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# A Simulation Model for Fixed Vane Rotary Compressor Using Real Gas Properties

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# A SIMULATION MODEL FOR FIXED VANE ROTARY COMPRESSOR USING REAL GAS PROPERTIES

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ABSTRACT

The model is based on a control volume for suction and pressure volume. The first law of thermodynamics and the law of continuity in dynamic form are used on these control volumes. This means that thermodynamic properties, mass flow, heat effect and compression power are calculated as a function of the time or angle of rotation instead of a static average value.

The model describes:

- Suction mass flow.
- Pressure drop and temperature rise in suction pipe.
- Gas leakage from pressure volume to suction volume.
- Oil leakage from shell and shaft to suction and pressure volume.
- Discharge valve (one-dimensional model)
- Discharge mass flow.
- Gas heat exchange with cylinder walls.
- Compression power.
- Re-expansion of residue gas.
- Counterflow though suction pipe.
- Shaft torque arising from gas forces.
- Pressure, temperature, internal energy and enthalpy from the refrigerant equations (real gas)

The differential equations for the first law of thermodynamics and the law of continuity are solved numerically by a simple Euler integration.

Calculated values of volumetric efficiency are compared with measurements.

THEORETICAL MODEL BASIS

Control volumes

Control volumes are used for suction and discharge sides, as shown on fig. 1. The first law of thermodynamics, the law of continuity, and the equation of state for the actual refrigerant are used for these control volumes.

First law of thermodynamics

For the suction volume, the first law assumes the following form, in that the kinetic and potential energy is ignored

$$\frac{dU_1}{dt} = \dot{Q}_1 + \dot{W}_1 + \dot{m}_1 \cdot h_I + (\dot{m}_{L1} + \dot{m}_{L2}) \cdot h_2 - \dot{m}_b \cdot h_1 \quad (1)$$

Correspondingly, for the control volume for the discharge side

$$\frac{dU_2}{dt} = \dot{Q}_2 + \dot{W}_2 - (\dot{m}_2 + \dot{m}_{L1} + \dot{m}_{L2}) \cdot h_2 \quad (2)$$

Law of continuity

For the suction volume

$$\frac{dM_1}{dt} = \dot{m}_1 + \dot{m}_{L1} + \dot{m}_{L2} + a \cdot (\dot{m}_{o11} + \dot{m}_{o12}) - \dot{m}_b \quad (3)$$

Thus, for the discharge volume

$$\frac{dM_2}{dt} = - \dot{m}_2 - \dot{m}_{L1} - \dot{m}_{L2} + a \cdot (\dot{m}_{o21} + \dot{m}_{o22}) \quad (4)$$

These four differential equations give the total internal energy and mass as a function of time. The specific internal energy can be found from these values

$$u_k = \frac{U_k}{M_k} \quad , \; k = 1,2 \quad (5)$$

The volume of suction and discharge side as a function of the time is geometrically given, as we shall see later. The specific volume can now be calculated

$$v_k = \frac{V_k}{M_k} \quad , \; k = 1,2 \quad (6)$$

Equation of state

From the specific internal energy, specific volume, and the equation of state for the refrigerant used, the pressure, temperature and specific enthalpy can be calculated. The equations given in (1) are used to describe the refrigerant. From these equations, gas pressure p as a function of temperature T and specific volume v can be determined

$$p = F_p(T, v) \quad (7)$$

and specific enthalpy

$$h = F_h(p,T,v) \tag{8}$$

The definition of enthalpy is given by

$$h = u + p \cdot v \tag{9}$$

For known u and v, the three equations (7) - (9) have three unknowns, p, T and h, which can be calculated by iteration.

Geometric depended values

The geometric depended values as a function of the angle of rotation are calculated in appendix A. These are the values of suction and discharge volumes, surface areas, etc. The angle of rotation is given by

$$\theta = \omega \cdot t \tag{10}$$

The variables on the righthand side of differential equations (1) - (4) can now be determined from the calculated refrigerant properties and geometric values.

