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A STRESS ANALYSIS OF BENDING FATIGUE SPECIMENS USING THE FINITE ELEMENT METHOD

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ABSTRACT

A numerical analysis of stresses generated in bending fatigue specimens has been performed employing the commercially available FEM-program ADINA. The results show that transverse bending occurs, giving rise to a stress enhancement in the edge region. A stress maximum is obtained approximately 2 mm away from the specimen edge. This result is consistent with the fact that fatigue initiation predominantly takes place in a zone 1-2 mm inside the edge. The stress maximum thus obtained is due to a combination of a large tensile stress component and large shear stresses. Standard stress formulae can neither predict correct stress level nor stress distribution. A stringent analysis therefore requires the use of advanced numerical computations.

INTRODUCTION

The assessment of a reliable S-N curve requires an accurate determination of stresses involved. If the stress state is complicated the stress for a certain displacement may not always be calculated using formulae available in standard handbooks. Furthermore, the stress may vary from one point to another in a manner which is difficult to predict. This situation arises for instance during bending of fatigue specimens. It is a well known phenomenon that flat specimens subjected to pure bending adopt a curvature in the transverse direction (1). As a result of this a saddle surface is created, sometimes termed anticlastic surface. The transverse curvature usually has a parabolic shape, but when the width (w) to thickness (t) ratio increases ($w/t \gtrsim 25$), there is a tendency to concentrate the deflection to the edge regions leaving the inner surface relatively flat (2). It was speculated in a previous report (3) that the high frequency of fatigue initiation points in the edge region could be ascribed to the deflection of edges. Therefore it was considered to be of vital importance to investigate this more thoroughly.

The stress formulae available in the standard literature do not in general incorporate the effects of transverse bending. In addition, a general formula cannot be expected to be valid for a wide variety of geometries. Therefore, there has long been a need for a more accurate determination of the stress state prevailing in bending fatigue specimens. It is not until the last few years that such an analysis has become feasible owing to the access to fast computers combined with appropriate finite element programs. One such FEM-program is ADINA, which was developed by K.J. Bathe and coworkers (4). ADINA was considered to be suitable in the present study due to its ability to handle geometric non-linearities in shell and plate problems.

EXPERIMENTAL

Modelling

A numerical analysis of stresses was performed on a geometry corresponding to our own bending fatigue specimen (3).

In order to make the computer simulation as realistic as possible an elastic boundary condition was used corresponding to bakelite plates of 0.2 mm thickness on either side of the specimen in the grip section. These plates are used in all bending fatigue tests to avoid stress concentrations and a resulting premature failure at the grip edge. Values of 0.35 for Poisson's ratio and $5 \cdot 10^3$ MPa for the elastic modulus were assumed for bakelite. The specimen itself was treated as an elastic continuum with the corresponding values of 0.3 and $2.1 \cdot 10^5$ MPa respectively. An implication of this is that no effects of for instance residual stresses are accounted for.

The finite element mesh consisting of 36 9-node isoparametric shell elements (5) is shown in figure 1. Here we have taken advantage of the lateral symmetry of the specimen and confined the calculations to the right half. This gives a correct description of the situation provided the symmetry line is subjected to the boundary condition that y-displacements are all zero.

According to experimental practice the force is applied along a line corresponding to nodes 195-201 (indicated in figure 1). Furthermore the force is always perpendicular to the specimen. This was accomplished in the calculations by choosing appropriate values of the x- and z-components of the force. The force and the corresponding deflection angle were measured in the testing device (see table 1).

The boundary condition corresponding to the situation in the grip was represented by 27 brick elements, 9 of which represent the grip section of the specimen and 18 representing the bakelite plates.

Initially a linear analysis of the problem was attempted, but it soon became clear that this led to erroneous results. For instance, the radius of curvature of the specimen close to the grip became too small, and the concomitant stresses in this part became unrealistically large.

Linear calculations can only be expected to yield reasonable results for small deformations and strains. However, in order to describe the displacements and accompanying considerable axial and transversal curvatures encountered during our fatigue tests, we had to include geometrically non-linear effects. This can be done in ADINA by using the total Lagrangian formalism (5) in which all static and kinematic variables are referred to the initial configuration.

