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# An Analysis of Cylinder Overpressure Using the Method of Characteristics

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# AN ANALYSIS OF CYLINDER OVERPRESSURE USING THE METHOD OF CHARACTERISTICS

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## ABSTRACT

Behaviour of discharge gas flow through a cylindrical discharge port is analyzed with special attention to the effect of inertia of compressed gas. The flow is assurmed to be an unsteady one-dimensional homertropic flow of compressible fluid and a single degree of freedom approximation is applied to the valve motion. The method of characteristics applied to the port is described after introducing basic equations of compressible fluid.

The computer simulation analyzing the discharge flow behaviour of a 3/4 HP rolling-piston type compressor is performed. The calculated cylinder pressure variation and gas speed variation are shown. Discharge valve displacement is also computed on the basis of the gas speed variation. Some of the results are investigated comparing with experimental results of the 3/4 HP rotary compressor. Although the analysis model is simple, it is shown that the computed results relatively correspond to the experimental ones.

## INTRODUCTION

During the last two decades the computer simulations of positive displacement type refrigerant compressors have been greatly extended[1][2], and it has enabled precise prediction of the cylinder overpressure. The basic mathematical model for the simulation consists of the following four sets of coupled equations.

i) volume equations
ii) thermodynamic equations
iii) mass flow equations
iv) valve dynamic equations

The assumption has been made that the steady flow equations can be applied to calculate the instant values occurring during unsteady flow. Flow forces on valves have been considered to result from pressure difference across the valve and the effects of the flow through the valve. As to the prediction of cylinder overpressure,it is recognized that a few sets of laboratory information, such as damping factors and stiction forces of discharge valves, are indispensable.[3][4]

This paper reports a different approach to predict cylinder overpressure. The approach was studied from the viewpoint that cylinder overpressure is still present even if the discharge valve and port have no flow resistance, because the velocity of compressed gas through the port is zero when cylinder pressure reaches discharge pressure and it takes some time for the flow to get enough speed to keep cylinder pressure at discharge pressure. Therefore,the method of characteristics is applied only to the region of the discharge port, and it is assumed that flow forces on valves mainly result from the thrust of discharge gas flow. Exclusive of the combining the flow equation and the method of characteristics and of the applying the assumption mentioned above to the valve dynamic equation, basically the former mathematical models were applied to compute the cylinder overpressure. Since one of the main goals of this study is to examine the validity of the prediction of cylinder overpressure calculated from minimum laboratory information, neither damping coefficiencies nor stiction forces of the discharge valve reed are taken into consideration. The coefficient of restitution is assumed to be zero.

