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DYNAMICS OF THE SWASH PLATE MECHANISM

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ABSTRACT

Several mechanisms have been used to convert the motion of a rotating shaft to the reciprocating motion needed to drive the pistons of a conventional compressor.

The most common mechanism is the crankshaft in which pistons are driven up and down in a direction normal to the drive shaft centerline. The dynamics of the crankshaft are well developed and readily available in design handbooks. The swash plate, another mechanism, produces reciprocating motion in a direction parallel to the centerline of the rotating driveshaft. Compressors of this type are called axial piston compressors.

This paper investigates the dynamics of the swash plate and develops the mathematical relationships necessary for evaluating stresses, bearing loads, and for sizing and locating counterweights for a proper running balance.

INTRODUCTION (See Figure 1)

The swash plate can be made up in many different configurations. This paper deals with perhaps the most common construction. That being a bearing where the inner race is fixed to the shaft at an angle and rotates with it. The outer race is restrained from rotating by the thrust rider. As the shaft rotates, the outer race precesses with a wobble producing a back and forth motion in an axial direction at every point on the bearing periphery. A compressor can be formed by linking the compression pistons to the outer bearing ring.

Controlling the distribution of masses of pistons, thrust riders, and counterweights, provides a means for obtaining a dynamic balance. The primary and secondary unbalanced forces and moments can be reduced to zero by geometry. The effect is a smooth running machine with a capability of high speed operation. In a multistage compressor, piston size and location should be determined for an even distribution of load. This reduces wear, lowers horsepower input, and reduces weight and cost by eliminating the need for a flywheel.

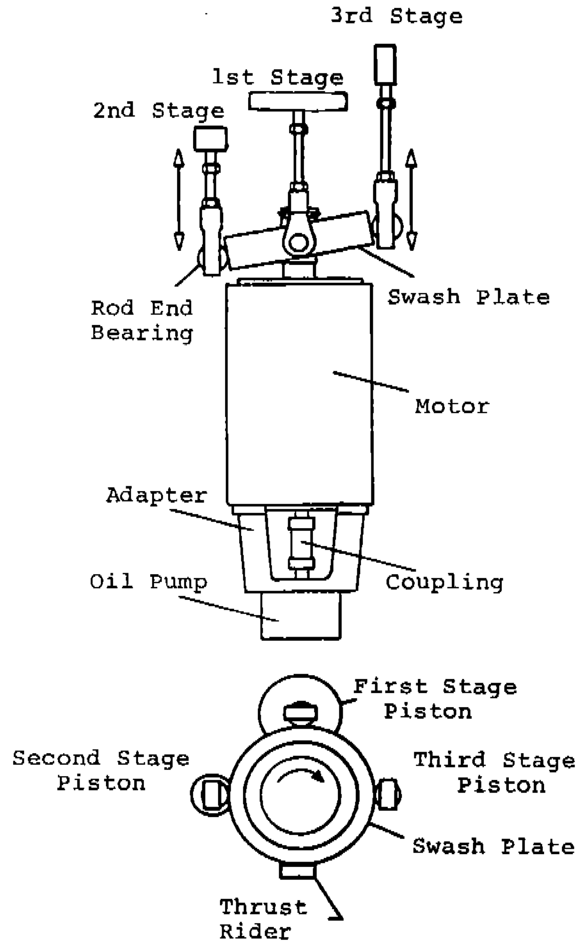

Figure 1

A Three Stage Swash Plate Compressor

THE COMPRESSION PROCESS

Modelling the compression process allows for a determination of local loads and stresses. These can be combined based on the orientation of the pistons on the swash plate to obtain the resultant or overall loads and stresses. This is necessary to properly size the bearings, crankshaft, and other load transmitting parts of the compressor.