Significant deviations from the linear results appeared due to the up-dating of the stiffness matrix after each of the 10 discrete steps in which the force was applied. In order to restore static equilibrium, 20-30 iterations were subsequently required in each such step. To illustrate the computational complexity of the problem we might mention that 80 minutes of IBM 3032 CPU-time were necessary to analyse the 0.381 mm fatigue specimen with element geometry as shown in fig. 1, 229 modes and 867 degrees of freedom. The equivalent CPU-time for 0.508 mm thickness was 34 minutes. Calculations were attempted also for 0.254 mm thickness but were unsuccessful due to numerical divergence.

RESULTS

Calculations were performed for specimen thicknesses of 0.381 mm and 0.508 mm. The results for both thicknesses clearly show that, already for small displacements, there is clear evidence of a transverse curvature.

t = 0.381

When the displacement in the x-direction is 0.83 mm at the point where the force is applied (indicated in figure 1 with arrows) the deflection at the edge is 22 μm for the section indicated by the dotted line in figure 1. Increasing the displacement enhances the effect of transverse bending. When the displacement is 2.50 mm and the center line assumes the shape of the lower curve in figure 2, the corresponding edge deflection becomes 43 μm. The curve produced by the ADINA calculations is shown in figure 3 as the curve labelled A. For comparison the experimental curve for the same section obtained using the Talysurf tester is drawn (labelled X). The agreement between the theoretical calculations and the Talysurf measurements is excellent, the discrepancy at the outermost edge being approximately 5 μm. The corresponding situation when Δx is 5 mm (B in figure 2) is shown in figure 4. In this case the agreement is not as good as in the previous example but it should be pointed out in this context that the Talysurf method used in the present study involves an experimental error. Furthermore there is an inaccuracy involved in evaluating the small diagrams obtained in the Talysurf tester.

The transverse curvature is accompanied by a variation in terms of stresses, which tend to increase towards the edge region. Even this effect is enhanced when the displacement increases. One striking result is that the stress maximum is not assumed on the edge itself but in a narrow band of width ~2 mm away from the edge. For instance, when $\Delta x = 5$ mm the z-component of the stress (σ_z) is 392 MPa at point 1 in element 13 (see figure 1). This should be compared to $\sigma_z = 480$ 2 mm away from the edge in element 15 (point 2) and $\sigma_z = 425$ MPa (point 3) in the outermost gaussian point. The dominating term is the z-component but the other stress-components are non-vanishing, indicating that the stress state is by no means uniaxial. Therefore, the von Mises effective stress is perhaps a better parameter for making comparisons. The stresses according to von Mises for the three points above at $\Delta x = 5$ mm are 411 MPa 562 MPa and 466 MPa respectively. A contributory reason for the large stress obtained in point 2 is the enhanced shear stresses created in the vicinity. In general, there was clear evidence of shear stress maxima in a region 1-2 mm away from the edge. By way of contrast, shear stresses were usually considerably lower in the center and the outermost edge region.

For the maximum displacement considered, $\Delta x = 6.92$ mm, corresponding to a force of 24.3 N, the calculated von Mises stresses in points 1, 2, 3 are 568 MPa, 792 MPa and 647 MPa respectively. It is obvious from those values that the stress maximum is even more pronounced at large displacements.

It may be qualitatively stated that the discrepancy between the FEM results and the results obtain by using standard formulae increased towards larger displacements.

Standard handbooks suggest $\sigma = \frac{6P}{t^2}$ where P is force applied and t is specimen thickness. The triangular shape of the specimen is accounted for in the factor 6. When P = 24.3 N, a stress of 1004 MPa is readily calculated for t = 0.381. By way of comparison, the maximum stress obtained in the FEM calculations is 792 MPa at this force.

t = 0.508

The results for t = 0.508 mm are in qualitative agreement with results for t = 0.381 mm. No significant dissimilarity in terms of transverse curvature could be observed for the two thicknesses considered. It is obvious that thinner specimens have to be analysed if thickness effects are to be studied. When the displacement was 2.5 mm at the point where the force is applied, the edge deflection became 52 μm. This value should be compared to 43 μm calculated for the corresponding cross section for t = 0.381 mm. Increasing the displacement yields slightly larger deflections. For instance, at Δx = 3.18 mm, which is the last step in our simulation, the edge deflection is 57 μm.

