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A MODEL FOR VALVE FLOW TAKING NON STEADY FLOW INTO ACCOUNT

Part I

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1.SUMMARY

The object of this paper is to discuss a model accounting for non steady flow effects in valve channels. Gas mass inertia is found to be the most important effect. Adequate simplifications are introduced to allow for a manageable system of equations. To demonstrate the order of magnitude of inertia effects, analytical solutions are presented for simple boundary conditions.

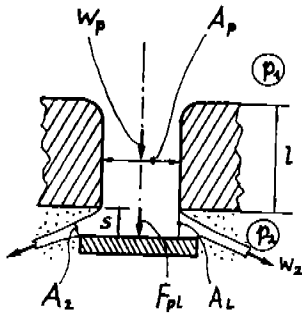
equ(2.1),(2.2),(2.3),(2.4) are very simple, problems arise when looking for adequate discharge and force coefficients C_D, c_p . Experiments may give an answer, but it is not always so easy to judge whether results from simple experiments can be properly transferred to conditions in real compressor operation(e.g. influence of viscosity or compressibility).

2.THE STEADY STATE FLOW CONCEPT

We may summarize the steady state flow concept for compressor valve flow as given in Table 1(incompressible flow).While

Discussing these questions of valve flow in a scientific way makes it necessary to build up a framework of theoretical insights, supporting experiments and numerical data for the designer. The author has contributed to this framework in a paper presented to the 1982 Purdue Conference [1].

TABLE 1 STEADY STATE FLOW CONCEPT, BASIC EQUATIONS



- ρ ...density of gas
- A_p ..port area
- A_2 ..actual flow area
- w_p ..port velocity
- A_L ..opening area
- s ...valve plate lift
- C_D ..discharge coeff.
- c_p ..force coefficient
- $p_1-p_2 = \Delta p$...pressure difference across valve

$w_2 = \sqrt{\frac{2(p_1-p_2)}{\rho}}$ (2.1) velocity of emerging jet

$\dot{V} = A_2 w_2 = A_L C_D w_2$ (2.2) volume flow rate

$\dot{m} = A_L C_D w_2 \rho$ (2.3) mass flow rate

$F_{pl} = A_p c_p \Delta p$ (2.4) gas force on valve plate

The aim of the present paper is to extend this framework to non steady flow effects.

In the steady state flow concept a change in pressure difference Δp causes an instantaneous change of the velocity field in the valve channel without any delay. Exactly this would be true only if the gas has no inertia. However, changes in pressure difference Δp and in valve flow area A_L are in many cases "slow enough" that the steady state equations can give a reasonable approach. But if one looks at plots of valve lift-time histories in literature -often with valve flutter included- one may be in doubt if steady state equations are justified in every case.

3. DESCRIPTION OF NON STEADY FLOW PHENOMENA IN VALVE CHANNELS

Before going to equations let us describe non steady flow effects in a phenomenological way. Having in mind the existing fluid flow theory, one has to imagine a valve opening and closing process as follows, fig.1:

Phase I: When the plate begins lifting gas enters from both sides and fills up the gap. A small depression area is built up between plate and seat. After the gap has been filled up (in a very short time) a unidirectional flow field builds up. At the beginning of this flow no separation at the edges take place, because the fluid has lost nearly no energy in the just being formed boundary layers. At the end of phase I separation starts at the seat edge

and a wake begins to form. The duration of phase I may be estimated from a comparison with related problems in external flow (e.g. flow around a cylinder which starts moving with constant acceleration, see e.g. [2]) to

