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Analytical Model of an Oil-Free Screw Compressor

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ANALYTICAL MODEL OF AN OIL-FREE SCREW COMPRESSOR

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ABSTRACT

The use of analytical modelling is a significant way in reducing of experimental costs in research and development. An analytical model of an oil-free screw compressor is described and used to study the effects of rotation speed, compression losses and difference of built-in and external pressure ratio on compression process. The influence of gas pulsation in the discharge piping upon the working process of the screw compressor is included.

INTRODUCTION

With the development of the computer art analytical models are asserting themselves in an ever increasing measure. By them costly experiments can be replaced to a certain extent. Among positive displacement compressors the modelling technique has been most highly developed on reciprocating compressors. Their present standard permits the course of the compression and the motion of the valves to be comprehensively treated and also the effect of the connected piping to be included. Dependences thus calculated are usually the basis for optimalization of the planned equipment.

The good experiences gained from the modelling of reciprocating compressors were the incentive for the creation of a similar model of an oil-free screw compressor. The concept of the analytical model corresponds to screw compressors which the ČKD Praha, Compressor Division, has been manufacturing already for a number of years (Fig. 1) in the licence of SVENSKA ROTOR MASKINER A.B. Their active parts are two rotors engaging with each other the mutual rotation of which is synchronized by a set of gears. The rotors, together with the compressor shell, form four paired chambers which are the working spaces. The cycle of suction, compression and discharge takes its course, with these chambers, with a phase displacement of one quarter of a revolution and causes that the fourth harmonic component, referred to also as chamber frequency, manifests itself in both the noise spectrum and the vibration in a significant way.

Discounting the possibility that vibration by the chamber frequency may be caused by contact of the rotors - such a case would lead very quickly to a breakdown - the most probable cause of vibration is pulsation in the discharge piping. Similarly as with piston compressors its level will depend on the dimensions of the piping and will, moreover, be markedly affected by the pressure in the chamber at the moment the discharge port is opened. These facts are well known and have been experimentally proved many times.

Experimental research becomes, however, very costly when one endeavours to find an optimum design or optimum operating conditions for a given case. A suitable means to rationalize this work is the modelling technique. However, it is only rarely that an analytical model can be produced which takes all substantial influences into account. Only some of the dependences can be studied, depending on the purpose which the model is intended to serve, all others which affect the process under observation in a smaller measure being disregarded.

The proposed analytical model of a screw compressor was built with the aim of observing the working process of the compression in conjunction with the dynamic processes in the discharge piping. The relatively uniform behaviour of the suction of a screw compressor and thus also its small effect on the dynamics of the gas in the suction piping were the reason why the dynamic phenomena in the suction piping were not considered.

Fig. 2 shows the diagram of a screw compressor with a discharge piping which leads into a noise damper. The diagram represents a model the individual parts of which will be analytically described below.

COMPRESSION SPACE

The compression space consists of four paired chambers the action of which is mutually displaced in phase by 90°. A change of the volume of one paired chamber can be expressed by means of the model shown in Fig. 3. The actual working space of the chamber is replaced by an equal volume in the shape of an oblique prism in which a partition is moving. The volume of the working space determined in this way changes from α_1 to α_4 and its change is protted against angle α, also in Fig. 3.

With screw compressors the compression process is affected by untightnesses to such an extent that, for purposes of the calculation of the compression, it cannot be disregarded. The calculation is made by steps and one calculating step corresponds to time interval Δt in which the main rotor turns through angle $\Delta\alpha$. Thus

$$\Delta t = \frac{\Delta\alpha}{\omega}$$

The volume quantity of gas which escapes from the chamber to the suction space in time interval Δt is determined by the relation

$$\Delta U_i' = \left(\frac{\eta_s\, p_s}{p_i}\right)^{\frac{1}{\varkappa}} \cdot A \cdot \Delta t$$

where

$$A = \frac{\pi D^2}{4} \xi \sqrt{2 \frac{\varkappa}{\varkappa - 1} R\, T_i \left[1 - \left(\frac{\eta_s\, p_s}{p_i}\right)^{\frac{\varkappa - 1}{\varkappa}}\right]}$$

The resultant pressure in the chamber in the course of the compression will be, after the main motor has turned through angle $\Delta\alpha$ and after time Δt

$$p_{i+1} = p_i \left(\frac{V_i}{V_i - \Delta V_i + \Delta U_i'}\right)^{\varkappa}$$

where $V_i = f(\alpha_i)$ and ΔV_i in interval

$$\begin{aligned}
\alpha_1 \leq \alpha_i < \alpha_3 \qquad & \Delta V_i = K_1 (\alpha_i - \alpha_1) \Delta\alpha \\
\alpha_3 \leq \alpha_i < \alpha_2 \qquad & \Delta V_i = K_2\, \Delta\alpha \\
\alpha_2 \leq \alpha_i \leq \alpha_4 \qquad & \Delta V_i = K_1 (\alpha_4 - \alpha_i) \Delta\alpha
\end{aligned}$$

K_1 and K_2 are constants corresponding to the dimensions of the rotors.