The stresses according to von Mises at points 1, 2 and 3 in figure 1 when Δx = 3.18 mm are 332 MPa, 436 MPa and 365 MPa. This situation corresponds to an applied force of 24.3 N and yields a stress of 564 MPa using the simple formula quoted above. Once again, the values calculated in the present investigation become significantly lower than the standard formula.

Finally it should be pointed out that, although it have been shown that the stresses vary considerably in the transverse direction, the longitudinal variations are in all cases very small. In general this variation was less than 10% if points at comparable distances from the edge were considered.

DISCUSSION

Experience from tests performed in our laboratory has led to the suspicion that the stress distribution in our bending fatigue specimens is far from homogeneous. This is primarily due to the observation that initiation of fatigue fracture predominantly occurs in the edge region (3). A detailed fractographic study performed recently showed that approximately 90% of all fractures had initiated 1-2 mm away from the edge (6). This observation is consistent with the FEM-calculations presented in this paper, resulting in a stress maximum in a narrow region close to but clearly distinct from the edge. On the basis of our theoretical calculations it is evident that the shear stresses are significant in this region. This effect combined with a large value of σ_z yields the large von Mises stress. Fatigue cracks are most commonly observed to be perpendicular to the edge. This observation is explicable in terms of large values of the tensile component σ_z and the shear component τ_{xy}.

Since the stress state is nowhere uniaxial it is considered that the von Mises stress gives the most relevant representation. However, due to the large variation of stresses over the specimen surface - both in terms of direction and magnitude - it is difficult to envisage how the situation should be specified in an S/N-curve. In the light of the present simulation there appears to be no way in which the problem can be uniquely described by one stress parameter. This is illustrated by the examples given above for t = 0.381 where stresses of 568 MPa, 792 MPa and 647 MPa were obtained in the same section but at different positions relative to the edge. The situation is further complicated by the fact that the standard manuals predict a stress of 1004 MPa under these conditions. Obviously, the present calculations systematically yield lower stress values than those previously calculated. This statement is valid for both t = 0.381 mm and t = 0.508 mm.

By way of contrast, there are rather small stress gradients in the longitudinal direction. As pointed out earlier these variations are less than 10%. The design of the bending fatigue specimen was originally based on educated assumptions but at that time no detailed numerical analysis was feasible. In our opinion the choice of specimen geometry was a good one since the present calculations show that the stress state exhibits rather small longitudinal variations. In contrast to longitudinal stress variations, transverse stress variations can only be influenced marginally by specimen geometry since they are due to an inherent material property, the so called Poisson contraction.

Although it has been shown that there are considerable discrepancies between previous and present calculations it must be emphasized that our testing procedure is <u>not</u> disqualified. They merely show that, despite the rather simple specimen geometry, powerful computational techniques have to be employed in a thorough stress analysis. However, since our testing technique with strain gauges provides an accurate and reproducible way of adjusting the amplitude it is a very good tool for classifying different materials. The results obtained in the present report must be regarded as very important since they enable a correct prediction of fatigue initiation in a zone 1-2 mm inside the edge. Similarly, they support the suggestion made by Nilsson and Persson (3) that this effect is explicable in terms of stress enhancement in this region due to edge deflections.

CONCLUSIONS

1. The calculations show that the maximum stresses are attained a small distance away from the edge. This is consistent with experimental observations of fatigue initiation points in this region.
2. Standard formulae available in the literature can neither predict correct stress level nor stress distribution. This is due to geometrically non-linear effects which elementary theory cannot account for.
3. The geometry of our bending fatigue specimens is very good since longitudinal stress gradients are small.
4. Since stress cannot be uniquely defined other variables should be considered in the representation of the S-N-curve.