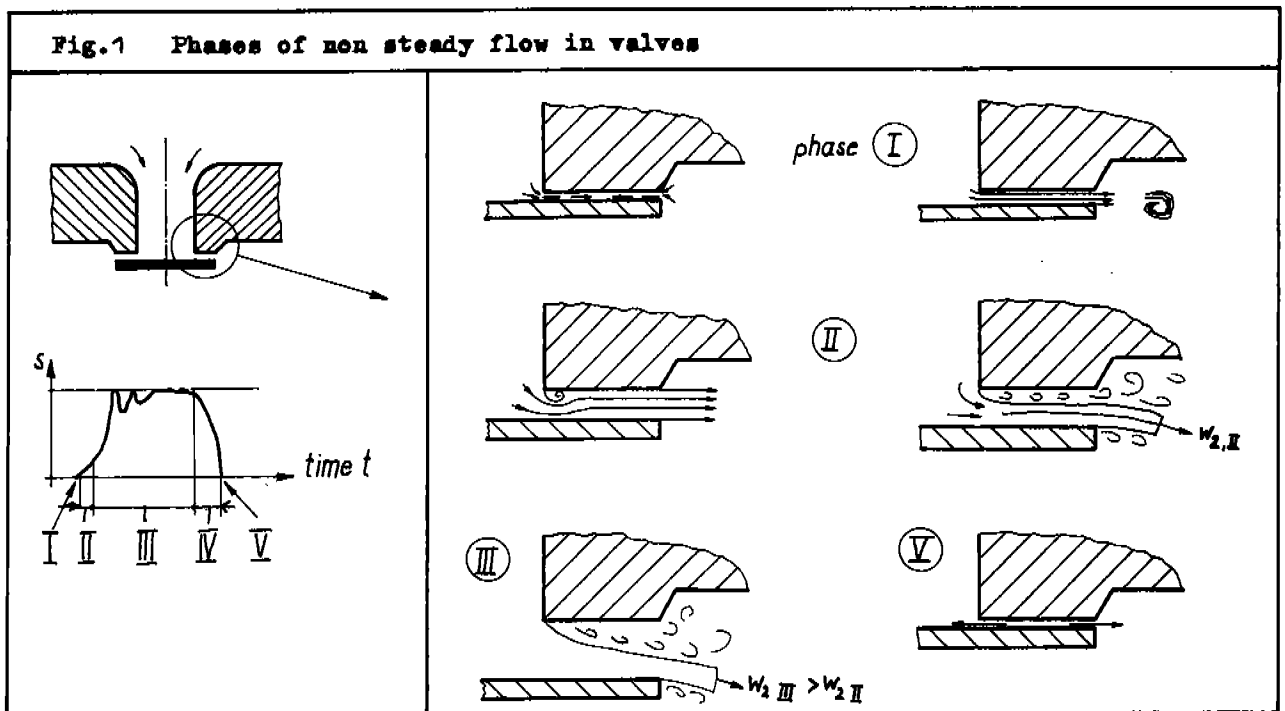
$$T_I = l/\bar{w}_2 \quad (3.1)$$

- 1...Thickness of seat plate
- \bar{w}_2 ...steady state velocity

Taking for example $l=1\text{cm}$ and $w_2=100\text{m/s}$ one gets $T_I=10^{-5}$ s. This corresponds to about 1 degree crank angle at 1800rpm.

Phase II: In this phase boundary layers have to pass the seat edge which have lost a good deal of their kinetic energy by friction. They cannot follow a way around the sharp seat edges and separation starts. Immediately wakes are formed and the flow pattern gets the typical appearance which we already know from steady state flow (see e.g. [1]). This appearance similar to a steady state flow, however, does not mean, that the flow has already reached steady state conditions. The duration of phase II -from beginning of separation to full wake formation- is of the same order of magnitude as phase I.

Phase III: Though the appearance of flow pattern at the beginning of phase III looks like steady state flow, the velocity level is still far from the final steady state flow level. The flow accelerates within its more or less invariable contours



(outside wakes) and reaches the steady state flow value asymptotically.

Phase IV: When the valve plate moves against its seat during closing period the flow is decelerated (even at constant Δp) and again causes an inertia effect. The principal flow pattern of phase III with separation and wakes is maintained.

Phase V: During the final approach of the valve plate a gas squeezing effect takes place which in some special cases may reduce the impact velocity.

Phase III and IV are by far the longest periods and of primary interest for modeling the compressor process. Fortunately these periods of non steady flow are accessible to a simplified theoretical treatment (which is outlined in section 4).

Theoretical treatment of phase II is very complex and involves 2 and 3 dimensional fluid flow theory. Phase I and V may be treated also theoretically but they are of secondary importance in comparison with phases III and IV.

Non steady phenomena outside the valve channels -e.g. the forming of a starting vortex- may be of scientific interest but what counts are only the phenomena inside the valve channel i.e. the region which "feels" the pressure difference $p_1 - p_2 = \Delta p$.

The implication of non steady flow within phases III and IV may be attributed to some distinct effects:

a) Effect of the valve plate as a moving boundary. As the plate velocity in most cases is at its maximum typically only 2-3% of gas velocity, this effect may be neglected.

b) Effect of gas inertia. If we e.g. suddenly apply a certain pressure difference Δp to an open valve, the steady state velocity according to equ(2.1) is obtained only asymptotically, because the gas in the flow channels has a certain mass and hence inertia. This inertia effect is by far the most important of all non steady effects. Gas accelerations may be caused by increases of Δp or A_L or by both. Deceleration of flow in a valve channel at the closing period may lead to exit velocities greater than predicted by steady state theory.