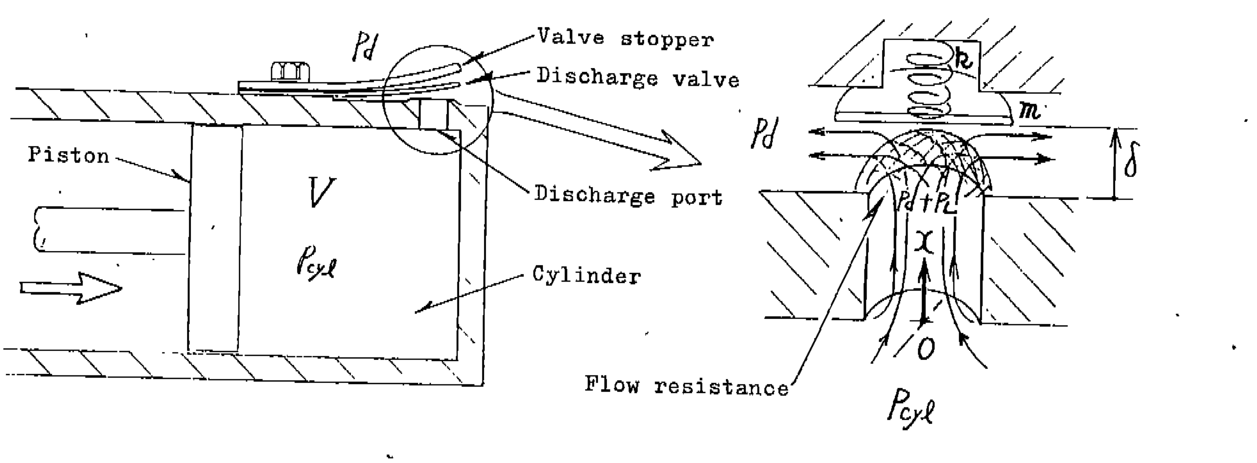


Fig. 1 Analysis model

## ANALYSIS MODEL

### Simplified discharge port and valve model

Fig. 1 shows the analysis model applied in this study for the prediction of cylinder overpressure of positive displacement type compressors. The time rate of change of cylinder volume depends on the type of compression mechanism and on rotation speed of the shaft. Polytropic process approach is used for the derivation of the cylinder volume equation. The discharge port is regarded as a simple cylindrical short tube. The discharge gas flow is considered to contract at the inlet and to impinge on the discharge valve. The whole resistance against the discharge gas flow, involving the resistance at the inlet, is assumed to arise at the outlet of the discharge port.

### Assumptions

The following assumptions are made for the analysis.

1) The rotating speed of the crank shaft is constant.

2) As to the derivation of the cylinder volume equation of rolling piston type compressors the thickness of the vane is negligible.

3) The polytropic index is constant during compression and discharge process.

4) The discharge gas through the port is unsteady one-dimensional homertropic flow.

5) The upstream condition can be considered to be stagnation condition.

6) The characteristic impedance of the discharge gas , $\rho a$ , is constant during discharge process.

7) The coefficient of contraction at the inlet of the discharge port is constant during discharge process. In this study 0.5 is chosen as the coefficient by way of first trial, because the coefficients under unsteady condition are less than those under steady condition[5] Hence the minimum area of the flow is as follows.

$$S = 0.5\, S_p \qquad (1)$$

8) The resistance coefficient at the inlet of the discharge port is constant and the resistance coefficient of the valve reed is regarded as a function of the valve displacement, which is obtained by experiments under steady flow conditions. Consequently the lost pressure of the discharge gas flow is expressed by

$$P_L = \frac{1}{2}\{\xi_i + \xi_v(\delta)\}\rho u^2 \qquad (2)$$

9) The discharge vlave can be regarded as a simple dynamic system with one degree of freedom equipped with a stopper which limits the valve displacement.

10) There is no pressure difference across the valve and the force acting on the valve results only from the thrust of the contraction jet of the discharge gas.

### Basic equations

i) Volume equations
Referring to Fig. 2, schematic diagram of the crank shaft-rolling piston-vane arrangement, and to the assumption 1) and 2) described in the preceding section, the cylinder volume equations for rolling piston type compressors are as follows.

$$V = \frac{H}{2}\{R^2\theta - r^2(\theta+\phi) - (R-y)e\sin\theta\} \qquad (3)$$

where

$$y = R(1-\cos\theta) - r(\cos\phi - \cos\theta) \qquad (4)$$

$$\phi = \sin^{-1}\left(\frac{e}{r}\sin\theta\right) \qquad (5)$$

$$\theta = \omega t \qquad (6)$$

ii) Thermodynamic equation
The assumption 3) leads to the following equation.