The expression $\frac{\pi D^2}{4} \cdot \xi$ represents the total untightness of the chamber, coefficient ξ is the specific untightness referred to the area of the face of the main rotor.

The penetration of the gas through the untightnesses into suction space has the consequence that, in the suction space, gas of suction temperature T_s mixes with gas the temperatures of which correspond to the temperatures in the individual chambers. Gas which has escaped in time Δt will have temperature T_i and will occupy, on the suction side, space

$$\Delta U_i'' = \left(\frac{p_i}{\eta_s\, p_s}\right)^{\frac{\varkappa - 1}{\varkappa}} \cdot A \cdot \Delta t$$

The resultant temperature on the suction side will be determined by the relation

$$T_s'' \doteq \frac{\left(\eta_s V_T - \sum_{i=1}^{i=N} \Delta U_i''\right) \cdot T_s + \sum_{i=1}^{i=N} \Delta U_i'' \cdot T_i}{\eta_s V_T}$$

where $N = \dfrac{360}{\Delta\alpha}$.

Untightness coefficient ξ is an important factor which affects the course of the compression and may therefore be considered one of the criteria of the quality of the compressor. However, the determination of it is difficult, for the clearances measured while the machine is at rest need not correspond to the operating values. A method which permits at least an approximate appraisal of the untightness during operation consists in a comparison of the measured and calculated values of the delivered gas and of the discharge temperature of it.

The quantity of gas escaped from the working space to the suction space depends not only on the untightnesses but also on time. Therefore the course of the compression will depend also on the speed of the machine.

The connection of the working space with the discharge starts as the discharge port is being opened and takes its course within a range of angles $(\alpha_2 - \alpha_0) < \alpha_i < \alpha_4$, Fig. 4. In an ideal case there should be an equality of pressures in the working space of the compressor and in the discharge chamber at the moment of opening. Since such a case is an exception rather than the rule the analytical model reckons, in the initial phase of the opening of the discharge port, with a gradual equalization of pressures. The volume quantity of gas which escapes from the chamber to the discharge in interval Δt will be

$$\Delta Q_i' = \left(\frac{p_{Ki}}{p_i}\right)^{\frac{1}{\varkappa}} \cdot B \cdot \Delta t$$

where

$$B = F_E \sqrt{2 \frac{\varkappa}{\varkappa - 1} R\, T_i \left[1 - \left(\frac{p_{Ki}}{p_i}\right)^{\frac{\varkappa - 1}{\varkappa}}\right]}$$

This quantity will have temperature T_i and will occupy, in the discharge chamber, space

$$\Delta Q_i'' = \left(\frac{p_i}{p_{Ki}}\right)^{\frac{\varkappa - 1}{\varkappa}} \cdot B \cdot \Delta t$$

The resultant pressure in the chamber in the course of the dicharge will be, after the main rotor has turned through angle $\Delta\alpha$ and after time Δt,

$$p_{i+1} = p_i \left(\frac{V_i}{V_i - \Delta V_i + \Delta U_i' + \Delta Q_i'}\right)^{\varkappa}.$$

Values T_i, p_{Ki}, p_i are mutually dependent functions of time.

F_E is an expression for the product of the momentary cross section of the discharge port and of the flow coefficient. It changes in dependence on angle α. Treated by similar relations is also the case when the pressure in the working space is lower that the pressure in the discharge space and gas flows into the working space.

The application of the relations for the discharge of gas from the working space in the above form would lead in the majority of cases, at the usual values of Δt, to instability of the calculation. Therefore the iterative procedure was chosen which solves, in interval Δt, the successive emptying of the working space during a simultaneous rise of the back pressure in the discharge chamber.

In the closing phase of the discharge, i.e. for $\alpha_i \geq \alpha_2$ already the following simpler relation for the volume quantity of gas discharged into the discharge chamber is used:

$$\Delta Q_i'' = \Delta V_i - \Delta U_i' .$$

Similarly as in the suction space, also in the discharge space the resultant temperature of the gas is determined by the relation

$$T_v \doteq \frac{\sum_{i=1}^{i=N} \Delta Q_i'' \cdot T_i}{\sum_{i=1}^{i=N} \Delta Q_i''} .$$

The escape of gas through untightnesses into the suction space as well as its delivery into the discharge space were treated as isothermal processes. Joule-Thompson's effect was not considered, for the error thus involved is within the limits of the precision of the model.