c) Effect of compressibility on non steady flow in valve channels. Small pressure disturbances propagate with the velocity of sound a . For an estimation we may use a typical value of $a=400\text{m/s}$ and a valve channel length of $l=1\text{cm}$. Hence it takes a time of about

$$\Delta T_v = 0.01/400 = 0.25 \cdot 10^{-4} \text{ s}$$

for a pressure wave to travel across the valve. This corresponds to a crank angle $\Delta\alpha$ of about 0.5 degree at a speed of 3600 rpm and to proportionally smaller angles at smaller speeds. If $\Delta\alpha$ (resp. ΔT_v) is small compared with angles (resp. time intervals) in which changes of Δp or A_L take place, compressibility effects in valve flow may be neglected. A more or less arbitrary but reasonable guide line may be as follows:

Compressibility effects on non steady valve flow may be neglected if:

- valve opening (or closing) time is longer than the time which is necessary for a pressure wave to travel 5 times to and fro across the valve
- change in relative pressure difference $\Delta p/p_{1, \text{abs}}$ during an interval ΔT_v is smaller than 5%.

It should be mentioned that with very high flow velocities w in the valve a pressure wave travelling against flow proceeds only with a relative velocity $(a-w)$ and hence $\Delta T_v > l/a$. Therefore non steady compressibility effects are of course not negligible for flow velocities w near sonic velocity a .

To avoid misunderstandings it should be noted that neglecting compressibility effects in a system with very short distances -like in a valve- does of course not mean that these effects are not of great importance in a long system in the same compressor e.g. in the pipework.

In some specialized cases compressors may be operated in such a way, that the flow pattern in valves does not reach phase III at all. This is the case when a compressor is operated at very high pressure ratios Π (=delivery pressure : suction pressure), e.g. $\Pi=15$. Then a very small quantity of gas is taken in and pushed out during one cycle. For an estimation we introduce the following ratio of masses M :

$$M = \frac{\text{mass of gas resting in the valve passages just before opening}}{\text{mass taken in (=mass pushed out) during one cycle}}$$

For operating conditions with $M > 1$ the flow pattern will not reach phase III at all. Delivery valves are primarily concerned

4. BASIC EQUATIONS ACCOUNTING FOR GAS INERTIA EFFECTS

As we already have discussed in section 3 non steady compressibility effects in valve flow are usually of minor importance. Hence let us start with one dimensional non steady flow equation for an incompressible fluid.

In this section we consider valves with fixed lift s . For this reason non steady flow must be due to variations of pressure difference Δp .

4.1 NON STEADY FLOW IN VALVES WITH CONSTANT LIFT

Fig.2 gives a schematic sketch of the flow and velocity distribution along typical streamlines at two different points of time. The fluid accelerates from stagnant conditions (for simplicity assumed at line 1"-1') to port velocity w_1 . Further acceleration takes place near the valve plate, where the fluid gains the final velocity w_2 (=velocity of emerging jet). Principally this flow pattern is maintained at any time. The velocity level only is changed.

Looking at fig.2a it is evident that the acceleration of the fluid is finished at the first point of the streamline where the fluid "feels" the pressure p_2 . This is the seat edge 2" and the valve plate edge 2'. To simplify the procedure we now adopt a representative mean streamline 1-2 which divides the fluid stream into two equal halves. The equation for non steady flow for a frictionless incompressible fluid reads:

$$\frac{p_1}{\rho} = \frac{p_2}{\rho} + \frac{w_2^2}{2} + \int_{x_1}^{x_2} \frac{\partial W(x,t)}{\partial t} dx \quad (4.1)$$

Anyone not so familiar with valve flow may

be astonished about the fact that a frictionless flow equation is used. The author discussed in a previous paper [1] that energy losses are produced behind the valve by diffusion of the emerging jet in a turbulent mixing process and for this reason a frictionless flow equation is justified.

The integral term in equ(4.1) accounts for the non steady flow effect. If the velocity field does not vary with respect to time, the integral term vanishes and equ(4.1) leads immediately to equ(2.1).