$$P_{cyl} = P_s\left(\frac{V_s}{\mathcal{V}}\right)^{n_s} \qquad (7)$$

During the compression process the compressed gas volume $\mathcal{V}$ is expressed by

$$\mathcal{V} = V(\theta) \qquad (8)$$

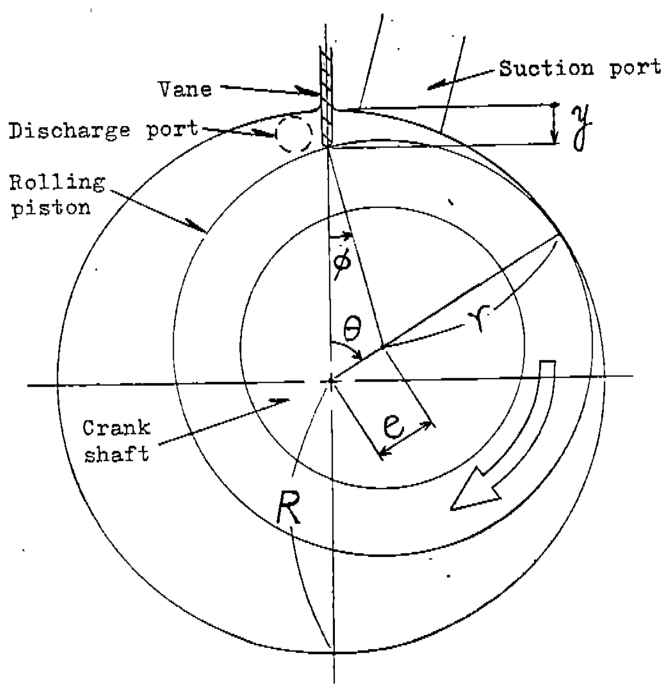


Fig. 2 Schematic diagram of crank shaft -rolling piston-vane arragement

The expression for $\mathcal{V}$ during the discharge process is described in the next section.

iii) Flow equations
The momentum equation for unsteady one-dimensional flow is expressed by [6]

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \frac{\partial P}{\partial t} = 0 \qquad (9)$$

Combining the continuity equation and the energy equation for homertropic flow gives the following equation. [6]

$$\rho a^2 \frac{\partial u}{\partial x} + \frac{\partial P}{\partial t} + u \frac{\partial P}{\partial x} = 0 \qquad (10)$$

The characteristic and compatibility equations corresponding to the coupled partial differential equations mentioned above are as follows.

$$\left(\frac{dx}{dt}\right)_{\pm} = u \pm a \qquad (11)$$

$$dP_{\pm} \pm \rho a \, du_{\pm} = 0 \qquad (12)$$

Those are the governing equations for the flow of discharge gas through the port. Numerical implementation of the method of characteristics is described in the next section.

iv) Valve dynamic equation
The assumptions 7), 9) and 10) lead to the following valve dynamic equation.

$$m\ddot{\delta} + k\delta = S\rho(u - \dot{\delta})^2 \quad (0 \leq \delta \leq h_v) \qquad \text{-----} \ (13)$$

This equation can be decomposed into two expressions as follows.

$$\dot{\delta} = v \qquad (14)$$

$$\dot{v} = \frac{S\rho(u - v)^2 - k\delta}{m} \quad (0 \leqq \delta \leqq h_v) \qquad \text{-----} \ (15)$$

## NUMERICAL CALCULATION

### The method of characteristics

In this study the numerical implementation of the method of characteristics is simplified as follows in order to avoid both complication of the simulation program and increase of the computer time. Fig. 3 shows schematically the finite difference grid for the unit process of the calculation. According to the Euler method, the characteristic and compatibility equations are transformed into the following computational equations.