Compression power

The momentary power for one of the control volumes is

$$\dot{W} = -p\,\frac{dV}{dt} = -p\,\frac{dV}{d\theta}\cdot\omega \tag{11}$$

Heat Transfer

The heat transfer in cylinder can be divided into two categories

1 - The gas heat exchange with the walls
2 - The gas heating/cooling by the oil leakages

For the suction side

$$\dot{Q}_{11} = A_{o1}\cdot H_{va1}\cdot(T_w - T_1) \tag{13}$$

$$\dot{Q}_{12} = (\dot{m}_{o11} + \dot{m}_{o12})\cdot C_{poil}\cdot(T_{oil} - T_1) \tag{14}$$

$$\dot{Q}_1 = \dot{Q}_{11} + \dot{Q}_{12} \tag{15}$$

There are corresponding equations for the discharge side.

Gas Leakages

Assuming that diverse gas leakages can be described by a nozzle flow, and under the assumption of adiabatic frictionless flow and ideal gas, the mass flow per unit of area can be expressed as

$$\dot{g} = \sqrt{\frac{2\cdot n}{n-1}\cdot p_2\cdot\rho_2\cdot\left(\frac{p_1}{p_2}\right)^{\frac{2}{n}}\cdot\left(1-\left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}}\right)} \tag{16}$$

Flows are critical when

$$\frac{p_1}{p_2} < \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$$

and the mass flow per unit of area is calculated from

$$\dot{g} = \sqrt{p_2\cdot\rho_2\cdot\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}} \tag{17}$$

Because of the pressure difference between discharge and suction sides there is gas leakage in the clearance between roller and cylinder wall and in the clearance between vane and cylinder top/bottom

$$\dot{m}_{L1} = 2\,\varphi_{L1}\cdot C_{VC}\cdot L_V\cdot\dot{g}_{L1} \tag{18}$$

$$\dot{m}_{L2} = \varphi_{L2}\cdot C_{RC}\cdot H\cdot\dot{g}_{L2} \tag{19}$$

$C_{RC}$ varies with the angle of rotation because of the varying force on the roller which is displaced in relation to the eccentricity. $C_{RC}$ is calculated from a balance of forces on the roller (2).

Oil Leakage

Oil leakage occurs from the shaft via roller and cylinder top/bottom out to the suction and discharge volume, and from the compressor pot via the clearance between vane and slot. For lamina flow between to infinitely developed parallel plates

$$\dot{m}_{oil} = \tfrac{1}{2}\cdot B\cdot U\cdot h\cdot\rho_o + \frac{B\cdot h^3}{12\cdot\mu}\cdot\frac{p}{L}\cdot\rho_o \tag{20}$$

The clearance considered has a very small height in relation to breadth and length, which confirms the assumption of infinitely developed plates. The oil leakage from the lubrication channels in the shaft to the suction side via the clearance between roller and cylinder top/bottom is

$$\dot{m}_{o11} = 2\,\frac{(D_1 - D_3)\cdot C_{RCT}^3\cdot(p_t - p_1)}{24\cdot\mu\cdot B_{\theta 1}}\cdot\rho_o \tag{21}$$

Correspondingly, for the discharge side

$$\dot{m}_{o21} = 2\,\frac{(D_1 - D_3)\cdot C_{RCT}^3\cdot(p_t - p_2)}{24\cdot\mu\cdot B_{\theta 2}}\cdot\rho_o \tag{22}$$

It is assumed that the oil leakage from the compressor volume via the vane only occurs to the discharge side because the vane will mainly be pressed over towards the suction side

$$\dot{m}_{o22} = \frac{H \cdot C_{VS}^3 \cdot (p_t - p_2)}{12 \cdot \mu \cdot L_s} \rho_{\bullet} + \frac{1}{2} H \cdot U_v \cdot C_{VS} \rho_{\bullet} \quad (23)$$

### Suction mass flow

The suction mass flow is calculated from the pressure difference between suction pressure and the pressure in the suction volume

$$p_s - p_1 = (\lambda \frac{L_p}{D_p} + \sum f_i) \cdot \frac{1}{2} \tilde{\rho} \, \bar{u}^2 \quad (24)$$

$$\dot{m}_1 = \frac{\pi}{4} \cdot D_p^2 \cdot \bar{u} \cdot \rho_I \quad (25)$$

Mass density $\rho_I$ is determined from tempe - rature $T_I$ at the suction volume inlet. The temperature rise through the suction pipe is found by assuming constant wall temperature $T_W$