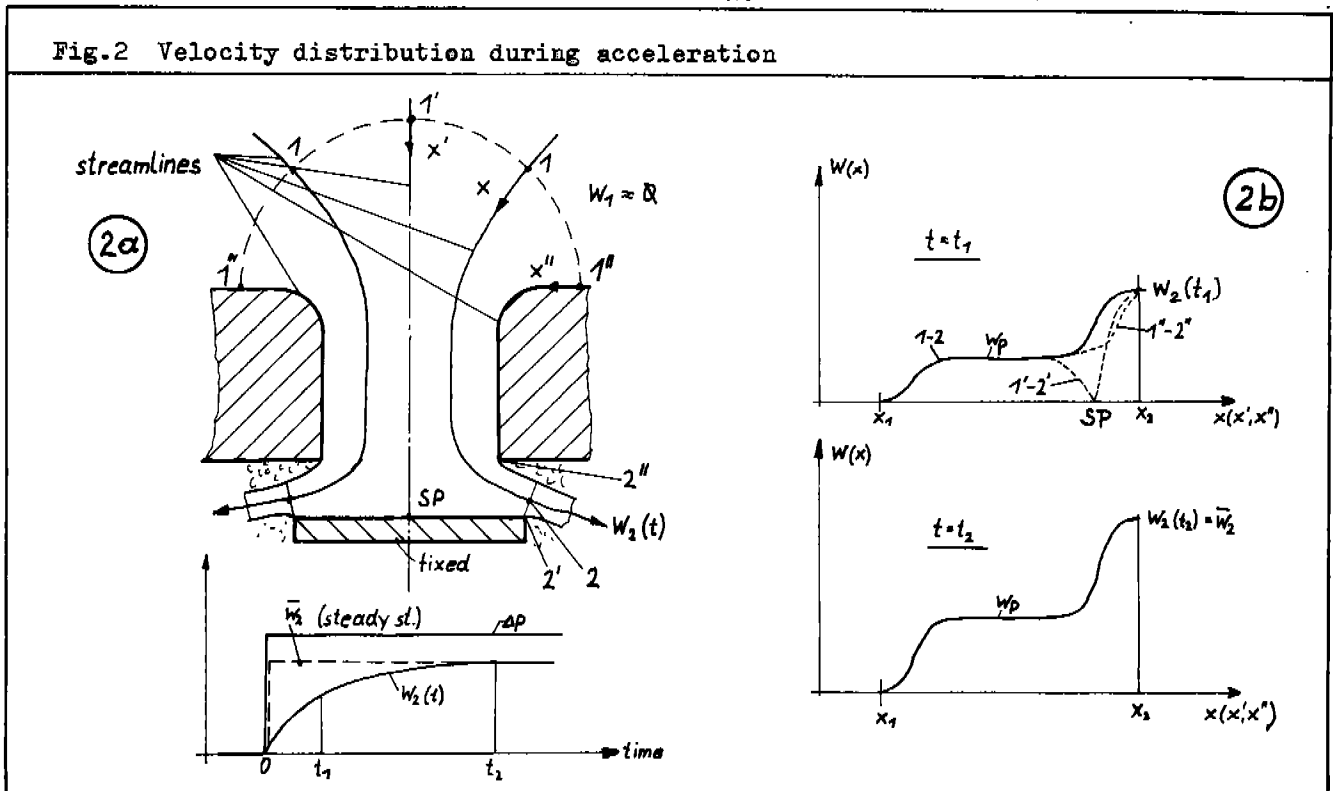
To evaluate the integral term in equ(4.1) the continuity equation is introduced which reads for an incompressible fluid with $A(x)$ as varying cross section along streamline 1-2

$$A_2 w_2(t) = A(x) \cdot W(x,t) \quad (4.2)$$

$w(x,t)$ stands for the time dependant velocity along the idealized streamline 1-2. To calculate A_2 the reasonable assumption is made that the steady state discharge coefficient C_D is valid also for non steady conditions in phase III and IV, hence $A_2 = A_1 C_D$. With equ(4.2) the integral term may be evaluated:

$$\int_{x_1}^{x_2} \frac{\partial W(x,t)}{\partial t} dx = \frac{dw_2}{dt} \int_{x_1}^{x_2} \frac{A_2}{A(x)} dx \quad (4.3)$$

Fig.2 Velocity distribution during acceleration



$$\int_{x_1}^{x_2} \frac{A_2}{A(x)} dx = I \quad (4.4)$$

The term I depends only on valve channel dimensions (and on lift s; $I=I(s)$) and represents the factor accounting for the inertia effect. For an evaluation we further idealize the flow path as indicated in fig.3 getting an area composed of two rectangles.

Using a function P(t) characterizing the time dependant pressure difference $\Delta p(t)$

$$P(t) = \frac{p_1(t) - p_2(t)}{\rho} \quad (4.5)$$

equ(4.1) reads

$$P(t) - \frac{w_2^2(t)}{2} - I \cdot \frac{dw_2(t)}{dt} = 0 \quad (4.6)$$

For the use with computers equ(4.6) may be written in terms of differences Δw_2 , Δt :