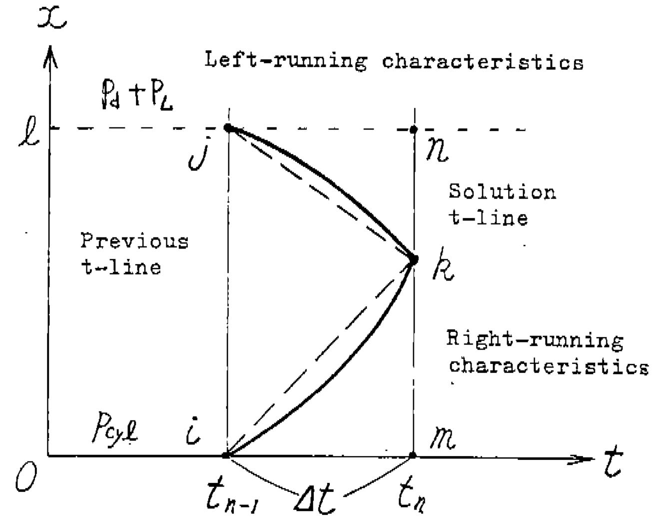


Fig. 3 Finite difference grid for a unit process of the method of characteristics

$$\frac{x_k - x_j}{\Delta t} = u_j - a \qquad (16)$$

$$\frac{x_k - x_i}{\Delta t} = u_i - a \qquad (17)$$

$$P_k - \rho a u_k = P_j + \rho a u_j \qquad (18)$$

$$P_k + \rho a u_k = P_i + \rho a u_i \qquad (19)$$

The assumption made for the simplification of the method of characteristics is that there is not very much change of the velocity of discharge gas along x-axis at any moment. In Fig. 3 this implies that

$$u_i = u_j \qquad (20)$$

$$u_k = u_m = u_n \qquad (21)$$

The assumption is based on the short length of the discharge port. This assumption yields another advantage which keeps the time interval, $\Delta t$, constant throughout the discahrge process. Fig. 4 illustrates the finite difference grids for the simplified computation process. The time interval, $\Delta t$, is given by

$$\Delta t = \frac{l}{(u+a) - (u-a)} = \frac{l}{2a} \qquad (22)$$

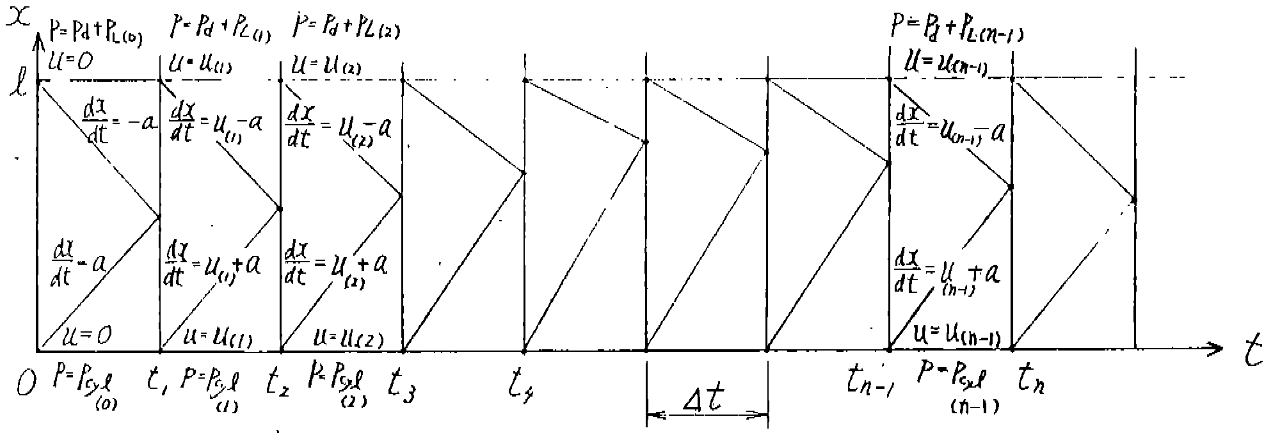


Fig. 4 Finite difference grids for analysis

Referring to Fig. 1, Fig. 4 and the assumption 8), the computational equations for the n-th step of the secant calculation are as follows.

$$\begin{bmatrix} 1 & -\rho a \\ 1 & \rho a \end{bmatrix} \begin{Bmatrix} P_{(n)} \\ u_{(n)} \end{Bmatrix} = \begin{Bmatrix} P_d + P_{L(n-1)} + \rho a u_{(n-1)} \\ P_{cyl(n-1)} + \rho a u_{(n-1)} \end{Bmatrix} \qquad (23)$$