The first step is to determine piston position, x, as a function of crank angle, θ. Refer to Figure 2 for this analysis. The analysis is performed for one stage, in this case the first stage. The constants are fixed by design. The variables change with crank angle.

$$\alpha = \arctan \frac{s}{(r^2 - s^2)^{1/2}} = \arctan \frac{x}{b}$$

therefore:

$$\frac{x}{b} = \frac{s}{(r^2 - s^2)^{1/2}} \qquad x = b \frac{s}{(r^2 - s^2)^{1/2}}$$

$$c = (r^2 - x^2)^{1/2}$$

$$b = c \cos\theta = (r^2 - x^2)^{1/2} \cos\theta$$

by substitution

$$x = s\ (r^2 - x^2)^{1/2} \cos\theta\ (r^2 - s^2)^{1/2}$$

rearranging terms and squaring

$$(r^2 - s^2) = \frac{s^2}{x^2}(r^2 - x^2) \cos^2\theta$$

$$(r^2 - s^2) = s^2 \left(\frac{r^2}{x^2} - 1\right) \cos^2\theta$$

$$\left(\frac{r^2}{s^2} - 1\right) \cos^2\theta^{-1} = \frac{r^2}{x^2} - 1$$

$$\frac{r^2}{x^2} = \left(\frac{r^2}{s^2} - 1\right)\left(\cos^2\theta\right)^{-1} + 1$$

$$\frac{1}{x^2} = \left(\frac{1}{s^2} - \frac{1}{r^2}\right)\left(\cos^2\theta\right)^{-1} + \frac{1}{r^2}$$

$$x^2 = \left|\left(\frac{1}{s^2} - \frac{1}{r^2}\right)\left(\cos^2\theta\right)^{-1} + \frac{1}{r^2}\right|^{-1}$$

$$x^2 = \cos^2\theta\left|\left(\frac{1}{s^2} - \frac{1}{r^2}\right) + \frac{1}{r^2}\left(\cos^2\theta\right)\right|^{-1}$$

extracting r^2

$$x^2 = r^2 \cos^2\theta\left|\left(\frac{r^2}{s^2} - 1\right) + \cos^2\theta\right|^{-1}$$

or

$$x^2 = r^2 \cos^2\theta\left|\frac{r^2}{s^2} + \left(\cos^2\theta - 1\right)\right|^{-1}$$

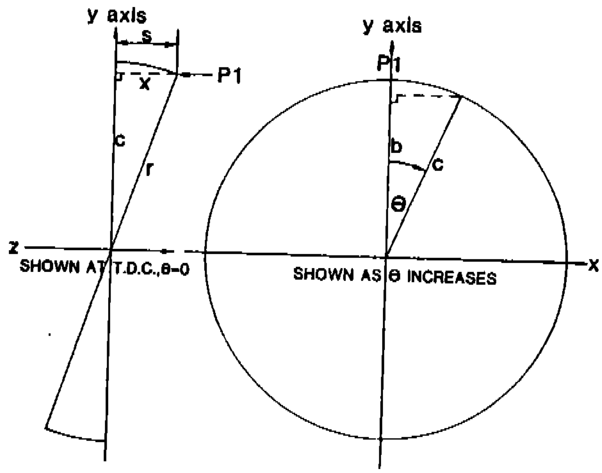

Figure 2 Swash Plate Geometry

substituting $-\sin^2\theta = \cos^2\theta - 1$ and taking the square root;

$$x = r \cos\theta \left(\frac{r^2}{s^2} - \sin^2\theta\right)^{-1/2}$$

since x varies between the limits defined by s a more appropriate form for this equation would be

$$x = s \cos\theta \left(1 - \frac{s^2}{r^2} \sin^2\theta\right)^{-1/2}$$

note from this expression that piston motion is not purely sinusoidal, yet sinusoidal motion would be a good first approximation.

For a multistage or multipiston compressor each cylinder can be analyzed by substituting $(\theta - \theta_n)$ for θ in this expression where θ_n equals this piston position on the swash plate (this also assumes r is equal for all pistons.)

For the purposes of this analysis we will assume ideal isentropic compression (ignoring losses due to valves, dead volume, or heat transfer within the cylinder.)

$$\frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^n$$

for an isentropic process n = k

$$\frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^k$$

cylinder volume = V = A(s - x) where A is piston area and s - x is the distance from piston to the head. For a given cylinder undergoing compression from position 1 to position 2

$$\frac{A_1(s - x_1)}{A_2(s - x_2)} = \left(\frac{P_2}{P_1}\right)^{1/k} \quad \text{where } A_1 = A_2$$

then

$$\frac{s - x_1}{s - x_2} = \left(\frac{P_2}{P_1}\right)^{1/k}$$

consider position 1 to be the start of the compression process where P_1 = inlet pressure and $x_1 = -s$, then for any position 2 (prior to when the discharge valve opens) the following relation holds:

$$\frac{2s}{s - x_2} = \left(\frac{P_2}{P_1}\right)^{1/k}$$

or

$$P_2 = \left(\frac{2s}{s - x_2}\right)^k P_1$$

it is useful to know at what point the discharge valve opens (i.e. P_2 equals the discharge pressure)
solving for :

$$x = s - 2s\left(\frac{P_1}{P_2}\right)^{1/k}$$

in this case $\frac{P_1}{P_2}$ represents the compression ratio of that stage.
With the relation for pressure, P, as a function of piston position, x, and piston position as a function of crank angle, θ, the thrust load on the bearings can be determined by summing the forces P x A, where A is the piston area, for all stages through one revolution of the crankshaft.

ARRANGEMENT OF PISTON MASSES FOR DYNAMIC BALANCE

A good dynamic balance can be achieved by proper mass and load distribution coupled with adequate counterweighting.

Consider a three stage compressor with a single thrust rider. Positions on the swash plate are as shown in Figure 3.

A = 1st Stage Piston
B = Thrust Rider
C = 2nd Stage Piston
D = 3rd Stage Piston
$\phi = 15^{\circ}$
r = radius

An equal load distribution occurs when $\phi = 30^{\circ}$ leaving 120° between A, C, and D. However, such a configuration would require the thrust rider B to have zero mass for a good mass balance. Thus the existence of a thrust rider at a position other than co-

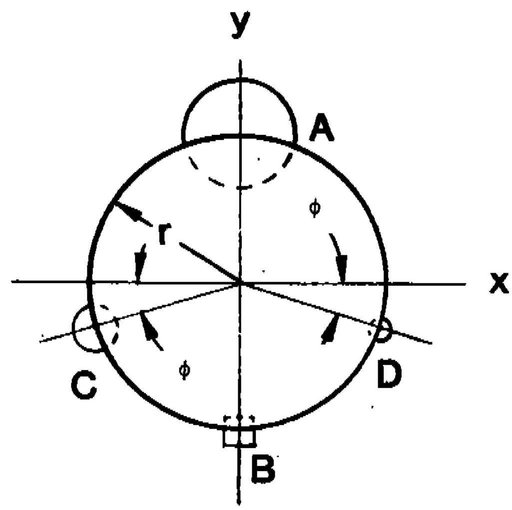

Figure 3

Mass & Load Distribution

incident with one of the pistons gives a less than ideal load distribution.

To minimize weight and product of inertia effects, the locations of four masses favors a 90° symmetrical distribution ($\phi = 0^{\circ}$). Since both configurations are used, four masses and three loading points, a compromise must be reached.

The kinetics and kinematics of the system lead to relative size requirements of the pistons and the thrust rider. Starting with an adequately designed thrust rider, the piston masses and counterweights can be sized.

It is important to note how the compromise was made to arrive at $\phi = 15^{\circ}$. Particularly, the size of the thrust rider bearing in comparison to the optimal first stage size (as determined by load capacity), coupled with the most even spacing between A, C, and D, resulted in $\phi = 15^{\circ}$ as the best trade-off (i.e. the lighter the thrust rider the better the load distribution.

Having established the relative positioning, a moment balance and moment of inertia balance is considered. See Figure 4.

For balance: $\Sigma M_x = 0$, $\Sigma M_y = 0$ (Statically)

and $I_{xx} = I_{yy}$

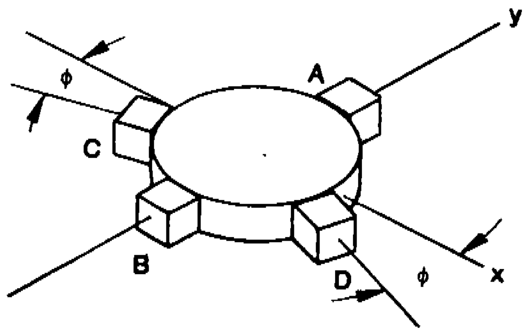

Figure 4

Inertia & Moment Balance

Assume A, B, C, and D are at an equal radius r from the axis of the machine.

Let m_A, m_B, m_C, and m_D equal the masses at each position, respectively.

g = gravitational acceleration.