DISCHARGE PIPING

A non-stationary flow through the discharge piping can be expressed by the relations

$$\rho \frac{\partial v}{\partial t} + \frac{\partial p_x}{\partial x} = -\rho z v ,$$

$$\rho a^2 \frac{\partial v}{\partial x} + \frac{\partial p_x}{\partial t} = 0 .$$

Member $-\rho z v$ expresses the pressure losses which are linearly dependent on the rate of flow of the gas. These relations can, however, be even further simplified. With pipes without major changes of cross sections the pressure losses are relatively low compared to those which occur at an open end of it. There will therefore be no great error when the losses in the discharge pipe (between the compressor and the noise damper) are disregarded and when it is assumed that they occur only at its end. Summarily they are expressed by coefficient Z. This coefficient indicates the relative loss of the velocity of a wave as it is reflected at the open end of the pipe. For a calculation of the propagation of waves in the pipe the simpler wave equation may be used.

The solution of the wave equation with inclusion of the effect of the losses at the end of the pipe, requires a knowledge of the boundary and initial conditions. Simplest of them is the boundary condition of the pipe at the noise damper end. There a substantial increase of the cross section takes place and therefore there is no great error when the pressure inside the damper is considered constant.

The boundary condition of the pipe at the compressor end is the course of the discharge of gas in dependence on time. It is not, however, a static time dependence, for the course of the discharge is linked also to the pressure changes in the discharge piping.

Quite unknown in advance are the initial conditions of the pulsation in the discharge piping.

DESIGN OF ANALYTICAL MODEL

From the setting of the problem and the mutual linkages it is obvious that it is unavoidable to deal with all relations of the analytical model of the screw compressor simultaneously. Chosen as a suitable method of the calculation of a non-stationary flow through a pipe was the method of characteristics with a modification for the calculation of losses at the open end of the piping. The relations concerning the course of compression were converted to a system of differential equations and constitute the boundary condition of the pipe at the compressor end.

Ignorance of the actual initial conditions leads to the choice, at the beginning, of a zero rate of flow and a constant pressure corresponding to the rated discharge pressure, in the whole discharge piping. By repetition of the calculation and substitution of the final states of the previous cycles for the initial conditions of the subsequent cycles the desired solution is arrived at. The problem solved on an automatic computer converges relatively quickly to settled values. It is usually sufficient to perform five to ten cycles.

COMPUTATION OF ČKD-SRM TYPE ZK 204 COMPRESSOR

A numerical computation is always linked to a concrete setting. In our case computation by means of the model was used on the ČKD-SRM Type ZK 204 screw compressor.

Principal particulars:

rotor diameter	D	= 204 mm
volume of paired chamber	V_T	= 0.1668 10^{-2} m^3
speed	n	= 132.1 Hz
built-in compression ratio	π_{in}	= 3.6
operating compression ratio	π_{ex}	= 4.0
pressure on suction side	p_s	= 98.1 kPa
temperature on suction side	T_s	= 297° K
working medium		air

Chosen values:

untightness coefficient	ξ	= 1.53 10^{-3}
damping coefficient	Z	= 0.4

The dimensions of the discharge pipe between the compressor and the noise damper - the discharge chamber being considered as well - were chosen for the computation in two variants. After reduction of the piping to the constant inside diameter of 125 mm the first of them corresponds to the length

$$L_{red} = 0.666\ \lambda \qquad \text{(Fig. 5)}$$

and the second one to the length

$$L_{red} = 0.755\ \lambda \qquad \text{(Fig. 6)}$$

λ is the wave length corresponding to the chamber frequency and the speed of sound in the gas in the discharge piping under operating conditions.

In Figs. 5 and 6 the pressure - time diagrams of the two variants are calculated. Since, in the closing phase, the chamber is connected to the discharge piping their courses are, at the same time, also the behaviour of the pressure pulsation in the discharge piping.

Both pictures are supplemented with the behaviours of the pressure pulsation given in the relative value $\frac{p_K - p_V}{p_V}$ and the behaviours of the flow velocity in the discharge chamber of the compressor. Both values are repeated with a 90° period. Since the built-in compression ratio is lower than the operating one a reverse flow into the working space as well as a sharp pressure drop in the discharge chamber occur at the moment of opening of the discharge port. A consequence thereof is an increased level of pulsation in the discharge piping. When the two variants are compared it is obvious that the non-uniformity of flow depends considerably on the phase of the pressure wave at the moment of opening of the discharge port. The opposite case, when the built-in compression ratio considerably exceeds the operating one, is also a source of increased pulsation.