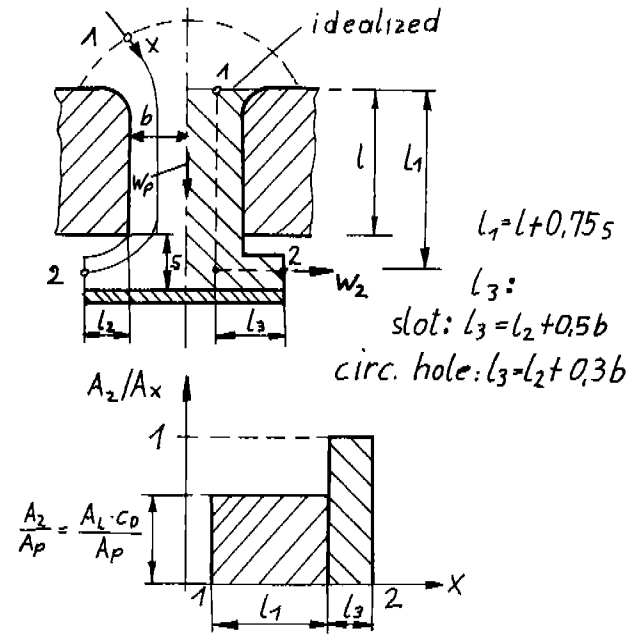
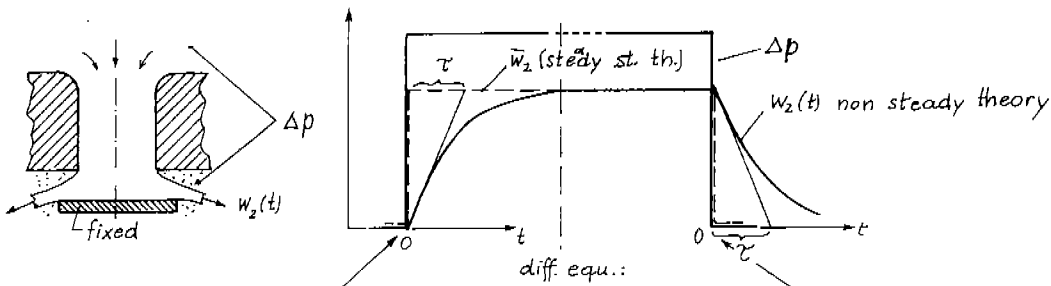


Fig.3 Calculation of inertia parameter I

TABLE 2 A simple example for inertia effect in valve flow



$t \leq 0: P=0 \quad w_2=0; \quad t > 0: P = \frac{\Delta p}{\rho} = \text{const.}$

$t \leq 0: P = \frac{\Delta p}{\rho} \quad w_2 = \frac{2\Delta p}{\rho} = \bar{w}_2$

$t > 0: P=0$

$t > 0:$

$$\frac{dw_2}{dt} = \frac{2P - w_2^2}{2I} \rightarrow \frac{dw_2}{2P - w_2^2} = \frac{dt}{2I}$$

$t > 0:$

$$\frac{dw_2}{dt} = -\frac{w_2^2}{2I}$$

solution: $w_2(t) = 2P \cdot \tanh\left(\sqrt{\frac{P}{2I}} \cdot t\right)$

solution: $w_2(t) = \frac{2I}{t + 2I/\bar{w}_2} \quad (4.10)$

using the asymptotic steady state value $\bar{w}_2 = \sqrt{2P}$

$\tau = 2I/\bar{w}_2$... time constant of transient at $t=0$

$$w_2(t) = \bar{w}_2 \cdot \tanh \frac{\bar{w}_2 \cdot t}{2I} = \bar{w}_2 \cdot \tanh \frac{t}{\tau} \quad (4.8)$$

$$\tau = \frac{2I}{\bar{w}_2} \quad (4.11)$$

$\tau = 2I/\bar{w}_2$... time constant of transient at $t=0$
(4.9)

$$\Delta W = \frac{1}{I} [P(t) - \frac{W_2^2(t)}{2}] \cdot \Delta t \quad (4.7)$$

$$W_2(t + \Delta t) = W_2(t) + \Delta W_2$$

4.2 Simple Examples

Let us at first demonstrate the inertia effect with exact solutions of equ(4.6). Consider a valve with fixed lift on which a constant pressure difference Δp is applied for a certain time interval ΔT . The reader may easily follow the different steps to the solution in Table 2. The transient of velocity $w_2(t)$ for a stepwise pressure rise is represented by a hyperbolic tangent function, the transient for stepwise pressure drop by a simple hyperbola. The transients may be roughly characterized by their time constants τ at the beginning. For both transients this is given by the very simple formula

