B_i^t}$. Then, the system is described by a 4-dimensional Markov chain $(N_1^t, N_2^t, B_1^t, B_2^t)$. To study ergodicity we must introduce a little more sophisticated Lyapunov function $V(k,b)$ namely:

$$V(k,b) = c_1k_1 + c_2k_2 + a_1f_1(b_1) + a_2f_2(b_2) \quad (4.3)$$

where $f_i(b_i) = 2^{b_i}$ for $b_i > b_i^*$ and $f_i(b_i) = 2^{-b_i}$ for $b_i < b_i^*$, and b_i^* is a small integer. Choosing appropriate c_i, a_i and b_i^* we are able to determine the largest stability subregion.

□

Using this approach we can find a subset of values $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_M)$ which assures that the system is stable (stability subregion). In most cases, it is a proper subset of *the stability region*, i.e., there exists values of λ for which the above criteria fail although the system is ergodic. This subset may be enlarged by an appropriate choice of the the Lyapunov function (in

particular, the constants c_i , $i = 1, 2, \dots, M$ as in Ex. 4.2 and 4.3). The main problem with Lyapunov function approach lies in the restriction of the set H to a class of finite sets. The paper of Rosberg [ROS1] studies sufficient condition for ergodicity of a multidimensional Markov chain when H is infinite. However, his criteria are applied only to a special class of Markov chains, which are not very interesting from a practical point of view. We shall show how to extend the criteria to a wider class of Markov chains.

Most of the above discussion deals with sufficient conditions for ergodicity. It is not reasonable to expect in the near future easily verifiable *sufficient and necessary* conditions for ergodicity of multidimensional Markov chains. A reasonable solution is to look for sufficient conditions for non-ergodicity (i.e. necessary for ergodicity). Following the work of Kaplan [KAP], recent work of Sennot et al [SEN1], [SEN2] and Szpankowski [SZP2] we shall show that in a multidimensional environment the Lyapunov function method may be applied. In particular, we shall prove that under some restrictions a Markov chain is not ergodic if there exists a Lyapunov function $V(k)$ such that $AV(k) > 0$ for $k \in C - H$, H is finite. We shall investigate a possibility to relax the assumption about finiteness of the subset H . Examples from the computer field of networks will be considered and easily variable stability and nonstability regions will be given.

Comparison tests

As mentioned above the Lyapunov function method has some restrictions in a multidimensional environment. We may overcome some of them using another approach called *comparison tests*. To introduce the idea let us assume Z^t is a one-dimensional Markov chain. If one finds Markov chains \underline{Z}^t and \bar{Z}^t such that $\underline{Z}^t \underset{st}{\leq} Z^t \underset{st}{\leq} \bar{Z}^t$ (where $\underset{st}{\leq}$ means *stochastically smaller* [STO]). Then, it is obvious that ergodicity (nonergodicity) of \bar{Z}^t (\underline{Z}^t) implies ergodicity (nonergodicity) of Z^t . We extend this idea to a multidimensional stochastic processes. The following problems arise: how do we define stochastic order in multidimensional case; which stochastic order

implies ergodicity (nonergodicity) (see [STO]); how do we find stability conditions for \underline{Z}^t and \bar{Z}^t and so on.

Example 4.5. *Continuation of Ex.4.2.*

We assume that M users compete for an access to a common channel according to the rule described in Ex. 4.2 (see also [SAA], [TSY1]). Then $N^t = (N_1^t, N_2^t, \dots, N_M^t)$ represents queue lengths in M buffers. Let \bar{N}_m^t be the queue length in the m -th buffer under the condition that all other buffers are never empty. Then, \bar{N}_m^t is a one-dimensional Markov chain, and it is ergodic if

$$\lambda_m < r_m \cdot \prod_{\substack{k=1 \\ k \neq m}}^M \bar{r}_k = r_m q_m. \text{ Naturally, } N_m^t \leq \bar{N}_m^t \text{ for } m = 1, 2, \dots, M. \text{ We prove that the } M\text{-}$$

dimensional Markov chain N^t is ergodic if $\lambda_m < r_m q_m$ for all $m = 1, 2, \dots, M$. This condition *cannot* be obtained by Lyapunov function methods.

□

Unbounded random walk method.

To explain the idea, let us consider a one-dimensional case. Intuition says that under some restrictions a Markov chain Z^t is not ergodic if average drift $d(k)$ is positive for sufficiently large k (in a multidimensional environment the expression " sufficiently large" needs precise definition) . In some cases this might be proved using the theory of unbounded random walk [HEY].