The effect of the untightness coefficient on the sucked volume per hour and discharge temperature is shown in Fig. 7. The curves were calculated for operating values of the ZK 204 compressor within a range of

$$0.92\ 10^{-3} < \xi < 2.45\ 10^{-3}$$

and for a suction efficiency $\eta_S = 0.95$. Also plotted in the diagram are the average values of sucked volume of air per hour and discharge temperature measured in the test shop on ten compressors (V_S^*, T_V^*). The example shows, at the same time, how the measure of untightness can inversely be determined from the measured values. It is obtained both from delivered volume and discharge temperature and a criterion of the correctness of the calculation or measurement is the mutual agreement of both values. In this case $\xi \doteq 1.43 \cdot 10^{-3}$.

The quantity of gas escaping through the untightnesses into the suction space depends on time and therefore also on the speed of the machine. The effect of a change of speed on the volumetric efficiency and discharge temperature for a given ξ is shown in Fig. 8.

CONCLUSION

The analytical model of a screw compressor makes possible a quick and inexpensive determination of dependences which would otherwise be obtained with difficulty experimentally. Even though it will not be possible to replace the experiment in its full extent analytical modelling will be a very significant supplement to it.

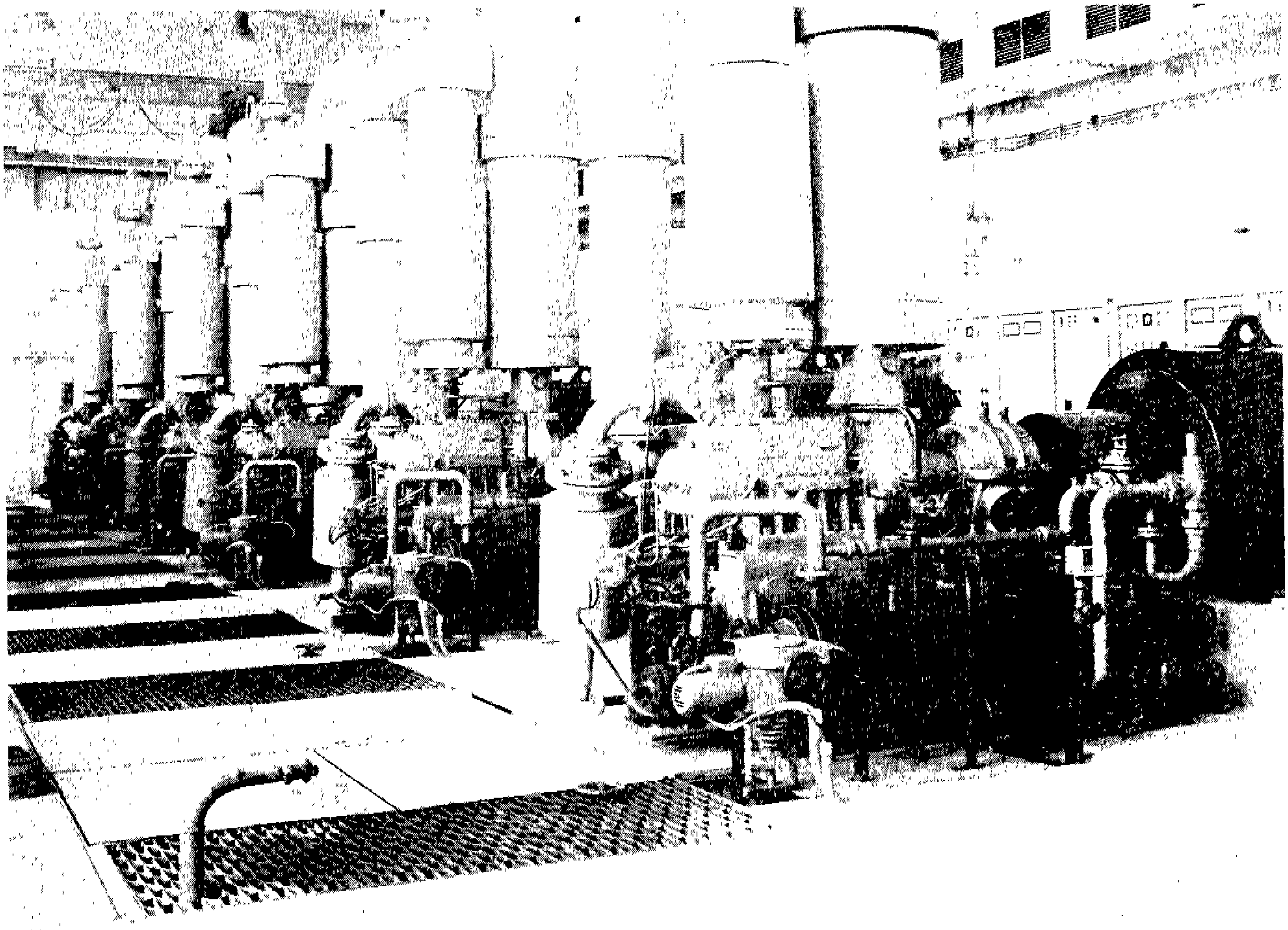
The computation carried out on a ČKD-SRM Type ZK 204 compressor is an example of the utilization of the analytical method in practice and the calculated dependences will demonstrate the possibilities afforded by this method. The results achieved justify the hope that the method will find application particularly in the seeking of the optimum design of a screw compressor and of its operating conditions.