$$\tau = \frac{2I}{W_2} \quad (4.12)$$

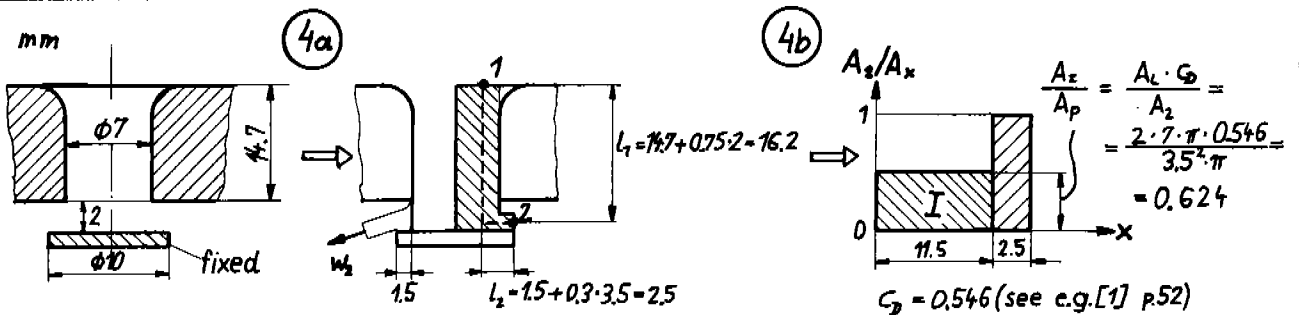
To get a real feeling for the relative importance of the inertia effect let us consider the delivery valve of an air compressor running at a speed of 1800rpm operating between 1 and 7 bar. The density of the gas is assumed to be $\rho = 5.38 \text{ kg/m}^3$. We apply a pressure difference $\Delta p = 17500 \text{ Pa} = 0.175 \text{ bar}$ for a time interval of 6 ms ($\cong 64.8 \text{ degr. c.a. at } 1800 \text{ rpm}$), see fig. 4. The transients have been calculated according to Table 2 and are plotted in fig. 4c.

Let us now consider another example. To a constant mean pressure difference Δp_m we superpose a sinusoidal pressure distribution with frequency f (period T) and amplitude $\pm \Delta p_a$ ($\omega = 2\pi f$)

$$\Delta p(t) = \Delta p_m + \Delta p_a \cdot \sin \omega t \quad (4.13)$$

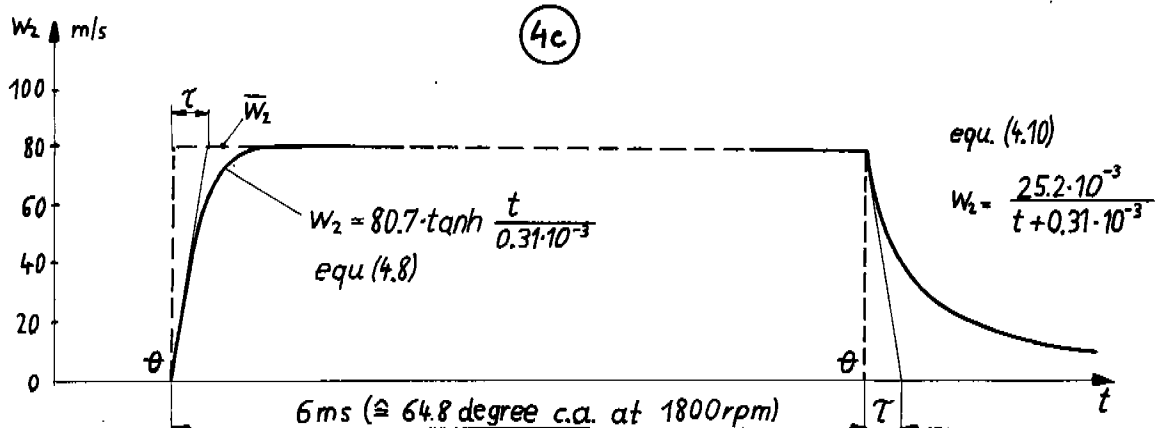
Equ(4.6) now becomes

Fig. 4 Numerical example to Table 2



Inertia parameter $I = 0.624 \cdot 16.2 + 1 \cdot 2.5 = 12.6 \text{ mm} = 12.6 \cdot 10^{-3} \text{ m}$; $\bar{w}_2 = \sqrt{2 \Delta p / \rho} = \sqrt{\frac{2 \cdot 17500}{5.38}} = 80.7 \text{ m/s}$

time constant $\tau = 2I / \bar{w}_2 = 2 \cdot 12.6 \cdot 10^{-3} / 80.7 = 0.31 \cdot 10^{-3} \text{ s} = 0.31 \text{ ms}$



$$\frac{dw_2}{dt} + \frac{w_2^2}{2} - \frac{\Delta p_m}{\rho I} - \frac{\Delta p_a}{\rho I} \cdot \sin \omega t = 0 \quad (4.14)$$

An analytical solution of equ(4.14) is not possible. Hence we look for an adequate approximate solution. We expect a sinusoidal velocity variation about a mean value and therefor state

$$w(t) = \sqrt{\frac{2\Delta p_m}{\rho}} + A \cdot \sin(\omega t + \varphi) \quad (4.15)$$

A and φ are two yet unknown parameters. Introducing equ(4.15) in equ(4.14) and neglecting the term with A^2 (which restricts our approximate solution to values $\Delta p_a < 0.3\Delta p_m$) leads to the following result

$$A = \frac{\Delta p_a}{\rho \sqrt{\omega^2 I^2 + \bar{w}_{2m}^2}} \quad (4.16)$$

$$\varphi = \arctan(-\omega I / \bar{w}_{2m}); \quad \bar{w}_{2m} = \sqrt{2\Delta p_m / \rho} \quad (4.17)$$

Compared with the steady state solution equ(4.16) and (4.17) indicate a reduction in amplitude A and a phase shift back by an angle φ . Fig.5 presents some results. The phase shift has a value of -33 degrees

(corresponding to $\Delta T = 0.14ms$ or 1.5 degr.c.a.)

