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Performance Characteristics of Fixed Volume Ratio Compressors

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PERFORMANCE CHARACTERISTICS OF FIXED VOLUME RATIO COMPRESSORS

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ABSTRACT

The effects of built-in volume ratio on the performance characteristics of fixed volume ratio, positive displacement compressors are demonstrated in an analysis using idealized pressure-volume diagrams. Power and adiabatic efficiency characteristics are computed and a discussion as to how the built-in volume ratio influences these characteristics is presented.

It is shown that the shapes of the performance curves are controlled by the mis-matching between internal compressor pressure and system pressure at the discharge. Internal losses, while dictating levels of performance, are of secondary importance in determining the characteristic shapes of the performance curves.

The analysis is applied to fully loaded and unloaded compressor operation. Examples indicate the importance of selecting the right built-in volume ratio and the performance penalties that can be expected when a fixed volume ratio compressor is required to operate over a wide range of system pressure ratio.

INTRODUCTION

Investigation of fixed volume ratio, positive displacement compressors reveals that they have distinctive performance characteristics. A typical set of power curves for a rotary screw compressor (Reference [1]) is shown in Figure 1. Data for compressors with built-in volume ratios (v_i) of 2.6 and 4.8 are presented, showing the powerful effect that this design parameter has on compressor performance. Increasing the condenser temperature has about the same effect on the 2.6 and 4.8 compressors, shifting the power curves to higher levels but having little effect on the slopes. Raising the volume ratio, however, results in an increase in the slope of the power curves.

The purpose of this study is to explore the performance of fixed volume ratio compressors in an attempt to explain the characteristics noted above. An expression for compressor power as a function of suction pressure, pressure ratio and built-in volume ratio is developed using idealized pressure-volume diagrams for both fully loaded and unloaded operation. No provision for internal losses is included, and thus the analysis does not reflect actual compressor performance levels. However, the model does show how a compressor responds to changes in operating conditions and how the built-in volume ratio affects performance characteristics.

The following assumptions are used in the analysis:

1) The fluid is an ideal gas with constant specific heats.
2) The pressure within the compressor adjusts instantaneously to the system imposed discharge pressure when porting begins.
3) The volumetric efficiency is 100%.

NOMENCLATURE

C_p	Specific heat at constant pressure	BTU/lbm-°F
f	Fraction of fully loaded suction volume at which unloader closes	
g	Fraction of fully loaded suction volume at which unloader opens	
h	Enthalpy	BTU/lbm
k	Isentropic exponent	
$\dot{m}$	Mass flow rate	lbm/min
N	Compressor speed	rev/min
P	Pressure	lbf/ft^2
R	Universal gas constant	lbm-°F/ft-lbf
r	Pressure ratio	
T	Temperature	°F
V	Volume	ft^3/rev
v	Specific volume	ft^3/lbm
v_i	Built-in volume ratio	
$\dot{W}$	Compressor power	ft-lbf/min
η_a	Adiabatic efficiency	

SUBSCRIPTS

1	Suction conditions
3	Discharge conditions
f	Point of unloader closing
g	Point of unloader opening
i	Isentropic state point

ISENTROPIC COMPRESSION ANALYSIS

Before studying the performance of fixed volume ratio compressors, it is useful to compute the power required for an isentropic compression between specified suction and discharge pressures. From the first law of thermodynamics, the isentropic power is

$$W_i = \dot{m}\Delta h_i \quad (1)$$

Since we are considering a positive displacement compressor, the mass flow rate can be expressed as a function of the compressor's constant volume flow rate and the specific volume of the vapor

$$\dot{m} = \frac{NV_1}{v} \quad (2)$$

For an ideal gas:

$$v = RT/P \quad (3)$$

$$\Delta h_i = C_p(T_{3_i} - T_1) = C_p T_1\left(\frac{T_{3_i}}{T_1} - 1\right) \quad (4)$$

$$\frac{T_{3_i}}{T_1} = (P_3/P_1)^{(k-1)/k} \quad (5)$$

Using equations 2 through 5 in equation 1 results in the following expression for power:

$$W_i = \frac{NV_1}{R} C_p P_1 \left[\left(\frac{P_3}{P_1}\right)^{(k-1)/k} - 1\right] \quad (6)$$

For constant speed operation, NV_1 is a constant and for an ideal gas $C_p/R=k/(k-1)$. Equation 6 can thus be written as

$$\frac{W}{NV_1} = \frac{k}{k-1} P_1 \left[\left(\frac{P_3}{P_1}\right)^{(k-1)/k} - 1\right] \quad (7)^*$$

A family of performance curves computed using equation 7 is illustrated in Figure 2. The conditions are the same as those for which the screw compressor data is presented in Figure 1. Comparison of the isentropic curves with performance of the screw compressor reveals little similarity.