NOTATION

- a speed of sound
- i subscript of calculation step
- p pressure in working space
- p_K pressure in discharge chamber
- p_S rated pressure in suction space
- p_V rated pressure in discharge space
- p_X pressure in piping
- t time
- v velocity of gas
- D diameter of rotor
- R gas constant
- T temperature
- V volume
- V_T volume of working chamber
- α angle of turning of main rotor
- η volumetric efficiency
- η_S suction efficiency
- $\varkappa$ adiabatic coefficient
- ρ specific mass of gas
- ω angular velocity

REFERENCES

Bráblík, J., "Computer Simulation of the Working Process in the Cylinder of a Reciprocating Compressor with Piping System", Proc. 2nd Compressor Technology Conference, Purdue, July 1974.



SCREW COMPRESSORS CKD-SRM ZK 204

FIGURE 1

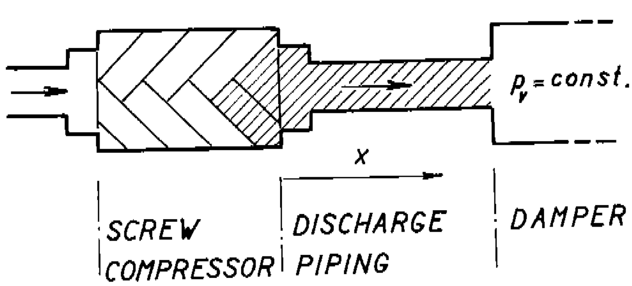


FIGURE 2

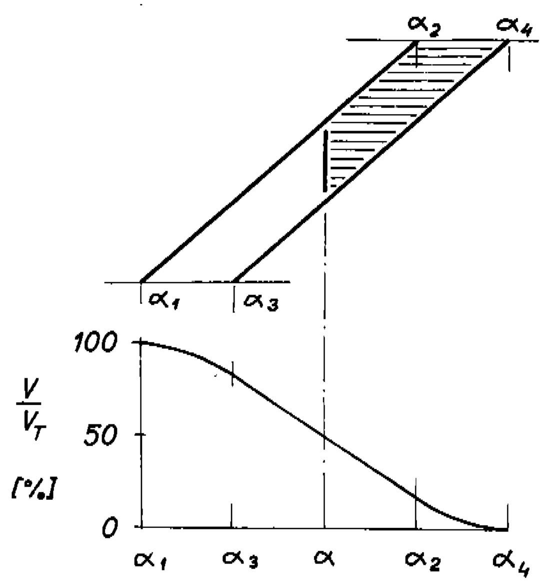


FIGURE 3

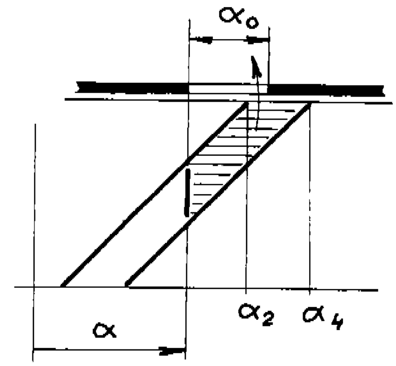


FIGURE 4

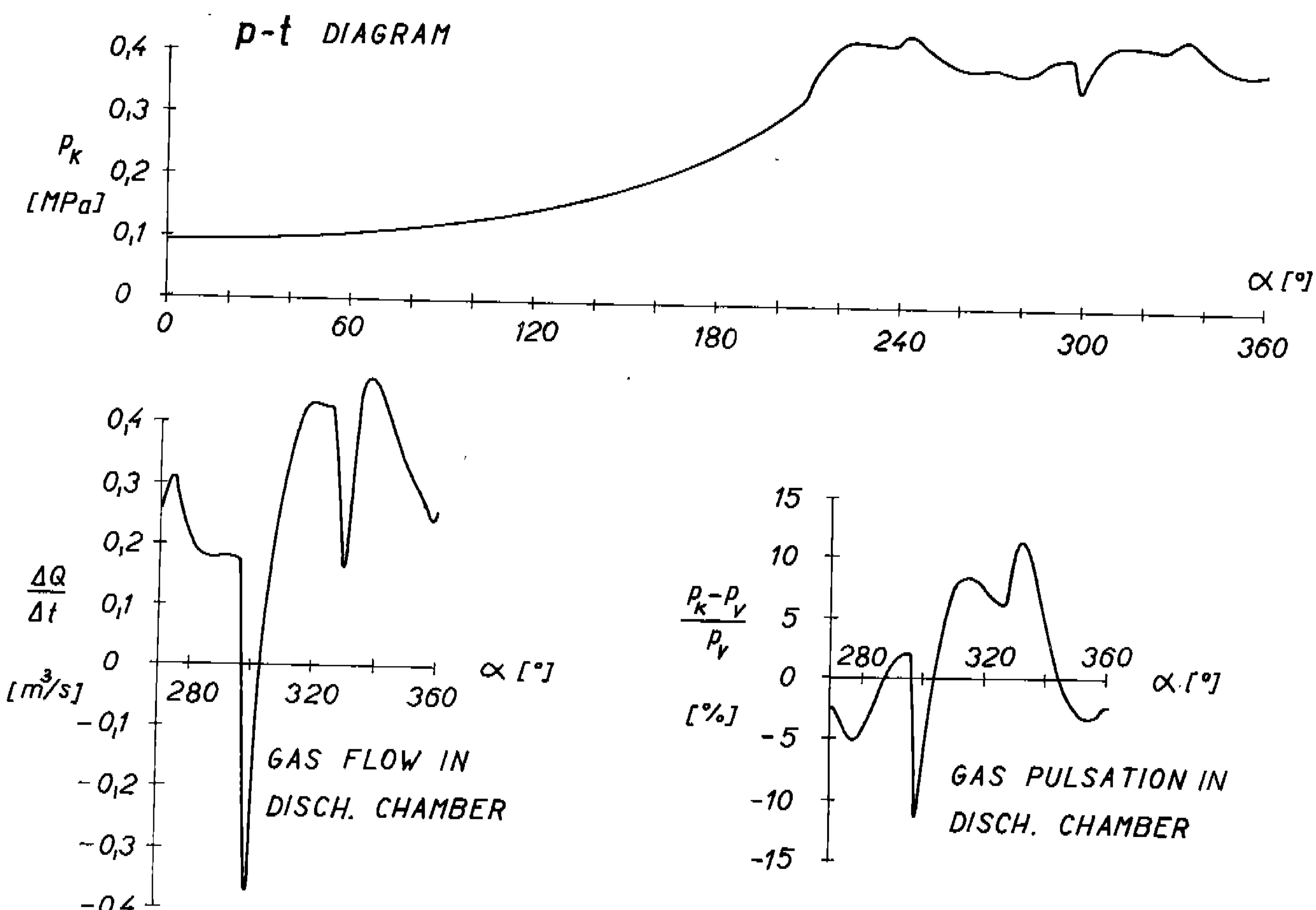


FIGURE 5

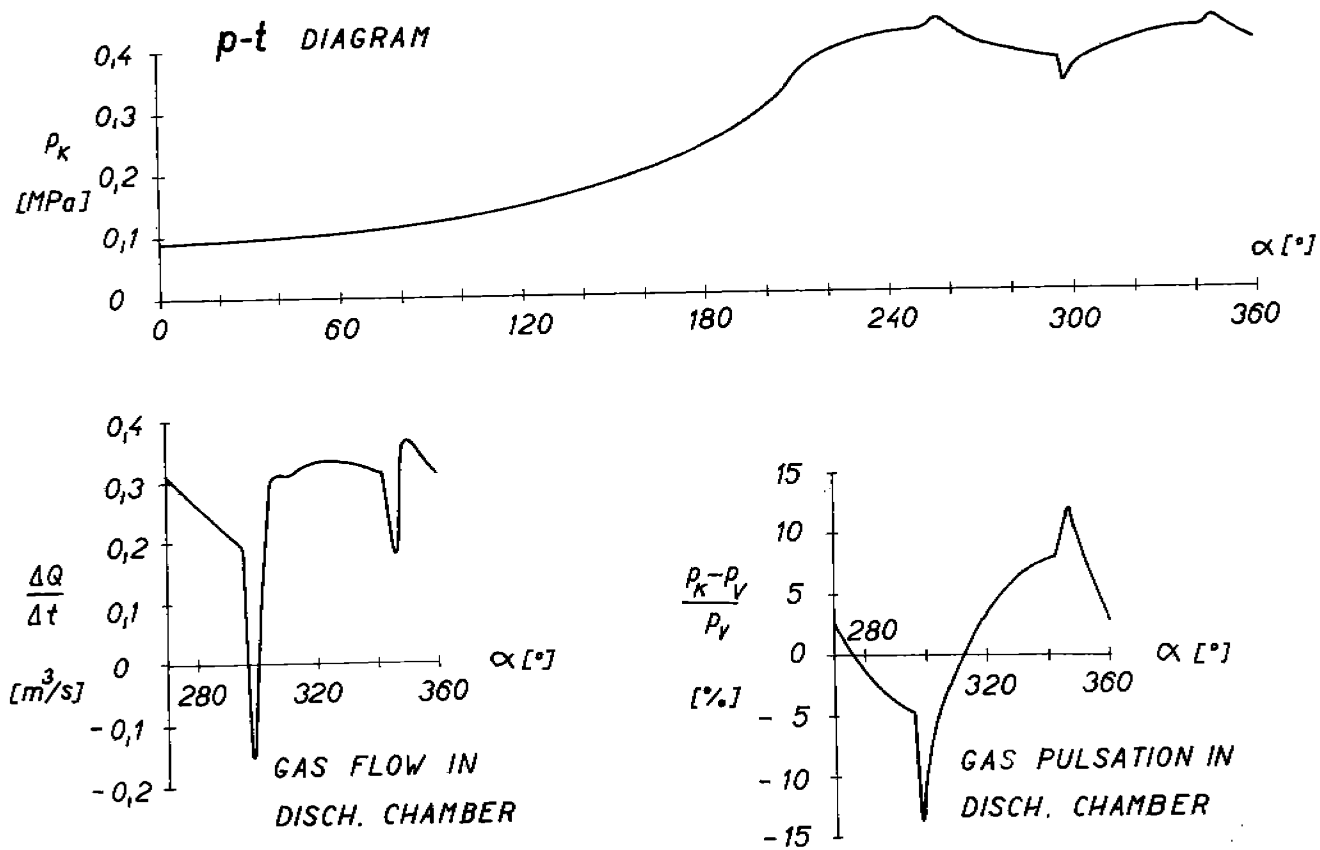