Example 4.6.

Let N^t, X^t, Y^t represent queue length, arrival process and number of available servers in a queueing system. Then

$$N^{t+1} = [N^t - Y^t]^+ + X^t \tag{4.4}$$

where $a^+ = \max\{a, 0\}$. The process N^t is defined on nonnegative integers I_+ , and we call it

bounded random walk since state $\{0\}$ is a boundary state. If we introduce a new process N_μ^{t+1} as

$$N_\mu^{t+1} = N_\mu^t - Y^t + X^t \quad (4.5)$$

then N_μ^t is unbounded random walk defined on all integers I . Using a number of already proved theorems on unbounded random walks we know that N_μ^t "drifts" to $+\infty$ if $EX > EY$, (in other words, the average drift is positive). What can we say about N^t ? In fact, it is easy to show that N^t is also not ergodic.

□

The above nonergodicity condition could also be derived from the Kaplan's criterion [KAP] [SZP2]. However, Kaplan's condition (as generalized drift criterion) requires that the condition on the average drift (in our case $d(k) > 0$) is violated only on a finite subset of the state space. It is a critical issue (as we have seen before) in a multidimensional environment. Using the unbounded walk approach we are able to relax it. Under some restrictions on the process (e.g., the drift $d(k)$ is constant for $k > d$, where d is a constant vector) we prove that positivity of each component of the drift $d(k)$ for $k \in C - H$, where H is infinite subset of C , is sufficient for nonergodicity. Further research should relax the imposed restriction on the process, establish conditions on the set H under which the criterion works, and extend the applicability of the criterion.

Asymptotic analysis method

The criteria discussed above are based on the sign of the generalized drift. However, in some cases the drift is difficult to compute or the formula on it is so complex that the condition is difficult to verify. Then asymptotic analysis may be useful.

Example 4.7. Conflict resolution algorithms

Let many (infinite) users compete for access to a broadcast communication channel. The main problem is how to efficiently share the channel among the users. A class of very efficient

algorithms are those known in the literature as conflict resolution algorithms (CRA) [MAT], [TSY2], [TSY3], [GAL], [CAP], [FAY]. The idea is to split each conflict of multiplicity n (n packets involved into a conflict) into smaller conflicts until n conflicts of multiplicity one (success) arise. Then the conflict is resolved. Let L_n be average conflict resolution interval with the initial conflict of multiplicity n . If the partition of a conflict is done on the basis of a random variable (so called Capetanakis-Tsybakov-Mikhailov algorithm, in short GTM algorithm), then by Pakes Lemma [PAK] the algorithm is stable (ergodic) if $\lambda < \lambda_{\max}$, where $\lambda_{\max}^{-1} = \limsup_{n \rightarrow \infty} L_n/n$.

But L_n satisfies the following recurrence equation [HOF] [SZP5]

$$L_n = 1 + \sum_{k=1}^n \binom{n}{k} p^k q^{n-k} L_k \quad p + q = 1 \quad (4.6)$$

hence, the limit L_n/n as $n \rightarrow \infty$ is not easy to find. But it is proved [HOF] [SZP5] [MAT] that $L_n = \alpha n + f(n) + O(1)$ for large n (asymptotic approach), where α is a constant and $f(n)$ is a very small function of n . It immediately implies that $\lambda_{\max} \approx \alpha^{-1}$.

□

We restrict this investigation to conflict resolution algorithms. However, a new approach to stability analysis of CRA will be presented. Namely, knowing that the key problem here is to solve a special type of recurrence, we introduce a general type of the equation which covers a wide class of CRA algorithms. A form of recurrence we might consider here is as follows [SZP6]

$$f_n L_n = a_n + \gamma \sum_{k=0}^{n-N} p_{nk} L_k \quad (4.7)$$

where $\sum_{k=0}^n p_{nk} = 1$, a_n (additive term) is any sequence of numbers, f_n and N are either equal to $1 - \gamma p_{nn}$ and $N=0$ for CTM algorithms or $1 - \gamma p_{nn} - \gamma p_{n0}$ and $N=1$ for so called Gallager-Tsybakov-Mikhailov algorithm [GAL] [TSY] [SZP5] [SZP6]. The additive term a_n is any sequence of numbers and various modifications of CRA are modeled by the appropriate choice of

a_n .