$$T_I = T_W - (T_W - T_S) \cdot e^{-\frac{A_p H_p}{\dot{m}_1 C_p}} \quad (26)$$

### Exhaust mass flow

The exhaust mass flow is dependent on the discharge valve lifting height, the pressure difference and the discharge hole area. A simple valve model is used (3)

$$M_V \frac{d^2 x}{dt^2} + C_V \frac{dx}{dt} + K_J \, x = A_t \cdot (p_2 - p_t) \quad (27)$$

$$\dot{m}_2 = \varphi_2 \cdot A_k \cdot \dot{g}_2 \quad (28)$$

### Re-expansion of residual gas

It is assumed that at the moment when the roller is at top dead point momentary re-expansion occure. The residual gas is re-expanded in the whole suction volume. An internal energy balance gives

$$U_2 = M_1 \cdot u_1 + M_R \cdot u_R \quad (29)$$

$$M_2 = M_1 + M_R \quad (30)$$

$$u_2 = \frac{U_2}{M_2} \quad (31)$$

$$v_2 = \frac{V_1}{M_2} \quad (32)$$

Index 2 is used on the calculated values because the suction volume has now become discharge volume. Pressure, temperature and enthalpy after re-expansion can be calculated form specific energy and volume.

### Return Flow

After re-expansion of the residual gas, pressure $p_2$ will be larger than suction pressure $p_S$, which is why there will be a return flow through the suction pipe during the first $\theta_S$ degrees of rotation. The return flow must be through the clearance between roller and cylinder wall at the suction pipe opening. Distance S between roller and wall is calculated as a function of angle of rotation. The mass flow per unit of area is calculated as nozzle flow and we get

$$m_b = \varphi_b \cdot S(\theta) \cdot H \cdot \dot{g}_b \quad , \theta < \theta_S \quad (33)$$

### Final calculations

A numerical solution of the differential equations gives all variable values as functions of time. Average values are found by integration over a period of one revo - lution. Power consumption, performance and volumetric efficiency can then be calculated.

## COMPARISON OF MEASUREMENTS AND CALCULATIONS

To verify the computer model, comparisons are made of measurements and calculations on performance and power consumption. Computer model adjustment is made at check - point (-23.3/54.4/32C - 60Hz) as regards nozzle coefficients for gas leakages and heat transfer coefficients in the cylinder chamber. The coefficients so adjusted are kept constant by calculation with other operating conditions.

### Measuring object - Measuring methods

Measurements are taken on a fixed vane rotary compressor mounted in a flanged pot. Measurement of performance is by calorimeter and power consumption by wattmeter. The shaft power is calculated using motor efficiency. The performance measurements are given in accordance with CECOMAF con - ditions, i.e. without subcooling.

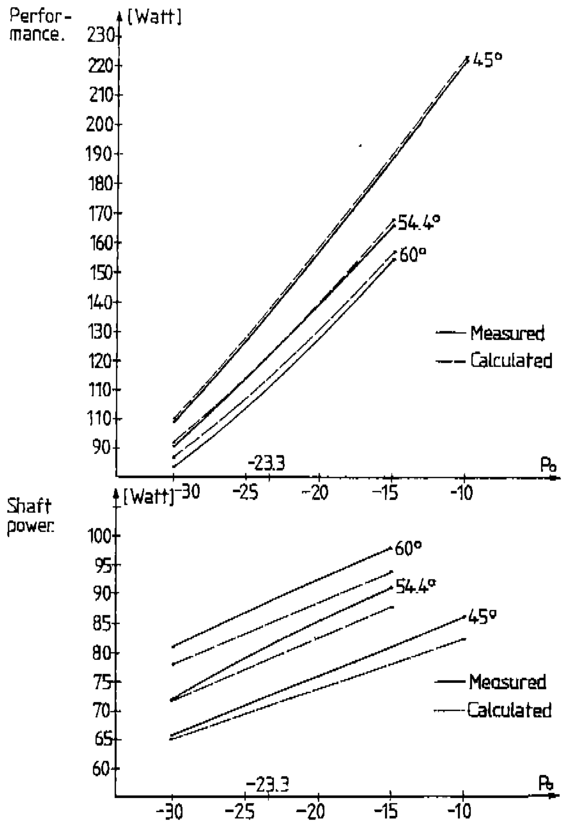
### Compressor data

| | |
|---|---|
| Displacement | : 3.0 $cm^3$ |
| Cylinder diameter | : 38.0 mm |
| Cylinder height | : 10.6 mm |
| Roller diameter | : 32.9 mm |
| Eccentric diameter | : 23.0 mm |
| Vane thickness | : 3.0 mm |
| Relative clearance volume | : 1.2 % |
| Motor efficiency | : 0.73 - 0.77 |
| Refrigerant | : R12 |
| Oil | : G46 |

### Characteristic for performance and power consumption

The characteristic for performance and

shaft power is plotted on the figure below. The curves show measured and calculated values.