Therefore

$$u_{(n)} = \frac{1}{2} \cdot \frac{P_{cyl(n-1)} - \{P_d + P_{L(n-1)}\}}{\rho a} \qquad (24)$$

where

$$P_{L(n-1)} = \frac{1}{2}\left[\xi_i + \xi_v\{\delta_{(n-1)}\}\right] \rho u_{(n-1)}^2 \qquad (25)$$

Cylinder pressure

Based on the assumption 3), cylinder pressure for the n-th step of the calculation, $P_{cyl(n)}$, is expressed by

$$P_{cyl(n)} = P_s \left\{ \frac{V_s}{\mathcal{V}_{(n)}} \right\}^{n_s} \qquad (26)$$

Compressed gas volume, $\mathcal{V}_{(n)}$, can be computed by the following expression on the basis of the assumption that the cylinder gas changes quasi-statically from one state to another by a polytropic process.

$$\mathcal{V}_{(n)} = \mathcal{V}_{(n-1)} + \{V_{(n)} - V_{(n-1)}\} + S u_{(n-1)} \Delta t \qquad (27)$$

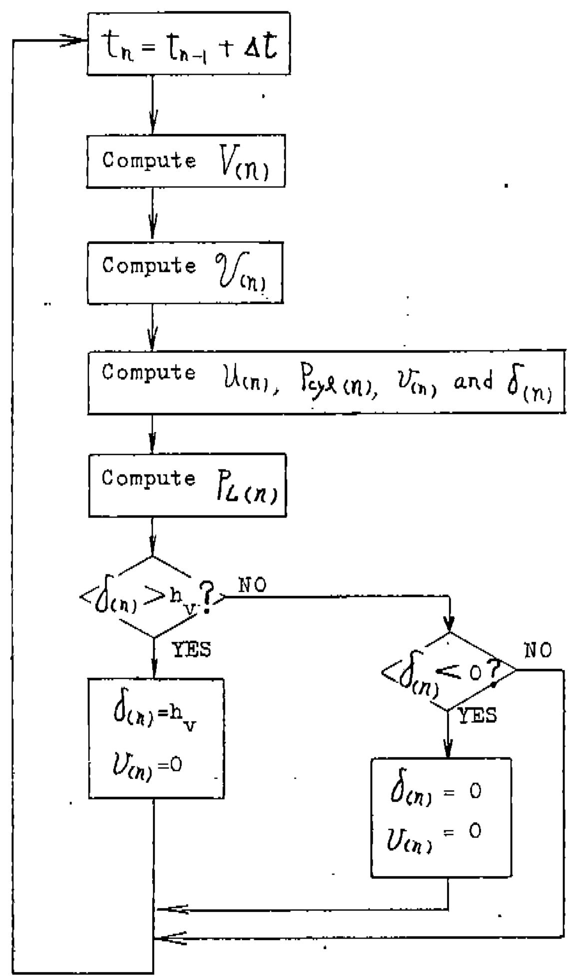


Fig. 5 Outline of the main program

Table 1 Specifications for laboratory compressor

| | | | |
|---|---|---|---|
| Compressor | Stroke volume (cc/rev) | | 2.5 |
| | Discharge port diameter (mm) | | 6.0 |
| | Discharge port length (mm) | | 4.0 |
| | Valve lift (mm) | | 4.0 |
| | Equivalent valve mass (gr) | | 0.16 |
| | Equivalent valve stiffness (N/mm) | | 0.2 |
| Moter | Pole number | | 2 |
| | Norminal output (w) | | 400 |
| | Power source | Voltage (V) | 100 |
| | | Frequency (Hz) | 60/50 |