Example 4.8 Modified CTM-Algorithm (continuation of Ex. 4.7)

If we split a conflict into two smaller conflicts and we know that the first one is of multiplicity zero, then for certain in the next slot we produce a conflict. To avoid it we may skip over the last step and immediately split the second conflict. This modification of CTM is known as modified CTM-algorithm. An appropriate recurrence for conflict resolution interval L_n is [HOF], [SZP5], [MAT]:

$$L_n = 1 - p^n + \sum_{k=1}^n \binom{n}{k} p^k (1-p)^k L_k \quad (4.8)$$

Comparing (4.6 and 4.8) we note that both algorithms might be analyzed by the common recurrence (4.7) with $a_n=1$ or $a_n=1-p^n$.

□

To establish easily computable criteria for CRA algorithms, we consider recurrence equation (4.7), solve it, and provide asymptotic analysis of it. We shall also study a functional equation associated with the recurrence (4.7) (see [MAT], [SZP5], [SZP6], [FAY]) and present an asymptotic analysis for it. Such an investigation is necessary for stability analysis of more sophisticated CRA algorithms such as interval-searching algorithms, e.g., Gallager-Tsybakov-Mikhailov algorithm [GAL], [TSY2], [TSY3].

4.2 Partial Ergodicity and Attainability

We begin with an example.

Example 4.9. Two users in the ALOHA system

Once again we look at the problem of two users in the ALOHA system (Ex. 4.2). It is proved [MAL], [TSY1],[SZP3] that the Markov chain (N_1^i, N_2^i) is ergodic if and only if $\lambda_1 < r_1 \bar{r}_2$ implies $\lambda_2 < r_2 [1 - \lambda_1 / \bar{r}_2]$ and $\lambda_1 \geq r_2 \bar{r}_2$ implies $\lambda_2 < \bar{r}_1 [1 - \lambda_1 / r_1]$. But, assuming the second queue is

infinite (non ergodic) we find that for $\lambda_1 < r_1 \bar{r}_2$ the first queue is ergodic, that is, the queue is finite with probability one. Moreover, there exist steady-state probabilities for the first queue. We may assume that the system still operates, and we want to know under what conditions it might happen. We say that the system is partially ergodic.

□

Concluding, we investigate the possibility that there exists a marginal distribution of an M-dimensional Markov chain even though there is no steady-state solution for the whole system. We conjecture that there is a relationship between partial ergodicity and attainability. This comes from the following facts. It is well known [CHU] that a Markov chain Z^t is ergodic if its mean value of the first return time to any state $k \in C$ starting from it, is finite. Let $\tau_H = \min \{ t : Z^t \in H, Z^s \in H, s < t \}$ and $E_k \tau_H = E \{ \tau_H | Z^0 = k \}$. Hence, $E_k \tau_k < \infty$ for any $k \in C$ implies ergodicity of Z^t . Under some assumptions the last is true if $E_k \tau_k$ is replaced by $E_k \tau_H$, where H is finite subset of C. Problems arise if one assumes H infinite. In most cases $E_k \tau_H < \infty$ does not imply ergodicity of $Z^t = (Z_1^t, \dots, Z_M^t)$, however, we expect that under appropriate choice of H the above condition implies partial ergodicity.

Example 4.10 (continuation of Ex. 4.9)

Assume $H = \{(0, i) : i \geq 0\}$ that is, H is the N_2 - axis. Then, $E_k \tau_H$ is the first return time for the second user to an empty queue starting from an empty buffer at the second user. This suggests a relationship between $E_k \tau_H < \infty$ and existence $\pi_2(0) = \lim_{t \rightarrow \infty} Pr \{ N_2^t = 0 \}$.

□

To study properties of $E_k \tau_H$ we apply the Lyapunov function as discussed in Section 4.1. We shall prove that if $V(k), k \in C$ is a Lyapunov function such that $V(k) > -\zeta, k \in C, AV(k)$ is the operator for Z^t satisfying $AV(k) < -\epsilon$ for $k \in C - H$, then $E_k \tau_H < [V(k) - \zeta] / \epsilon$ for $k \in C - H$ and $E_k \tau_H < 1 + [V(k) + AV(k) - \zeta] / \epsilon$ for $k \in H$. This implies that $E_k \tau_H < \infty$ if $AV(k) < -\epsilon, \epsilon > 0$ and

$|AV(k)| < \infty$ for $k \in H$, and it is true for H finite and infinite. Assuming H finite we immediately obtain sufficient conditions for ergodicity of Z' . For H infinite many situations may occur, which we shall investigate. Moreover, these inequalities can be used to establish upper bounds for $E_k \tau_H$ and lower bounds for $\pi_i = \lim_{t \rightarrow \infty} Pr\{Z^t = i\}$ (if Z' is ergodic). Finally, we use τ_H to study transient behavior of a system.