It can be seen from the curves that there is good agreement between measurements and calculations over a wide area. The measured shaft power is however greater than the calculated shaft power especially by high condensing pressure.

## Analysis of power consumption and volumetric loss

To obtain a deeper analysis of power con - sumption and volumetric loss a comparison is made of measurements and calculation in checkpoint -23.3/54.4/32C - 60Hz.

| | Measurement | Calculations |
|---|---|---|
| Performance : | 122 Watt | 121 Watt |
| Shaft power : | 81 Watt | 79 Watt |
| Vol. eff. : | 90 % | 89.5 % |
| Mass flow : | 1.01 g/s | 1.00 g/s |
| Pot temp. : | 78 C | 78 C |

Calculated distribution of vol. loss:

| | |
|---|---|
| External gas leakage | 1.0 % |
| Internal gas leakage | 2.5 % |
| Heating in suction pipe | 4.2 % |
| Heating in cylinder | 2.5 % |
| Re-expansion | 0.3 % |
| Total volumetric loss | 10.5 % |

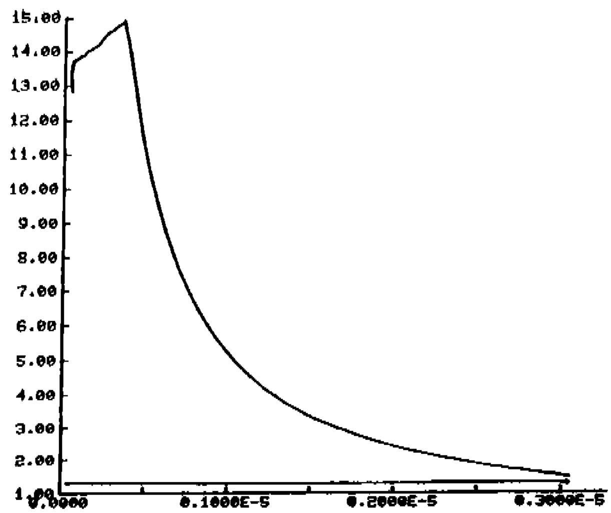
Calculated distribution of shaft power:

| | |
|---|---|
| Friction loss | 11.7 Watt |
| Gas compression power | 67.0 Watt |

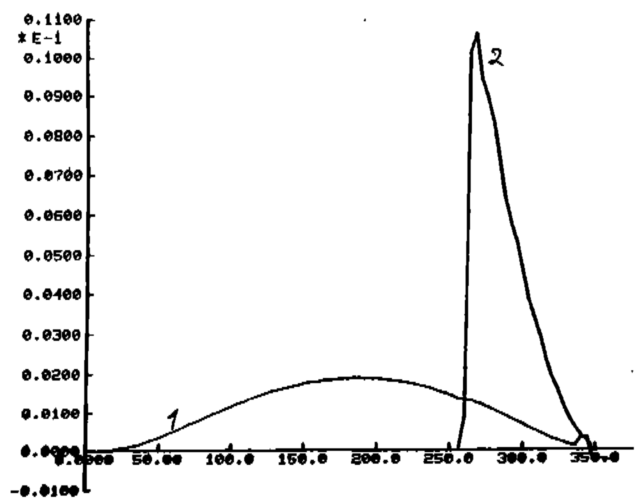
The gas compression power includes the following contributions:

| | |
|---|---|
| Overshoot, discharge valve | 1.0 Watt |
| Undershoot, suction pipe | 0.3 Watt |
| Re-expansion | 4.8 Watt |