4.3 FULL NON STEADY FLOW EQUATIONS

Now the restriction of a fixed lift is left. ($s=s(t)$). The continuity equation (4.2) and the non steady term(4.3) have now got to

$$W(x,t) = \frac{A_2(t)}{A(x,t)} \cdot w_2(t) = \frac{LsC_D}{A(x,s)} w_2(t) \quad (4.18)$$

$$\frac{\partial W(x,t)}{\partial t} = \frac{dw_2}{dt} \cdot \frac{A_2(t)}{A(x,t)} + w_2 \frac{ds}{dt} \cdot \frac{d}{ds} \left(\frac{LsC_D}{A(x,s)} \right)$$

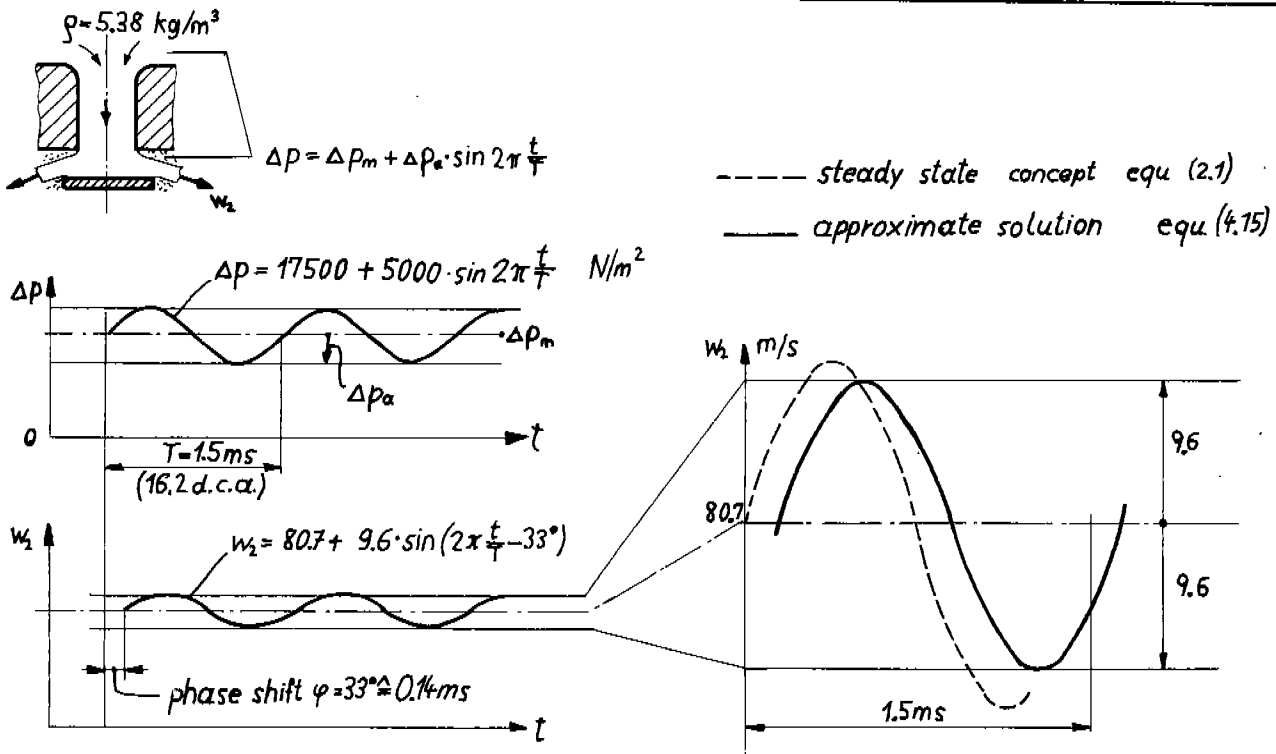
$$\frac{ds}{dt} = w_v \dots \text{valve plate velocity} \quad (+\text{for opening})$$

Using the same simplifications as in sections 4.1 and 4.2 for evaluating the integrals we get

$$\int_{x_1}^{x_2} \frac{\partial W(x,t)}{\partial t} dx = \frac{dw_2}{dt} \cdot I(s) + w_2 \cdot w_v \cdot \underbrace{\int_{x_1}^{x_2} \frac{d}{ds} \left(\frac{LsC_D}{A(x,s)} \right) dx}_{I_1(s)} = \frac{dw_2}{dt} \cdot I(s) + w_2 \cdot w_v \cdot I_1(s) \quad (4.20)$$

For a valve with dimensions as given in fig.4 the inertia parameters $I(s)$ and $I_1(s)$ may be easily evaluated (using C_D data from [1]), fig.6.