4.3 Shape of steady-state distributions

When a system is described by a process with finite state space, ergodicity is not interesting property, since every irreducible Markov chain with finite state space possesses a steady-state solution. In other words, such a system is stable from the ergodicity point of view. However, a number of new phenomena may occur, which have non-trivial description and analysis.

Example 4.11 *Unbuffered ALOHA System*

Let us consider M unbuffered users in the ALOHA environment (see Ex. 2.1). Each user has a buffer of capacity one, hence the system is governed by a one-dimensional Markov chain N' , representing the number of active users. Steady-state probabilities $\pi = [\pi_0, \pi_1, \dots, \pi_M]$ are solutions of a system of linear equation, $\pi P = \pi$ where P is the transition matrix. Under some circumstances the system possesses a bistable behavior, that is, it oscillates between a "good" state and a "bad" state. It turns out that these two states are approximately identified by so called most probable states, that is, such values $k_1, k_2 \in \{0, 1, \dots, M\}$ that the probability of being in state k_1 and k_2 is much higher than in other states. More precisely, let us consider the probabilities π_k as a function of k . Therefore, we write $\pi(k)$ instead of π_k . Then, the function $\pi(k)$ (for some values of input parameters) may be a bimodal function, that is, the function $\pi(k)$ possesses two local maxima. Obviously it is an undesirable property since it leads to bistability.

□

Generalizing the above example, we consider the following problem: Let $\pi(k)$ be a solution of a system of linear equation $\pi P = \pi$, where P is a transition matrix for a Markov chain, and we investigate properties of $\pi(k)$ as a function of k . An obvious way to accomplish it is to solve $\pi P = \pi$, but this is not acceptable from the qualitative analysis point of view since the set of linear equations might be too complex to solve, and - what is more important - solving the set of equations we restrict our considerations to a particular system not a class of systems. Note, however, that we want to study a property, a shape or characteristics of the function $\pi(k)$, so the exact values of $\pi(k)$ are not important. *In a qualitative approach to that problem we shall investigate properties of $\pi(k)$ without solving $\pi P = \pi$.* We illustrate this approach by the following three examples.

Example 4.12. (continuation of 4.11)

Let in the unbuffered ALOHA system $S_i(k)$ and $S_o(k)$ denote conditional input rate and conditional throughput, that is, average number of arrivals and departures in a unit of time (slot) under the condition that the system is at state k . These functions are easy to compute. For example, [SZP1] $S_i(k) = (M-k)p$ and $S_o(k) = kr(1-r)^{k-1}$ where p, r are probabilities of generating a new packet and transmitting a packet, respectively. Then, we may prove [SZP4]

$$\frac{S_i(k)}{S_o(k+1)} [0.632 + o(p)] \leq \frac{\pi(k+1)}{\pi(k)} \leq \frac{S_i(k)}{S_o(k+1)} [2 - p + o(p)] \quad (4.9)$$

where $\lim_{p \rightarrow 0} o(p)/p = 0$. Hence, the multimodality property of $\pi(k)$ depends on $S_i(k)$ and $S_o(k)$.

This fact is well known since a paper of Carleial and Hellman [CAR], but (4.9) gives some simple theoretical explanations.

Example 4.13. *Birth and Death Process*

Consider an $M | M | s$ queueing model with input arrival rate λ_k and output rate μ_k . Then

[GRO]

$$\frac{\pi(k+1)}{\pi(k)} = \frac{\lambda_k}{\mu_{k+1}}$$

and the property of $\pi(k)$ depends only on λ_k and μ_k .

Example 4.14. Diffusion Approximation

The type of queue length may be determined by a diffusion approximation. To show it, let $p(x)$ be a density function of a queue length, where x is a continuous variable, $0 \leq x \leq M \leq \infty$. If boundary conditions are not considered, then $p(x)$ satisfies the following differential equation

$$\frac{dp(x)}{dx} = \frac{1}{2\beta(x)} p(x) [\alpha(x) - \beta'(x)]$$

where $\alpha(x)$ and $\beta(x)$ are infinitesimal drift and variance, respectively [HEY]. For, by $p(x) > 0$, $\beta(x) > 0$, $0 \leq M$, the type of the distribution $p(x)$ depends on sign of $\alpha(x) - 0.5\beta'(x)$. In particular, maxima and minima of $p(x)$ are roots of the equation