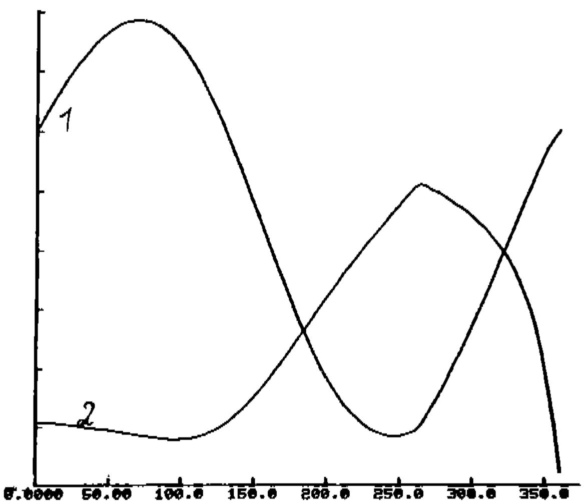
From the comparison between measured and calculated characteristics it is assumed probable that the distribution between gas power and friction and bearing loss is correct.
The loss distribution with regard to performance is only verified by measurements of loss in suction pipe. The distribution between the internal losses is difficult to demonstrate.
The reason for the deviations between measured and calculated shaft power is not explained.




P-V Graph




1 - Suction mass flow rate
2 - Discharge mass flow rate




1 – Clearance between roller and cylinder wall
2 – Roller eccentric displacement

APPENDIX A

Geometric values as a function of angle of rotation.

From fig. 2 the following values can be calculated:

$$E = R_2 - R_1 - C_{RC}$$

$$\beta = \mathrm{Arcsin}\left(\frac{E}{R_1}\cdot \mathrm{Sin}(\theta)\right)$$

$$\gamma = \theta + \beta$$

$$L = \sqrt{R_1^2 + E^2 + 2R_1E\cdot \mathrm{Cos}(\gamma)}$$

$$L_V = R_2 - L$$

$$A_\theta = \tfrac{1}{2}\theta R_2^2$$

$$A_\gamma = \tfrac{1}{2}\gamma R_1^2$$

$$A_T = \tfrac{1}{2}ER_1\cdot \mathrm{Sin}(\gamma)$$

$$A_V = \tfrac{1}{2}L_V B_V - A_R$$

$$V_1 = H\cdot(A_\theta - A_\gamma - A_T - A_V)$$

$$V_S = \pi\cdot(R_2^2 - R_1^2)\cdot H$$

$$V_2 = V_S(1+\varepsilon) - V_1 - 2A_V H$$

$$B_{\theta 1} = \theta R_2$$

$$B_{\gamma 1} = \gamma R_1$$

$$A_{o1} = H\cdot(B_{\theta 1} - \tfrac{1}{2}B_V + B_{\gamma 1} + L_V) + 2\frac{V_1}{H}$$

$$B_{\theta 2} = (2\pi - \theta)\cdot R_2$$

$$B_{\gamma 2} = (2\pi - \gamma)\cdot R_1$$

$$A_{o2} = H\cdot(B_{\theta 2} - \tfrac{1}{2}B_V + B_{\gamma 2} + L_V) + 2\cdot\frac{V_2}{H}$$

From fig. 3 we have:

$$H_2 = \sqrt{R_K^2 - \tfrac{1}{4}B_V^2}$$

$$H_1 = R_K - H_2$$

$$\alpha = \mathrm{Arccos}\left(\frac{H_2}{R_K}\right)$$

$$A_R = \tfrac{1}{2}B_V H_1 - \tfrac{1}{2}\alpha R_K^2 + \tfrac{1}{4}B_V H_2$$

SYMBOLS

- $a$ = Weight proportion of gas in the oil
- $f_i$ = Loss coefficient
- $g$ = Ideal mass flow per unit of area in a nozzle
- $h_I$ = Specific enthalpy of gas at inlet to the suction volume
- $h_1$ = Specific enthalpy of gas in suction volume
- $h_2$ = Specific enthalpy of gas in dis - charge volume
- $m_b$ = Return mass flow through suction pipe during re-expansion of resi - dual gas
- $m_{o11}$ = Oil leakage from lubrication chan- nel to suction volume
- $m_{o12}$ = Oil leakage from shell to suction volume
- $m_{o21}$ = Oil leakage from lubrication chan- nel to discharge volume
- $m_{o22}$ = Oil leakage from shell to discharge volume
- $m_{L1}$ = Gas leakage from discharge volume to suction volume over and under vane
- $m_{L2}$ = Gas leakage from discharge volume to suction volume between roller and cylinder wall
- $m_1$ = Suction gas mass flow
- $m_2$ = Exhaust mass flow through discharge valve
- $n$ = Polytropic index
- $p_s$ = Suction pressure outside compressor
- $p_t$ = Condensing pressure
- $p_1$ = Pressure in suction volume
- $p_2$ = Pressure in discharge volume
- $t$ = Time
- $u_R$ = Specific energy in clearance volume
- $u_1$ = Specific energy in suction volume
- $u_2$ = Specific energy in discharge volume
- $v_1$ = Specific suction volume
- $v_2$ = Specific discharge volume
- $A_k$ = Least discharge area
- $A_p$ = Surface area of suction pipe
- $A_t$ = Effective force area