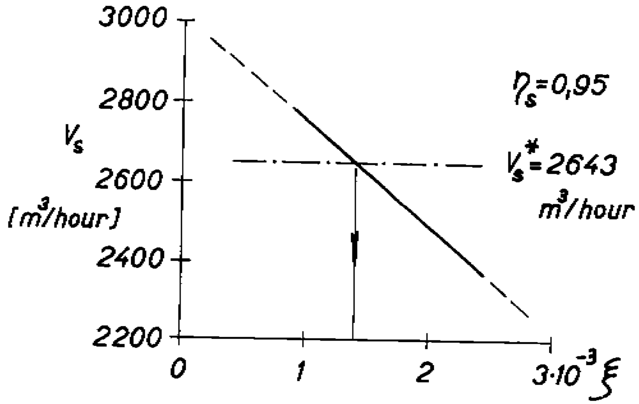

p-t DIAGRAM
p_K [MPa]
0,4
0,3
0,2
0,1
0
α [°]
0
60
120
180
240
300
360
$\frac{\Delta Q}{\Delta t}$ [m³/s]
0,4
0,3
0,2
0,1
0
- 0,1
- 0,2
280
320
360
α [°]
GAS FLOW IN
DISCH. CHAMBER
$\frac{p_K - p_V}{p_V}$ [%]
15
10
5
0
- 5
- 10
- 15
280
320
360
α [°]
GAS PULSATION IN
DISCH. CHAMBER

FIGURE 6

SUCKED VOLUME OF AIR PER HOUR



TEMPERATURE OF AIR IN DISCHARGE CHAMBER

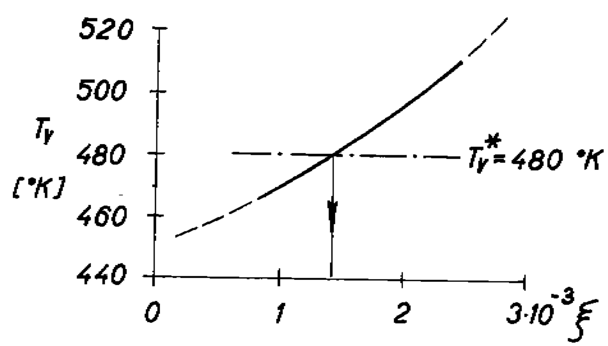


FIGURE 7

[V_s^*, T_V^* – measured values]