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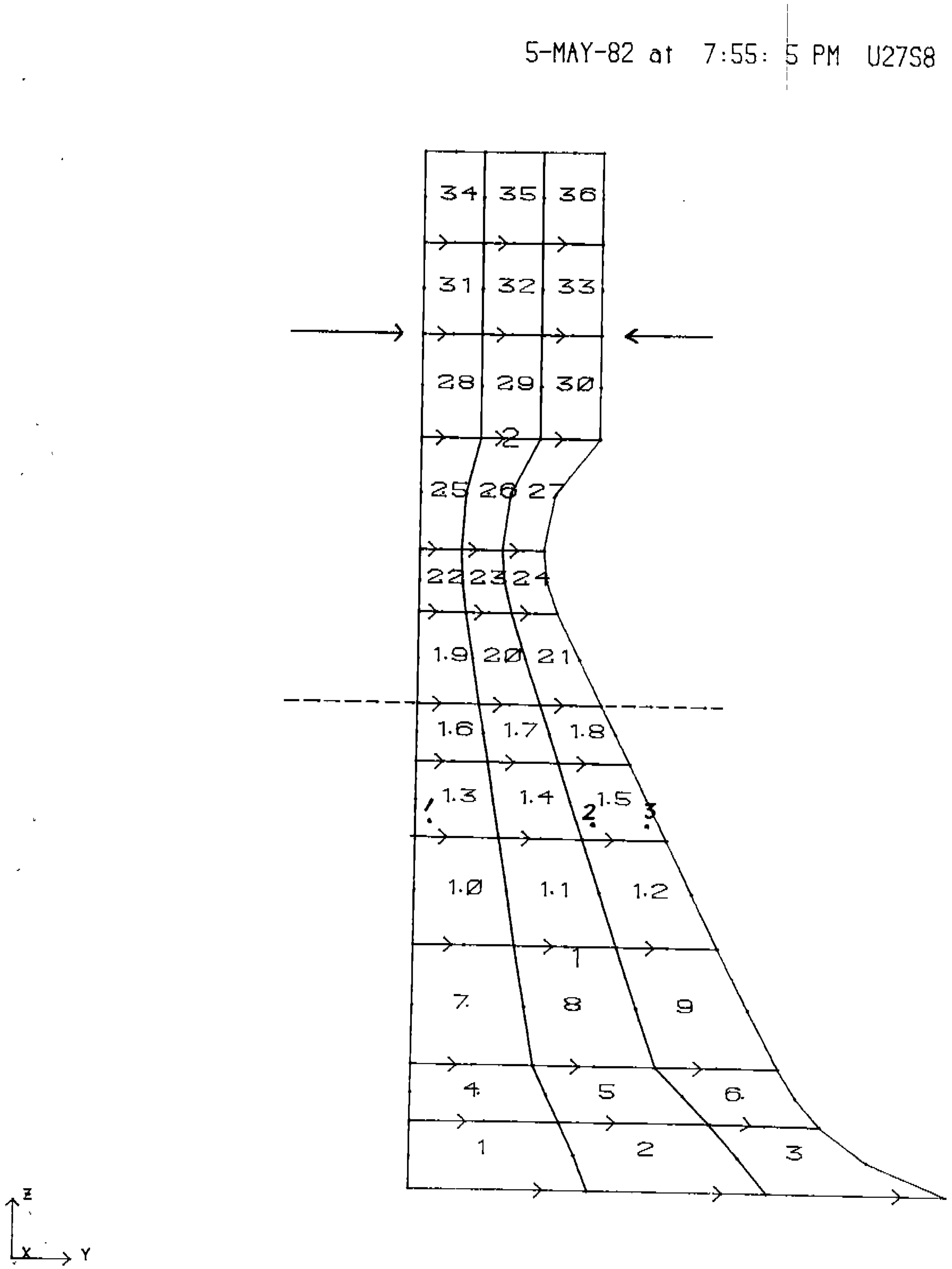
Table 1

Step No	F (N)	Θ (°)	F_x (N)	F_z (N)
0	0	0	0	0
1	2.63	1.7	2.629	-0.078
2	4.72	2.9	4.714	-0.239
3	7.51	4.5	7.487	-0.589
4	9.77	5.7	9.722	-0.970
5	11.86	6.4	11.786	-1.322
6	14.32	8.0	14.181	-1.993
7	17.20	9.4	16.969	-2.809
8	19.51	10.6	19.177	-3.589
9	21.94	11.7	21.484	-4.449
10	24.29	12.8	23.686	-5.381

FIGURE 1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34
35
36
31
32
33
28
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30
2
25
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1.9
20
21
1.6
1.7
1.8
1.3
1.4
1.5
1
2
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1.0
1.1
1.2
1
7
8
9
4
5
6
1
2
3
Z
X
Y