$B_V$ = Vane breadth
$C_d$ = Damping factor
$C_{poil}$ = Specific heat capacity of oil
$C_{RC}$ = Clearance between roller and cylinder wall
$C_{RCT}$ = Clearance between roller and cylinder top/bottom
$C_{VC}$ = Clearance between vane and cylinder top/bottom
$C_{VS}$ = Clearance between vane and slot
$D_p$ = Suction pipe diameter
$D_1$ = Roller outside diameter
$D_3$ = Roller inside diameter
$E$ = Eccentricity
$H$ = Cylinder height
$H_p$ = Heat transfer coefficient in suc - tion pipe
$H_{val}$ = Heat transfer coefficient in suc - tion volume
$K_J$ = Spring stiffness
$L_p$ = Length of suction pipe
$L_s$ = Length of slot
$L_V$ = Length of vane from roller to cylinder wall
$M_V$ = Valve mass
$M_R$ = Mass of residual gas
$M_1$ = Totale mass of gas in suction volume
$M_2$ = Totale mass of gas in discharge volume
$Q_1$ = Heat flow rate to suction volume
$Q_2$ = Heat flow rate to discharge volume
$R_K$ = Radius of curvature of vane tip
$R_1$ = Roller outside radius
$R_2$ = Cylinder inside radius
$S$ = Distance between roller and cylinder wall at the suction pipe opening
$T_I$ = Temperature at the suction volume inlet
$T_S$ = Temperature at the suction pipe inlet
$T_W$ = Wall temperature
$T_1$ = Temperature of gas in suction volume
$T_2$ = Temperature of gas in discharge volume
$U_v$ = Vane velocity
$U_1$ = Totale internal energy in suction volume
$U_2$ = Totale internal energy in discharge volume
$V_S$ = Stroke volume
$V_1$ = Suction volume
$V_2$ = Discharge volume
$W_1$ = Suction power
$W_2$ = Compression power
$\theta$ = Angle of rotation
$\omega$ = Angular velocity
$\alpha$ = Nozzle coefficient
$\rho_o$ = Oil density
$\varepsilon$ = Relative clearance volume
$\lambda$ = Frictional resistance of smooth pipes of circular cross-section