The performance curves in Figure 2 represent minimum compressor power requirements. Operation below these curves would require a decrease in entropy, violating the second law of thermodynamics. Compressors requiring power above that given by equation 7 will do so because of some loss in efficiency (leakage, friction, etc.).

The isentropic compression represented by equation 7 can be illustrated on a pressure-volume diagram such as that shown in Figure 3. The ideal compressor fills at constant pressure ($P/P_1=1$) as swept volume increases to V_1. The line 123 is the isentropic compression line which, for an ideal gas, is defined by

$$P/P_1 = (V_1/V)^k \quad (8)$$

* A non-dimensional power function, $W/(NV_1P_1)$ could be used. However, since most compressor data is presented using P_1 as the independent variable, the analysis will continue with P_1 as the independent variable.

For this idealized compressor, it is assumed that the compression proceeds isentropically until the pressure inside the compressor is equal to the imposed system pressure. Then the compressor discharges at constant pressure. The process 1a-1-2-2a is for operation at a pressure ratio of 2:1 while the process 1a-1-3-3a shows operation at 6:1 pressure ratio. The work of compression is the area bounded by the filling, compression and discharging curves and it is easy to show that evaluating this area results in equation 7.

The volume ratio at which the compressor begins discharging varies with the pressure ratio. This would be the case for an ideal reciprocating compressor where the discharge valve stays closed until the internal pressure reaches discharge pressure. It then opens and the mass within the compressor flows out with no losses. For a fixed volume ratio compressor, however, the story is quite different. The geometry of the compressor dictates the internal volume to which the gas is compressed. When the compression has reached this point, discharging begins through a fixed port, regardless of the system pressure (for high system pressures, the discharge process starts with a flow from the system into the compressor). This process is analyzed in the next section, resulting in an expression for power as a function of P_1, P_3 and v_i for fully loaded compressors. A more general analysis allowing for unloaded operation follows.

FULLY LOADED COMPRESSOR PERFORMANCE

Figures 4 and 5 show the pressure-volume diagrams for a fixed volume ratio compressor operating at high and low system pressure ratios, respectively. These figures represent a compressor with a built-in volume ratio of 3.5. Regardless of the system imposed discharge pressure, the pressure in the compressor rises to the level where $P_2/P_1=(V_1/V_2)^k$. At this point, the trapped volume can communicate with the system through a fixed positioned port. In the case of high system pressure as illustrated in Figure 4, the pressure rise in the compressor is too low. When the port opens, high pressure gas flows into the compressor until the internal and external pressures are equal (process 2-3 on figure). An isentropic compressor, on the other hand, would follow the path 1-2-3'. Thus, the shaded area 2-3-3'-2 represents the excess work required of the fixed volume ratio compressor.

For low system pressure ratios (Figure 5), the internal pressure rises above the system pressure before discharging begins. When the port is finally opened, the high pressure gas rushes out until the internal and external pressures are equalized. The remaining mass is discharged by reducing the internal volume to zero. The isentropic compressor would compress from 1 to 3' then discharge at constant pressure from 3' to 3a. The excess work required of the fixed volume ratio compressor is again represented by the area 2-3-3'-2.

In order to derive an expression for the reversible work of compression, valid for arbitrary system pressure ratios and compressor volume ratios, the area under the P-V diagram is computed (area 1a-1-2-3-3a-1a in either Figure 4 or 5).

$$\frac{W}{N} = \int_{1a}^{3a} P\,dv$$

$$= \int_{1a}^{1} P\,dv + \int_{1}^{2} P\,dv + \int_{3}^{3a} P\,dv \tag{9}$$

The process 2-3 is assumed to occur at constant volume and thus does not contribute to the total work. The filling and emptying phases of the compression (1a-1 and 3-3a) are assumed to occur at constant pressure. With $V_{1a}=V_{3a}=0$, the first and last integrals on the right hand side of equation 9 are $-P_1V_1$ and P_3V_2, respectively.