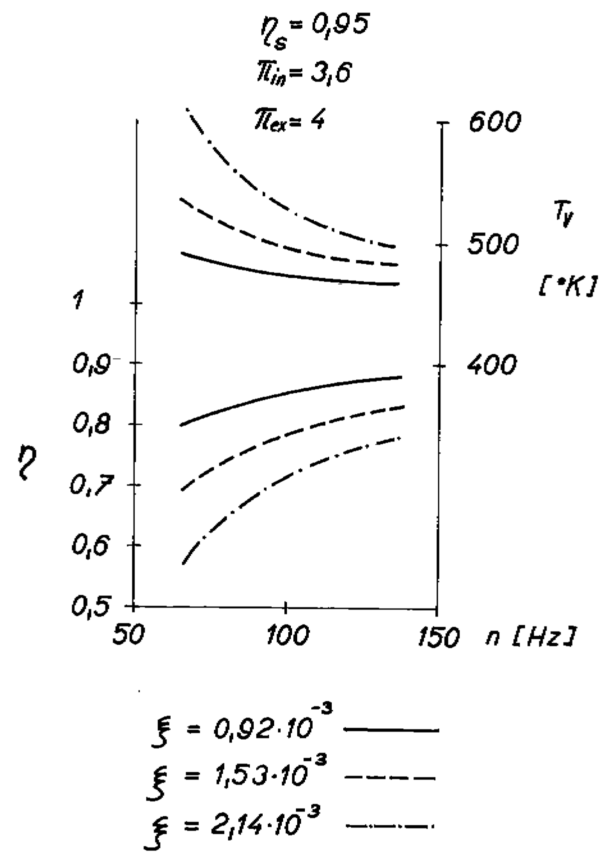


FIGURE 8

$p - t$ DIAGRAMS WITH DIFFERENT RATIO π_{in} / π

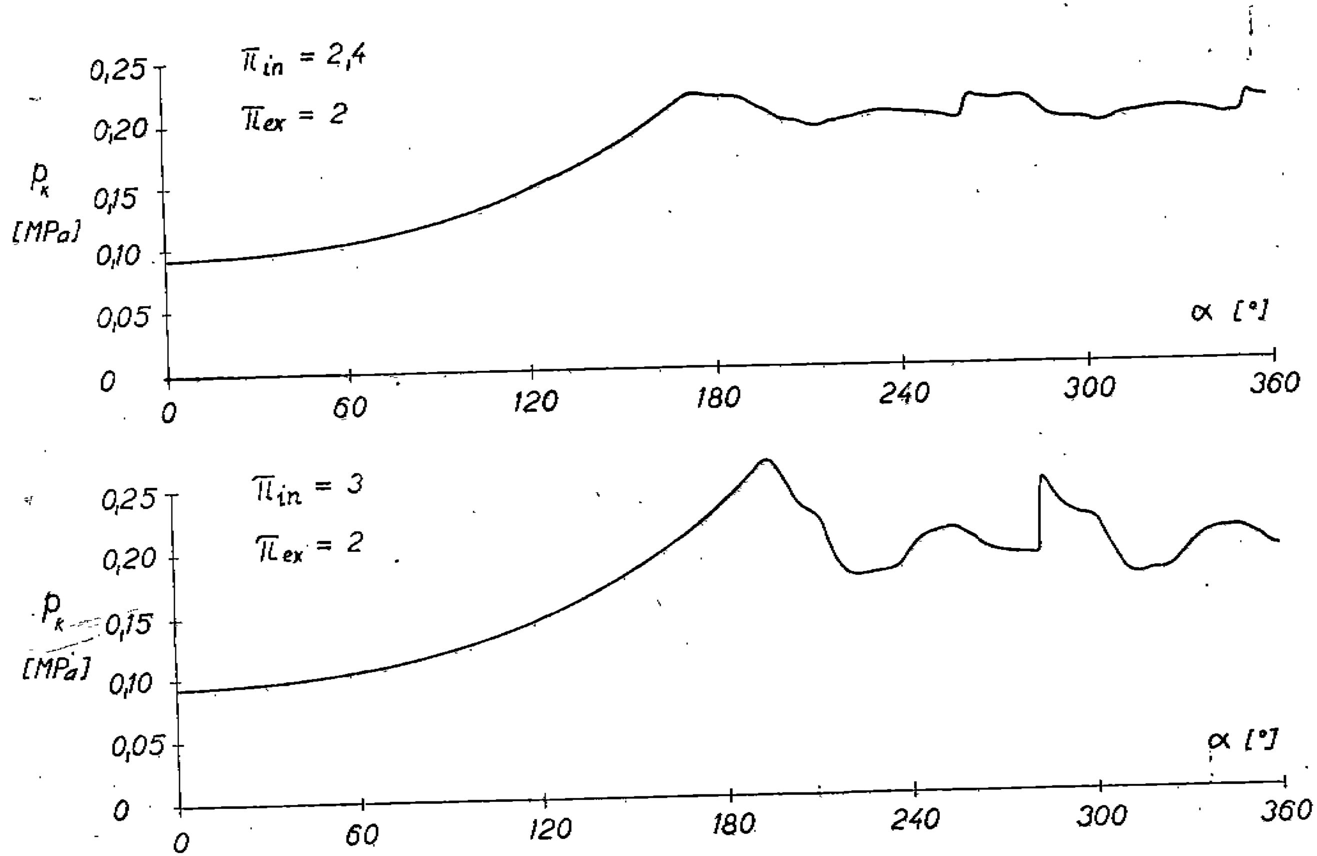


FIGURE 9