Fig.5 Another example: Velocity response to sinusoidal pressure variation



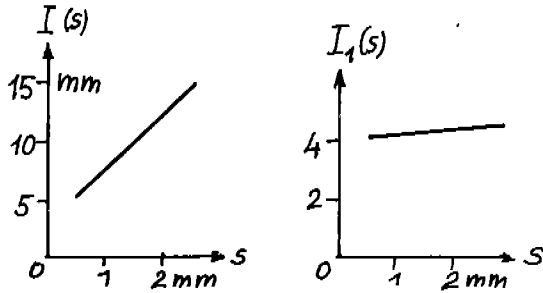


Fig.6 Inertia parameters for valve of fig.4

Equ(4.6) now reads

$$P(t) - \frac{w_2^2}{2} - \frac{dw_2}{dt} \cdot I(s) - w_2 w_v \cdot I_1(s) = 0 \quad (4.21)$$

This is the differential equation describing one dimensional non steady valve flow of an incompressible fluid. For the use with computers equ(4.21) may be integrated into a complete process modelling system and written in terms of differences Δw_2 , Δt :

$$\Delta w_2 = \frac{1}{I(s)} \left[P(t) - \frac{w_2^2(t)}{2} - I_1(s) \cdot w_2(t) \cdot w_v(t) \right] \Delta t$$

$$w_2(t + \Delta t) = w_2(t) + \Delta w_2 \quad (4.22)$$

A simple example

There is no chance to find a direct analytical solution of equ(4.21). In analogy to equ(4.15), (4.16), (4.17) we look for an approximate solution for a special case: sinusoidal velocity variations caused by forced sinusoidal valve movement, while pressure difference across valve shall be constant.

Forced valve movement:

$$s = s_m + a \cdot \sin(\omega \cdot t) \quad a/s_m \ll 1 \quad (4.22)$$

$$\omega = 2\pi f \quad f = 1/T \quad \text{frequency}$$

valve velocity: $w_v = ds/dt =$

$$a\omega \cos(\omega t) = w_{v, \max} \cdot \cos(\omega t)$$

approximate solution for velocity w_2 :

$$w_2 = w_{2m} + A \cdot \sin(\omega t + \varphi); \quad A/w_{2m} \ll 1 \quad (4.23)$$

Introducing this in equ(4.21) and omitting (second order) small quantities results in

$$P - \frac{w_{2m}^2}{2} - w_{2m} A \sin(\omega t + \varphi) - I(s_m) A \omega \cos(\omega t + \varphi) - I_1(s_m) w_{2m} a \omega \cos \omega t = 0 \quad (4.24)$$

with the following solution

$$w_{2m} = \sqrt{2P} = \sqrt{2\Delta p / \rho}$$

$$A = \frac{I_1(s_m) w_{2m} w_{v, \max}}{\sqrt{\omega^2 I(s_m)^2 + w_{2m}^2}} \quad (4.25)$$

$$\varphi = \arctan \left[-\frac{\omega I(s_m)}{w_{2m}} \right] - 90^\circ \quad (4.26)$$

It can be seen that the phase shift of the solution $w_2(t)$ with respect to $s(t)$ is greater than 90 degrees and depends on frequency and does not depend on amplitude. For an example we again use the valve of Fig.4 and the following data:

$$s = 1.5 + 0.5 \sin 2\pi f t \quad \text{in mm} \quad (f = 667 \text{ Hz})$$

$$w_{v, \max} = 2.09 \text{ m/s}, \quad w_{2m} = 80.7 \text{ m/s}$$

$$I(1.5 \text{ mm}) = 0.001025 \text{ m}, \quad I_1(1.5 \text{ mm}) = 4.36$$

Introducing these data in equ(4.25), (4.26) leads to the result

$$A = 8.05 \text{ m/s} \quad \text{amplitude of vel. variations}$$

$$\varphi = -118^\circ \quad \text{phase shift}$$

$$w_2 = 80.7 + 8.05 \cdot \sin(2 \cdot 667 t - 118^\circ)$$

Contrary to this result the steady state concept leads to

$$w_2 = 80.7 \text{ m/s} = \text{constant}$$