To evaluate the work from 1 to 2, use the relation between pressure and volume for isentropic compression of an ideal gas

$$(P/P_1) = (V_1/V)^k \tag{10}$$

Using 10 in 9 and evaluating the integral results in

$$\frac{W}{N} = \frac{P_2V_2 - P_1V_1}{k-1} = \frac{P_1V_2}{k-1}\left(\frac{P_2}{P_1} - v_i\right) \tag{11}$$

Again using 10 to express P_2/P_1 in terms of the compressor's built-in volume ratio

$$\frac{W}{N} = \frac{P_1V_1}{k-1}\,\frac{1}{v_i}\left(v_i^{k} - v_i\right)$$

$$= \frac{P_1V_1}{k-1}\left(v_i^{k-1} - 1\right) \tag{12}$$

The total work of compression is then

$$\frac{W}{N} = \frac{P_1V_1}{k-1}\left(v_1^{k-1} - 1\right) + P_3V_2 - P_1V_1$$

$$= P_1V_1\left[\frac{v_i^{k-1} - 1}{k-1} + \frac{P_3/P_1}{v_i} - 1\right] \tag{13}$$

Or, presenting the work per unit suction volume as was done for the isentropic compressor:

$$\frac{W}{NV_1} = P_1\left[\frac{v_i^{k-1} - k}{k-1} + \frac{P_3/P_1}{v_i}\right] \tag{14}$$

Equation 14 shows that for fixed P_3 and v_i, the W/NV_1 vs. P_1 characteristic is a straight line. The slope of the line can be found by differentiating 14 with respect to P_1:

$$\frac{d\left(\frac{W}{NV_1}\right)}{dP_1} = \frac{v_i^{k-1} - k}{k-1} \tag{15}$$

Equation 14 and 15 show that the power vs. suction pressure curves for a fixed volume ratio compressor have the following characteristics:

1) Power increases with increasing discharge pressure at constant suction pressure.
2) Discharge pressure does not affect the slope of the power curves.
3) Increasing the built-in volume ratio increases the slope of the power curves.

These characteristics are also exhibited by the screw compressor performance curves shown in Figure 1.

For any volume ratio, it can be shown that equations 7 and 14 give the same power when evaluated at a pressure ratio $r=P_3/P_1=v_i^k$. Since 7 represents isentropic compression, the line described by 14 cannot cross the curve described by 7. Thus, at $r=v_i^k$ the curves are tangent. At this point, the P-V diagrams for the isentropic compressor and the fixed volume ratio compressor are identical. For pressure ratios less than v_i^k, the fixed volume ratio compressor produces over pressure losses (Figure 5) which increase as the pressure ratio is reduced. Square carding losses occur in the fixed volume ratio compressor for pressure ratios greater than v_i^k (Figure 4). This loss also increases as the pressure ratio moves away from $r=v_i^k$.

Figure 6 shows the computed power characteristics at 172.9 psia condenser pressure (86°F in R-22) for ideal compressors with built-in volume ratios from 1.5 to 5.0. The isentropic compression curve computed from equation 7 is also included. This figure illustrates the powerful effect that the built-in volume ratio has on compressor performance characteristics.

Performance characteristics for compressors with built-in volume ratios of 2.6 and 4.8 were computed for the same conditions at which the screw compressor data is presented in Figure 1. The resulting curves are shown in Figure 7. The idealized curves and the actual performance characteristics are remarkably similar. Thus, while internal losses such as leakage or friction may be high, it appears that the underlying determinant of the shapes of the power characteristics for fixed volume ratio compressors is the excess work at the discharge associated with the mis-match between compressor and system pressures.

UNLOADED COMPRESSOR PERFORMANCE

Unloaded compressor performance was analyzed using the same techniques that were applied to the fully loaded analysis. The P-V diagram representing unloaded operation is shown in Figure 8. The point g represents the point at which the unloader opens. Ideally, g would be at V_1 so that the work associated with the area g'-1-g-g' would be 0. f is the ratio of the volume at the point of unloader closure to the fully loaded suction volume, V_1. For a given P_1 and P_3, f represents the fraction of full load at which the compressor is operating.