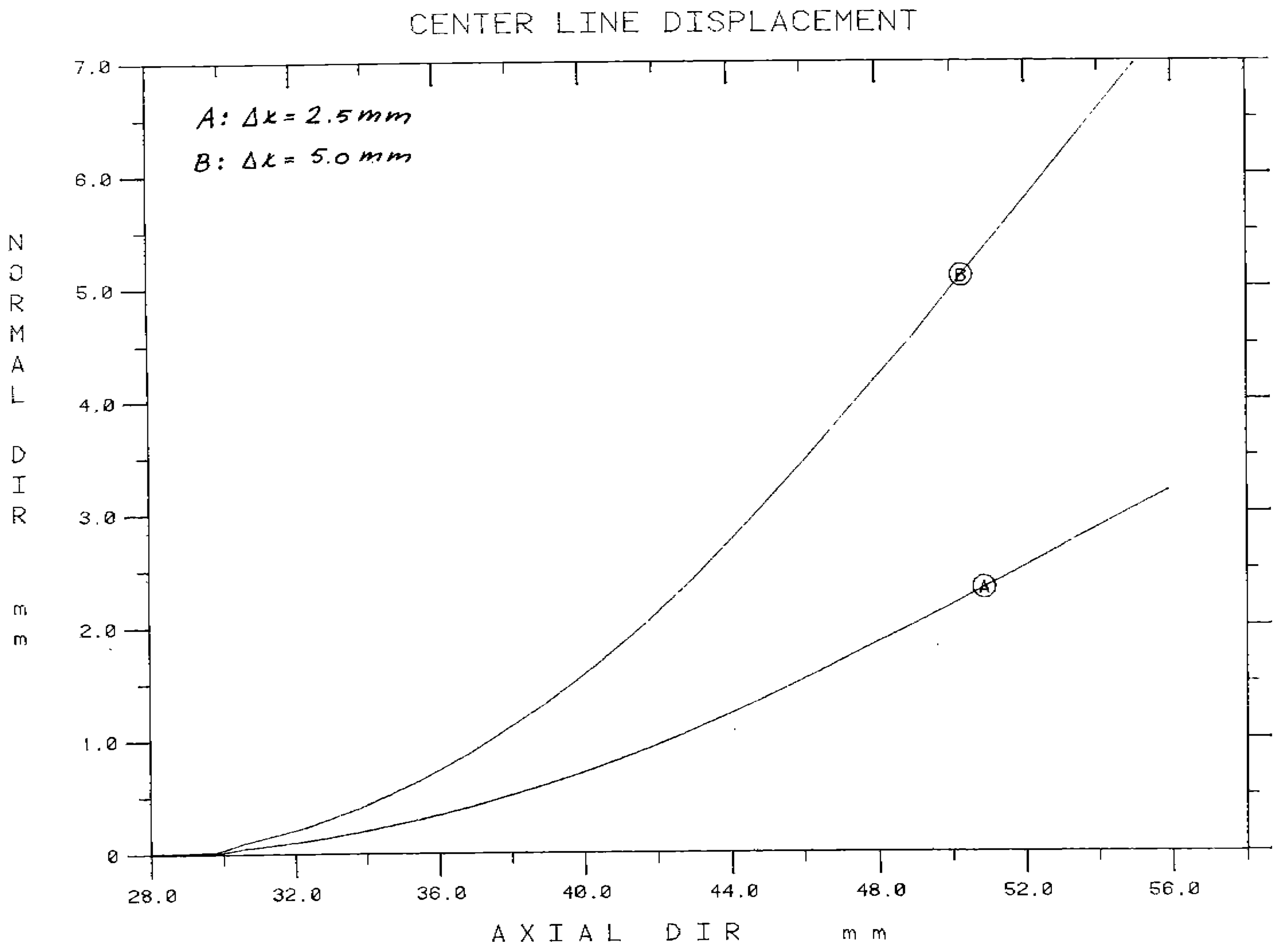
CENTER LINE DISPLACEMENT
A: Δℓ = 2.5 mm
B: Δℓ = 5.0 mm
Ⓐ
Ⓑ
NORMAL DIR mm
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
AXIAL DIR mm
28.0
32.0
36.0
40.0
44.0
48.0
52.0
56.0