$$\alpha(x) - 0.5\beta'(x) = 0 \tag{4.10}$$

If $\beta'(x) \ll \alpha(x)$, then (4.10) becomes $\alpha(x) = 0$ (fluid approximation). For queue length analysis, the infinitesimal drift may be represented as: $\alpha(x) = C [S_o(x) - S_i(x)]$, where $S_i(x)$, $S_o(x)$ are defined in Example 4.12 and C is a constant. An equation $S_i(x) = S_o(x)$ is called *local equilibrium equation*. Thus, the diffusion approximation states that the maxima and minima of $\pi(k)$, approximately cover the roots of the local equilibrium equation.

□

Concluding, for a system with finite Markov chain description we define stability with respect to shape of stationary distribution. We say that a system is stable if it has a unimodal steady-state distribution. Then, for a class of systems we propose a method to identify such a stability (using generalized approach from Ex. 4.12). In addition, we also apply a catastrophe theory [POS] to study multimodality property of a distribution. A catastrophe theory approach is a very powerful tool to study pathological behaviors (bifurcation, sudden jumps, hysteresis,

inaccessibility and divergence), though however, it may lead to very trivial conclusions. In the authors' opinion a paper of Nelson [NEL] is a step in the right direction. Nelson investigated the catastrophe theory approach to study properties of distribution functions through a diffusion approximation. However, he did not address the most important question ,i.e., whether the diffusion approximation is property-preserved. In other words, the question is whether diffusion approximation "loses" an investigated property of the exact model or not. This is the most difficult problem and we plan to explore it.

5. Summary of Research Program

The research program proposed here is concerned with qualitative approach to study performance evaluation of local area networks. Such an analysis seems to be necessary in the context of growing complexity of networks and their quantitative analysis (through a queueing theoretical approach). Most research in performance evaluation was so far concentrated on a quantitative description of some objective functions describing the quality of the network. This led to extensive development of various kinds of queueing models which became increasingly sophisticated and complex. On the other hand, only a little attention has been paid to describe structural properties and real behavior of networks, e.g., congestion and deadlock problems, stability, sudden changes in a network behavior, hysteresis, inaccessibility of some regions, divergence and so forth. Although such a phenomenological approach seems to be extremely difficult to analyze, there are some "light points" in that type of analysis. This comes from the fact that quite often gross behavior of a system is largely independent of quantitative values of system variables, and therefore, it does not need a detailed quantitative analysis.

As a first attempt we focus our attention on stability problems in local area networks. Stability definition adopted here is very broad and it covers such problems as existence of some long-run probabilities as well as fairness. Realizing that only stable systems may work in

practice we deal here with a problem of great importance. We are interested in establishing easily verifiable conditions for stability regions, that is, a set of system parameter values which assure a system is in a (desired) operational mode.

Let us now briefly mention the principal goals of the proposed research:

1. Defining stability of a system as possessing required properties in the presence of disturbances, we introduce and analyze stability in a number of senses. In particular, we consider:
 - ergodicity and partial ergodicity as a problem of whether some long-run probabilities exist or not.
 - finite moments problem refers to existence of average values of some quantities of interest.
 - attainability deals with the question of whether some states are accessible in a finite time or not.
 - type of distribution functions discusses some properties of a system with respect to shape of distribution function without explicit determination of the function.
 - fairness as a property required from the users' point of view.
2. Provide easily computable conditions for various kinds of stabilities (methodological studies described formally in the proposal).
3. Apply these criteria to stability analysis of the following local area networks.
 - buffered asymmetric packet broadcast contention (ALOHA-type) system.
 - multiaccess system with conflict resolution algorithms.
 - ETHERNET with exponential back-off algorithms.

– token-passing computer networks.

4. Extend these analysis to other objectives in a qualitative approach, that is, methodological backgrounds for various kinds of approximations, bounds and so forth as pointed out in Section 3.

In the first year of the proposal we expect to provide precise definitions for various kinds of stabilities and to establish some stability conditions. We apply these conditions to study stability regions of networks discussed in 3). We focus our attention on ergodicity problems, partial ergodicity and shape of steady-state distributions. In the next year we plan more advanced studies on ergodicity, mutual relationships between ergodicity and partial ergodicity, finite moments problems, fairness and other criteria. Finally, we intend that this study begins a trend or a general approach to computer network performance evaluation through qualitative analysis.

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