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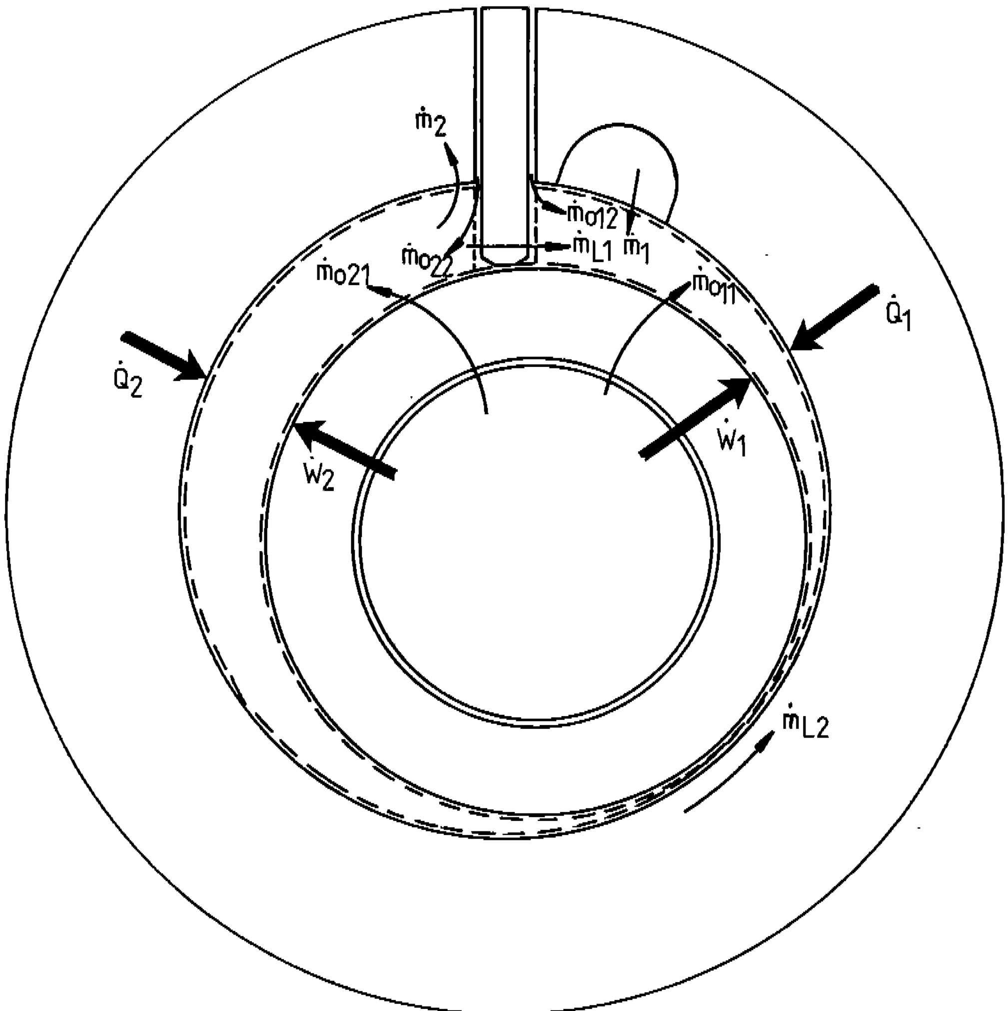


Figure 1: Control volumes

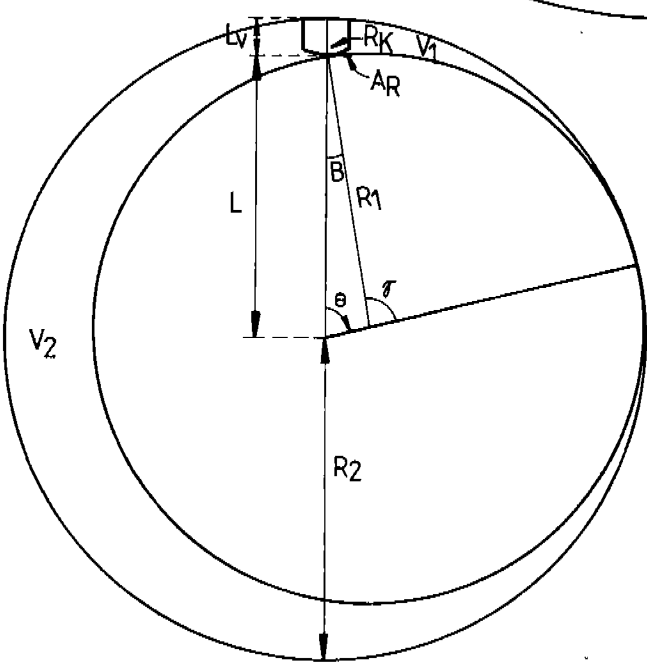


Figure 2: Geometric values

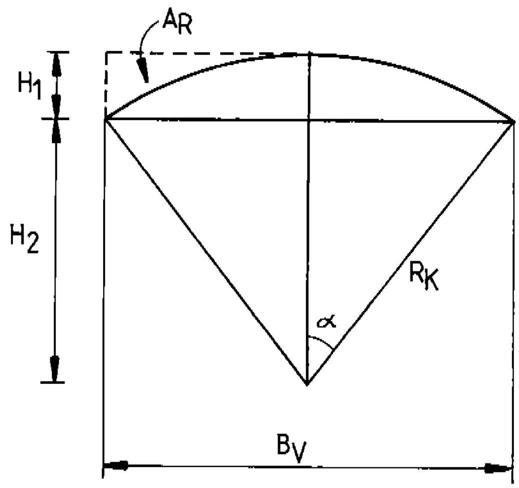


Figure 3: Vane tip