FIGURE 2

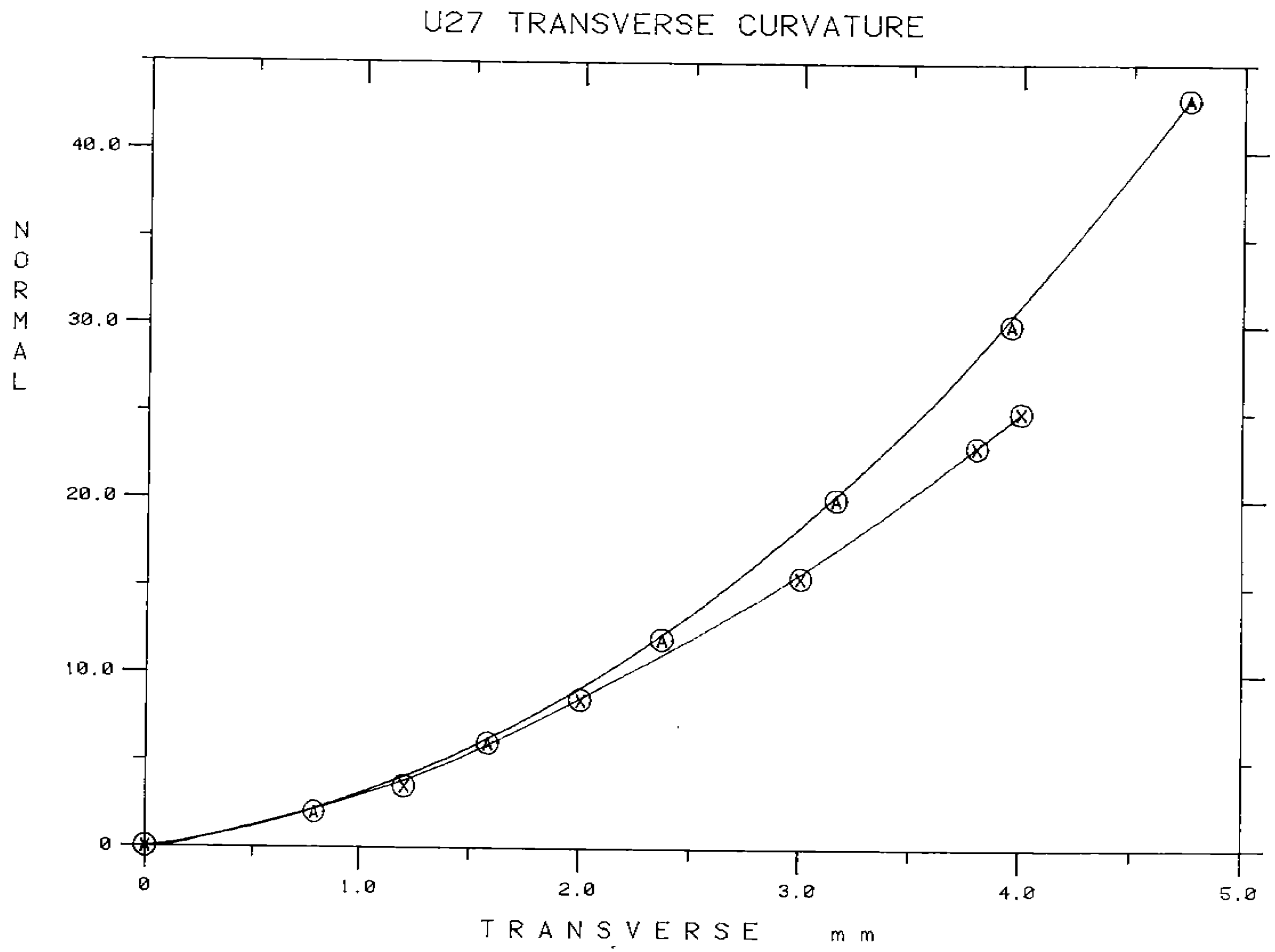
U27 TRANSVERSE CURVATURE
NORMAL
40.0
30.0
20.0
10.0
0
0
1.0
2.0
3.0
4.0
5.0
TRANSVERSE mm
A
X