From the assumptions

$$\begin{aligned} V_g &= V_g' \\ V_3 &= V_2 \\ P_g &= P_f = P_1 \\ P_g' &= P_1(V_1/V_g)^k \\ P_2 &= P_1(V_f/V_2)^k \end{aligned} \tag{16}$$

The work of compression from 1 to g' is

$$\left(\frac{W}{N}\right)_{1-g'} = \frac{P_g' V_g - P_1 V_1}{k-1} \tag{17}$$

and for the compression from f to 2

$$\left(\frac{W}{N}\right)_{f-2} = \frac{P_2 V_2 - P_1 V_f}{k-1} \tag{18}$$

Using equations 16 through 18 and noting that $V_f = fV_1$ and $V_g = gV_1$, the total work of compression for unloaded operation is

$$\frac{W}{NV_1} = P_1 \left\{ \frac{f\left[(fv_i)^{k-1} - 1\right]}{k-1} + \frac{g^{1-k} - 1}{k-1} + \frac{r}{v_i} - 1 + g - f \right\} \tag{19}$$

Fully loaded operation is represented by f=1 and g=1. When equation 19 is evaluated using these values, it reduces to equation 14.

Figures 9 through 11 show some part load operating characteristics computed using equation 19. In each of these figures, g=1.0 (no loss due to late opening of the unloader) and the pressure ratio is fixed. Power vs. f for several values of v_i are shown. Figure 9 shows operation at a pressure ratio of 5:1, Figure 10 operation at r=4:1 and Figure 11 shows operation at r=2:1.

The significant feature of these curves is the effect of v_i on the power vs. load characteristic. Figure 9 shows that the minimum power at a pressure ratio of 5:1 is achieved with the $v_i=4$ compressor at full load, but at part load the higher volume ratio compressors are more efficient. The same trend is seen for the lower pressure ratios illustrated in Figures 10 and 11, but the effect of v_i on unloaded power is not as great.

The implication of these unloaded characteristics is that the system operating characteristics must be known in order to select the correct built-in volume ratio. Compressors running at part load should have built-in volume ratios that are higher than required for minimum power at full load. For example, at a pressure ratio of 5:1, the $v_i=4$ compressor uses 25% more power at 50% load than the $v_i=8$ compressor, but uses 13% less at full load. The more hours the compressor is required to run at part load, the greater the excess operating cost that must be paid for selection of the lower built-in volume ratio.

ADIABATIC EFFICIENCY CALCULATIONS

To this point, the analysis has focused on the power characteristics of fixed volume ratio compressors. These compressors also have distinctive adiabatic efficiency characteristics, which are strongly affected by the built-in volume ratio. An expression for adiabatic efficiency can be derived for the ideal compressors. Equation 7 defines the isentropic work and equation 14 the actual work for a fully loaded fixed volume ratio compressor. The ratio of these work terms is the adiabatic efficiency, equation 20.

$$\eta_a = \frac{k\left[r^{(k-1)/k} - 1\right]}{v_i^{k-1} + \frac{r(k-1)}{v_i} - k} \tag{20}$$

Adiabatic efficiency characteristics for the screw compressor of Figure 1 (Reference [1]) are shown in Figure 12. Also shown in the figure are ideal compressor characteristics for the 2.6 and 4.8 volume ratio compressors computed using equation 20. As was the case with the power curves, the characters of the computed and actual efficiency curves are much the same. The shapes and the pressure ratio at which peak efficiency occurs are virtually identical. The main differences are the lower level of efficiency of the actual compressor and the sensitivity of its characteristics to discharge pressure. Both of these differences are probably due to the leakage inherent in the screw compressor.

As shown above, ideal fixed volume ratio compressors have efficiencies less than 1.0 (except where $r=v_i^k$, where they achieve isentropic compression with no excess work). Because of this, it is suggested that a relative efficiency, defined as the ratio of measured adiabatic efficiency to the ideal efficiency of equation 20, be used as a measure of design quality. In this way, the performance achieved is compared to the best performance that could be expected. The magnitude of the relative efficiency would thus be a measure of the losses that, in theory, could be controlled by the design.

The relative efficiency is computed from the data on Figure 12 for the 2.6 v_i compressor operating at 86° condenser temperature. Results are listed in Table 1. While the data shows significant variation in efficiency for the screw (from 0.76 at 2.9 pressure ratio to 0.62 at 6:1 pressure ratio), the relative efficiency changes less than 4 percentage points. This means that the controllable losses, amounting to 24 percentage points at peak efficiency, do not change much with changes in pressure ratio. Most of the drop in efficiency at high pressure ratio is due to the losses at the discharge, inherent in the fixed volume ratio compressor design.