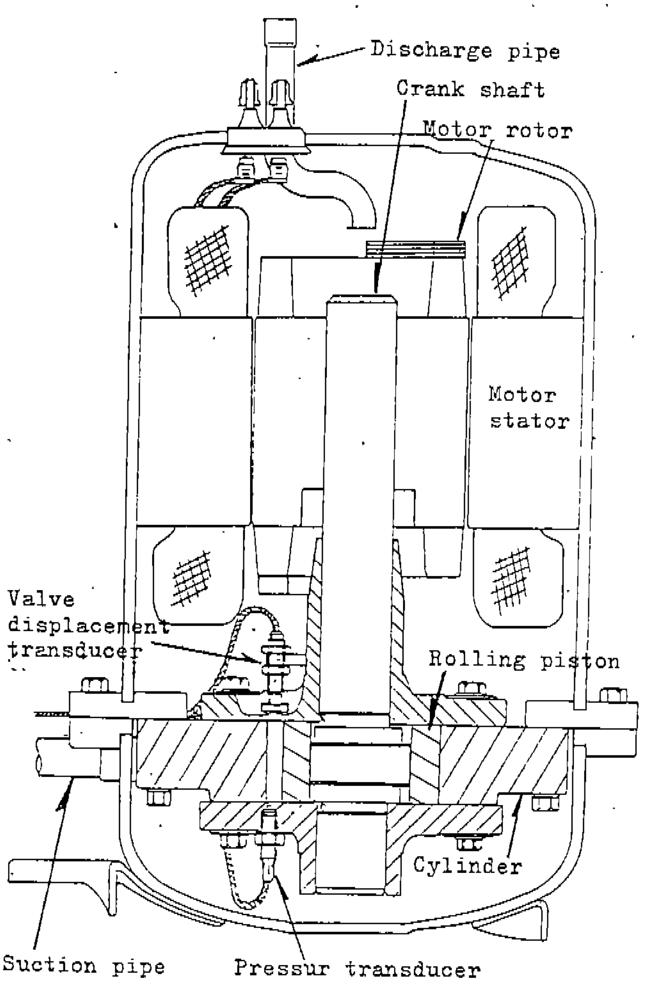


Fig. 6 Sectional view of laboratory compressor

Valve displacement

Both displacement and velocity of the discharge valve are numerically calculated by Euler's method. Differential equations (14) and (15) are transformed into the following computational equations.

$$\delta_{(n)} = \delta_{(n-1)} + v_{(n-1)} \Delta t \tag{28}$$

$$v_{(n)} = v_{(n-1)} + \frac{S\rho\{u_{(n-1)} - v_{(n-1)}\}^2 - k\delta_{(n-1)}}{m} \tag{29}$$

Numerical procedure

Fig. 5 gives the outline of the main structure of the computer program. It should be noted that the solution can be carried on even if the discharge valve is not open.

## MODEL EVALUATION

Experimental tests were run using a laboratory compressor very closely resembling the stock compressor from which it was derived Experimental-theoretical correlations were investigated in order to evaluate the mathematical model and to determine the validity of the various assumptions in the preceding sections.

Test compressor

The laboratory compressor was a rolling-piston type within hermetic shells which can be divided into two and be connected by flanges as shown in Fig. 6. This configuration made it easy to take out the lead wires from pressure and displacement transducers. The main modification was the increase of the space between the compression mechanism and the drive motor to accommodate the displacement transducer. The specifications for the laboratory compressor are given in Table 1.

Test condition

The experimental responces were obtained for one of the standard operating conditions given in Table 2 using Freon-22 as the refrigerant. A full-automatic compressor load stand was employed for the tests, which keeps test conditions completely constant. Automatically controlled expansion valve, water regulating valve and electric heaters adjust test conditions to any required operating conditions.