Thus:

$$\Sigma M_y = 0 \qquad m_C gr \cos\phi - m_D gr \cos\phi = 0$$

$$m_C = m_D \tag{1}$$

$$\Sigma M_x = 0 \qquad m_A gr - m_B gr - m_C gr \sin\phi - m_D gr \sin\phi = 0$$

$$m_A - m_B - m_C \sin\phi - m_D \sin\phi = 0$$

Since $m_C = m_D$ $\quad m_A - m_B - 2m_C \sin\phi = 0$

Solving for m_A

$$m_A = m_B - 2m_C \sin\phi \tag{2}$$

For a moment of inertia balance:

$$I_{xx} = I_{yy}$$

This produces a situation where the masses are evenly distributed about the center of rotation. This is necessary for obtaining balance by counterweighting alone.

Thus:

$$I_{xx} = m_A r^2 + m_B r^2 + m_C (r \sin\phi)^2 + m_D (r \sin\phi)^2$$

$$I_{xx} = m_A r^2 + m_B r^2 + 2m_C r^2 \sin\phi \tag{3}$$

$$I_{yy} = m_C (r \cos\phi)^2 + m_D (r \cos\phi)^2$$

$$I_{yy} = 2m_C r^2 \cos^2\phi \tag{4}$$

Setting $I_{xx} = I_{yy}$

$$m_A r^2 + m_B r^2 + 2m_C r^2 \sin^2\phi = 2m_C r^2 \cos^2\phi$$

$$m_A + m_B + 2m_C \sin^2\phi = 2m_C \cos^2\phi$$

From the moment balance

$$m_A = m_B + 2m_C \sin\phi \tag{2}$$

Substituting for m_A from eq. (2)

$$m_B + 2m_C \sin\phi + m_B + 2m_C \sin^2\phi = 2m_C \cos^2\phi$$

rearranging

$$2m_B = 2m_C (\cos^2\phi - \sin^2\phi - \sin\phi)$$

$$m_B = m_C (\cos^2\phi - \sin^2\phi - \sin\phi)$$

or

$$m_C = \frac{m_B}{(\cos^2\phi - \sin^2\phi - \sin\phi)} \tag{5}$$

Experience has shown that m_B, the mass of the anti-rotation device (thrust rider), is the most convenient starting point when initiating a new design. Therefore, the piston masses are expressed in terms of m_B.

for $\phi = 15^\circ$ $\qquad m_C = 1.647\ m_B$ (6)

$m_C = m_D$ $\qquad m_D = 1.647\ m_B$ (7)

Since $m_A = m_B + 2m_C \sin\phi$ (2)

$\qquad m_A = 1.852\ m_B$ (8)

or setting $m_B = 1$ (9)

$\qquad m_A = 1.852$ (10)

$\qquad m_C = 1.647$ (11)

$\qquad m_D = 1.647$ (12)

for dynamic balance.

COUNTERWEIGHTING

For a disc shaped body fixed at an angle ψ to the axis of a rotating shaft a gyratory moment results. This moment rotates with the shaft at angular velocity ω.

For a swash plate mechanism where the inner race rotates and the outer race wobbles without rotating (restricted by the thrust rider) two moments are of primary concern.

The radial moment of inertia of the inner race which rotates with the shaft tends to decrease the angle ψ because of the gyratory restoring moment produced (M_I). The moment of inertia of a wobbling outer race tends to increase the angle ψ because of the inertial moment (M_O) generated by the change of direction (i.e. axial reciprocation). The difference between these two moments is an unbalanced moment M_G which results in rough running (i.e. $M_G = M_O - M_I$). (See Figure 5).

From the dynamics of a rigid body rotating about an axis we have

$$M_I = I_I\omega^2 \sin\psi \cos\psi \qquad (1)$$

$$M_O = I_O\omega^2 \sin\psi \cos\psi \qquad (2)$$

where I_I = radial moment of inertia of the rotating inner race.

and I_O = radial moment of inertia of the reciprocating (non-rotating) outer race (includes piston and thrust rider masses).