Data at 86°F Condenser Temp.
2.6 v_i Screw Compressor

Pressure Ratio	Relative Efficiency
1.75	.738
2.25	.750
3.00	.760
4.00	.753
5.00	.747
6.00	.723

Table 1

CONCLUSIONS

A procedure for analyzing the performance of an idealized fixed volume ratio compressor has been presented. The analysis explains the characteristic shapes of fixed volume ratio compressor power and efficiency curves and can be used to study the effect of built-in volume ratio and late opening unloaders on unloaded performance.

An expression for adiabatic efficiency was developed and it was shown that the idealized fixed volume ratio compressor efficiency varies with pressure ratio. The losses in efficiency are the penalty paid for use of a fixed volume ratio concept. It is suggested that the efficiency computed including these losses (equation 20) is a proper reference performance level and comparison of actual performance with this reference is a good measure of design quality.

REFERENCES

[1] Sullair Corp., "Packaged Refrigeration Screw Compressors, Engineering Data", September, 1972.

FIGURE 2
ISENTROPIC POWER RATIO VS P1
REFRIGERANT R-22
K = 1.135
EQUATION 7

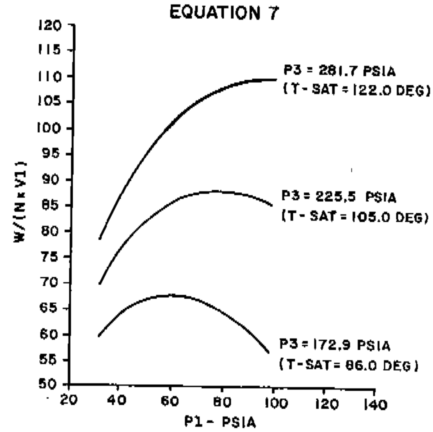

FIGURE 1
ROTARY SCREW COMPRESSOR
POWER CHARACTERISTICS
DATA FROM REFERENCE (1)

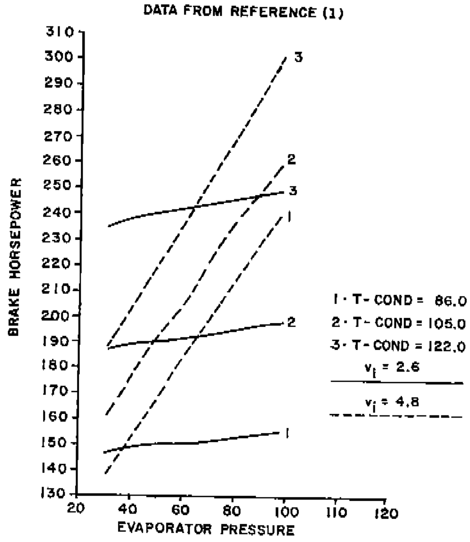

FIGURE 3

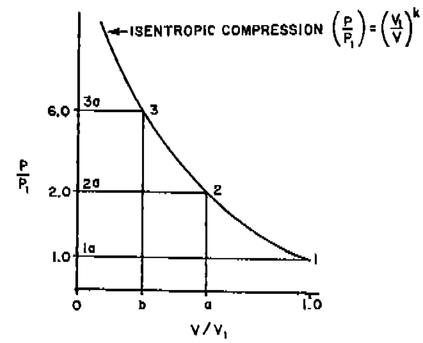

FIGURE 4

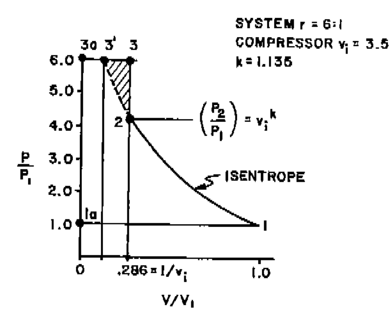

FIGURE 5

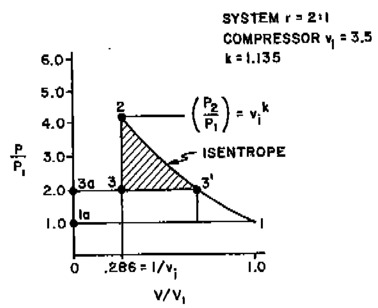

FIGURE 6

POWER RATIO VS P1

CONDENSER PRESSURE = 172.9 PSIA

CONDENSER TEMPERATURE = 86.0 DEG.