Table 2 Test conditions

| | | |
|---|---|---|
| Suction pressure ($\times 10^5$ Pa) | 6.0 | |
| Suction gas temparature ($^{\circ}$F) | 60 | |
| Discharge pressure ($\times 10^5$ Pa) | 21.0 | |
| Power source frequency (Hz) | 60 | 50 |
| Number of revolutions (RPM) | 3420 | 2870 |
| Discharge gas temparature ($^{\circ}$F) | 199 | 196 |

## Theoretical and Experimental results

Experimental cylinder pressure obtained is shown in Figs. 7 and 8 for two different rotating speed of the crank shaft. Theoretical-experimen.tal correlations of cylinder pressure and valve displacement are illustrated in Figs. 9 to 12, during the discharge process. Figs. 13 and 14 give theoretical velocity traces of discharge gas through the port.

## Evaluation

Referring to Figs. 9 and 10 it can be seen that good correlation was obtained for cylinder pressure response. The major deviation between theoretical and experimental results consists of an oscillation of the theoretical cylinder pressure after the initial pressure peak. This phenomenon corresponds to the fluctuation of the velocity of discharge gas shown in Figs. 13 and 14. Relatively

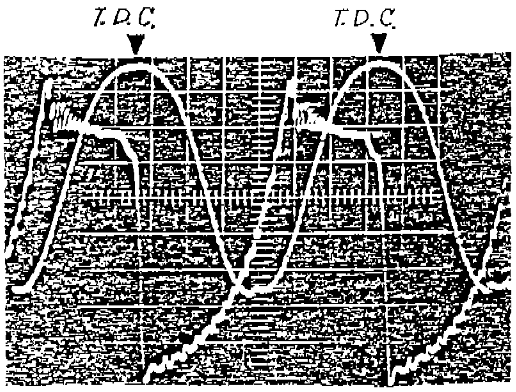


Fig.7 Measured cylinder pressure
(3420 RPM)

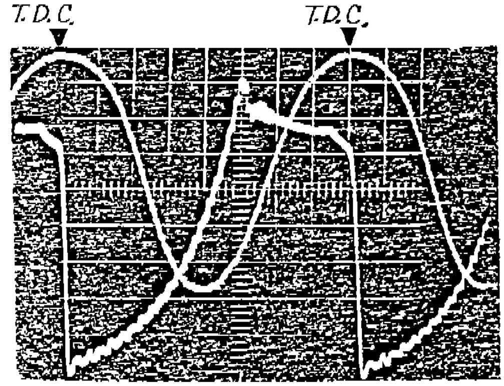


Fig.8 Measured cylinder pressure
(2870 RPM)

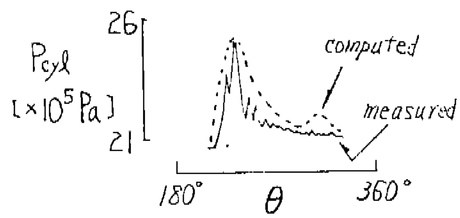


Fig.9 Computed-measured cylinder overpressure
(3420 RPM)

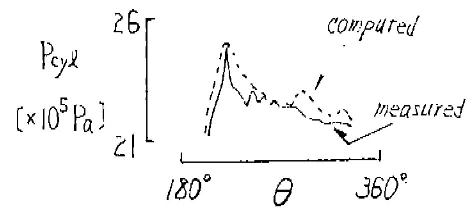


Fig.10 Computed-measured cylinder overpressure
(2870 RPM)

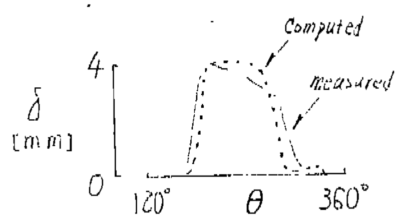


Fig.11 Computed-measured valve displacement
(3420 RPM)

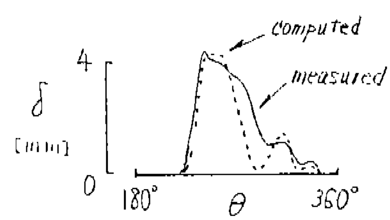


Fig.12 Computed-measured valve displacement
(2870 RPM)