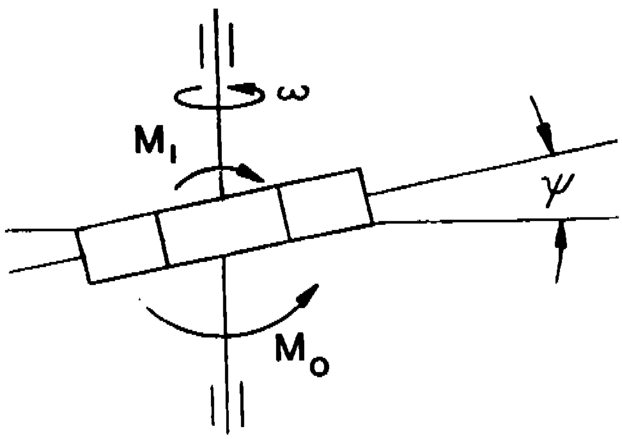

Figure 5

Gyratory Moments

Since $M_G = M_O - M_I$

Then: $$M_G = (I_O - I_I)\omega^2 \sin\psi \cos\psi \qquad (3)$$

By physically weighing and dimensioning parts, accurate values of I_I and I_O can be determined.

Since the direction of moment M_G rotates with the shaft and its magnitude is constant it is possible to reach a smooth running balance by counterweighting.

Consider two equal masses eccentrically mounted to the shaft, 180° opposed at separation b and radius a.

a = constant
b = constant
m_E = constant

See Figure 6.

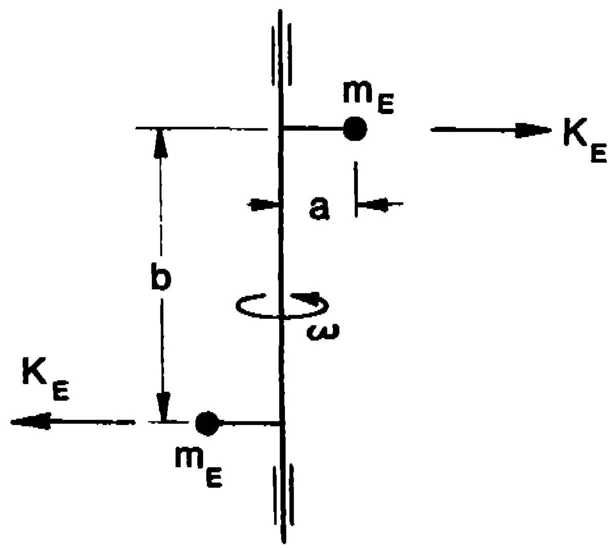

Figure 6

Counterweights

As these masses rotate about the shaft a centrifugal force K_E is generated for each.

$$K_E = m_E\omega^2 a \qquad (4)$$

A couple $M_E = 2K_E b/2$ results

$$M_E = m_E\omega^2 ab \qquad (5)$$

For a running balance the moments must be equated.

$$M_G = M_E \tag{6}$$

Thus:

$$(I_O - I_I)\omega^2 \sin\psi \cos\psi = M_E \omega^2 ab \tag{7}$$

$$(I_O - I_I)\sin\psi \cos\psi = M_E ab \tag{8}$$

Since I_I, I_O can be calculated and ψ is known m_E, a and b can be proportioned to best suit the machine design.

1. Note that counterweighting is independent of running speed.
2. Note that the greater the distance **b**, the smaller the counterweights need to be.
3. Note also that the center of M_G need not coincide with the center M_E to obtain balance. Where they are not coincident a bending moment is induced in the shaft. If the shaft is designed with an adequate stiffness (giving deflections of approximately .001 inch or less due to the bending moment) any increase in vibration should be negligible.

NOTATIONS (See Figure 2)

The constants are fixed by design.
The variables change with crank angle.

θ = crank angle $\theta = 0$ at T.D.C. (variable)
s = ½ piston stroke length (constant)
x = piston position, x = s at $\theta = 0$ and $x = 0$ at $\theta = 90$ (variable)
r = distance from center of crankshaft to piston (constant)
α = angle of the swash plate to the crankshaft (constant)
c = distance from piston to center of wobble plate normal to x-axis = $(r^2 - x^2)^{\frac{1}{2}}$ (variable)

CONCLUSIONS

1) It is possible to analyze the dynamic behavior of the swash plate mechanism to optimize piston loading and running balance.

2) Swash plate piston motion is not purely sinusoidal, yet it is close enough to be a good first approximation.

3) When the number of masses on the swash plate does not match the number of loading points (as occurs when a separate thrust bearing is used) a compromise must be reached when determining their distribution. Here an optimization technique may be used.

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