REFRIGERANT R-22

K = 1.135

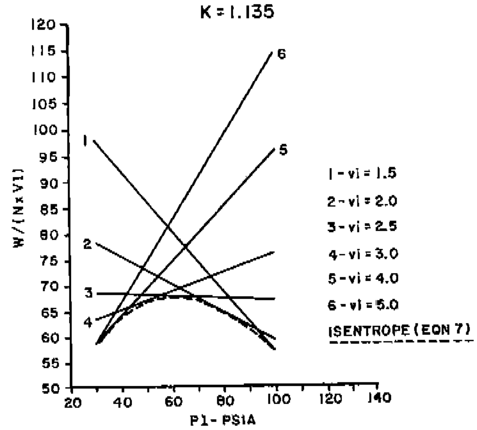

FIGURE 7

POWER RATIO VS P1

REFRIGERANT R-22

K = 1.135

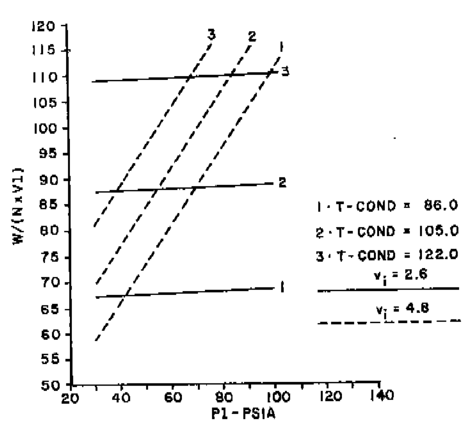

FIGURE 8

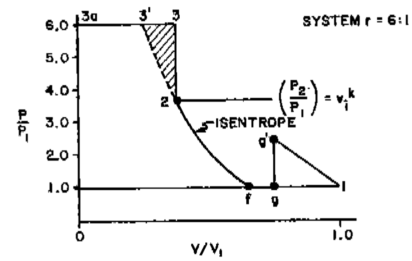

FIGURE 9

PART LOAD PERFORMANCE

5:1 PRESSURE RATIO

P1 = 50.0 PSIA

K = 1.135

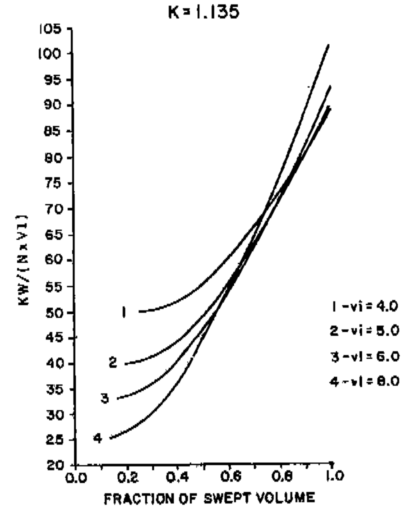

FIGURE 10

PART LOAD PERFORMANCE

4:1 PRESSURE RATIO

P1 = 50.0 PSIA

K = 1.135

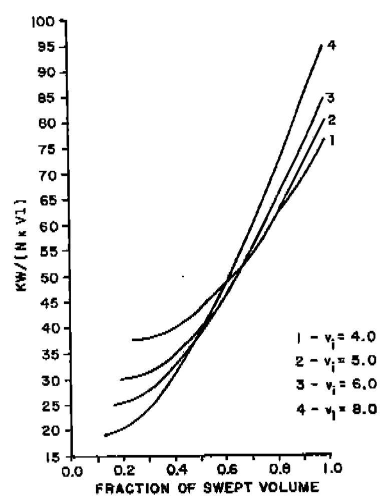

FIGURE 11
PART LOAD PERFORMANCE
2:1 PRESSURE RATIO
P1 = 50.0 PSIA
K = 1.135

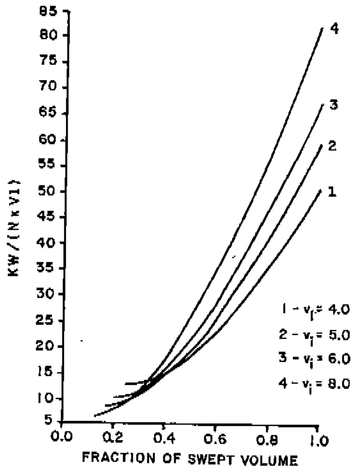

FIGURE 12
ADIABATIC EFFICIENCY
DATA FROM REFERENCE (1)