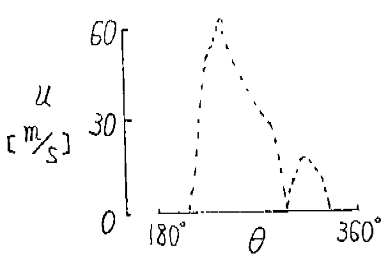


Fig.13 Computed velocity of contraction
(3420 RPM)

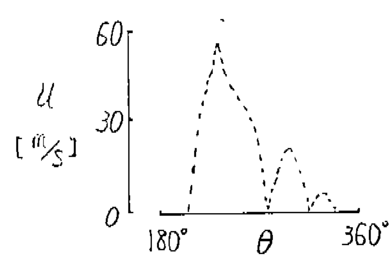


Fig.14 Computed velocity of contraction
(2870 RPM)

good correlation was also obtained between computed and measured valve displacement. Especially it should be noted that re-opening of the discahrge valve was definitely simulated by the computer program, and re-opening of the valve corresponds to the fluctuation of the velocity of discharge gas. Therefore, it was concluded that the actual trace of the cylinder pressure in close vicinity to the discharge port also had an oscillation after the initial pressure peak and that the oscillation was not observed in the experiment because the pressure transducer had been located in the opposite side to the discharge port. Those oscillations and fluctuations mentioned above are considered to be due to the effect of inertia of the high speed gas flow, in other words, inertia super-"dis"charging of the compressed gas.

In general, the experimental results verified that the analysis model successfully predicted the cylinder overpressure and the valve motion in spite of the approximate methods for the calculation. It should be noted that the model is simplified and does not require values for damping factors, coefficients of restitution and stiction forces.

## CONCLUSIONS

The basic equations and convenient approximation to predict the cylinder overpressure using the method of characteristics have been described and applied to a rolling-piston type refrigerant compressor. From the comparison between theoretical-experimental results the following conclusions may be drawn.

1) An approximate method has been determined to simulate the cylinder overpressure, the valve displacement and the velocity of discharge gas during discharge process on the basis of the method of characteristics applied only to the region of the discharge port.
2) As to the cylinder overpressure, good agreement between theoretical and experimental results of the initial pressure peak has been obtained taking no stiction forces into consideration.
3) The theoretical overpressure trace has an oscillation after its initial pressure peak, and the oscillation exactly corresponds to both fluctuation of the velocity of discharge gas and re-opening of the discharge valve.
4) As to the valve motion, good correlation has been obtained between computed and measured results of the re-opening phenomenon of the discharge valve.
5) The effect of inertia of compressed gas is very important, and it must be included in the analysis of cylinder overpressure.

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## NOMENCLATURE

a = velocity of sound of discharge gas
e = crank radius
H = cylinder height
hv = valve lift
k = equivalent discharge valve stiffness
l = discharge port length
m = equivalent discharge valve mass
Ns = polytropic index
P = discharge port pressure
Pc = cylinder pressure
Pd = discharge pressure
Pl = lost pressure
Ps = suction pressure
r = rolling piston radius
R = cylinder radius
S = sectional area of contraction
Sp = discharge port area
t = time
$\Delta t$ = time interval
u = velocity of contraction through discharge port
v = velocity of discharge valve
V = cylinder volume
Vs = maximum cylinder volume
x = axis along discharge port
$y$ = vane displacement
$\delta$ = valve displacement
$\rho$ = density of discharge gas
$\xi_i$ = resistance coefficient (inlet of discharge port)
$\xi_v$ = resistance coefficient (discharge valve)
$\theta$ = crank angle (see Fig. 2)
$\omega$ = angular velocity of crank shaft
$\mathcal{V}$ = compressed gas volume (see expression (8) & (27))