FIGURE 3

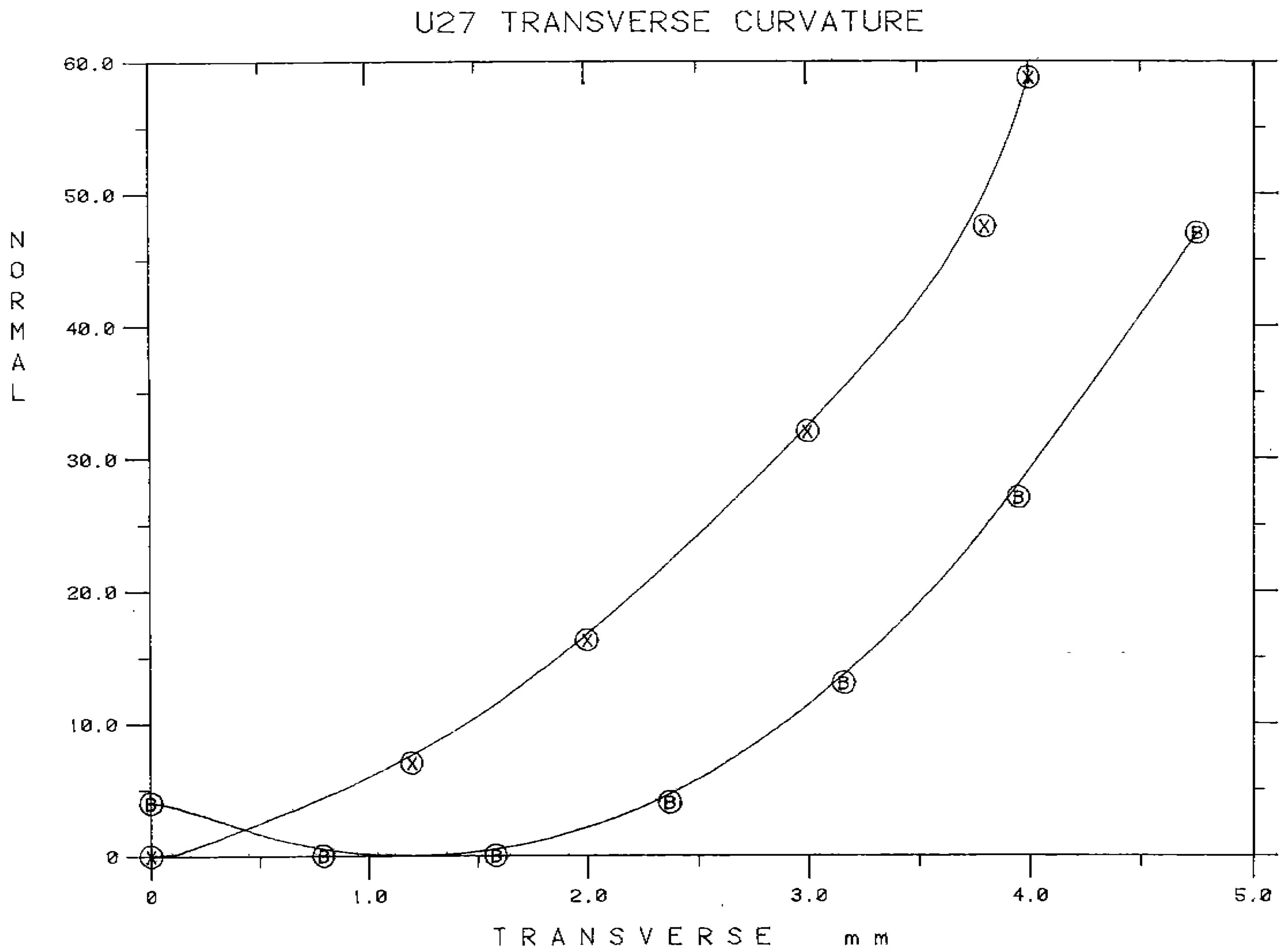
U27 TRANSVERSE CURVATURE
NORMAL
60.0
50.0
40.0
30.0
20.0
10.0
0
0
1.0
2.0
3.0
4.0
5.0
TRANSVERSE mm
X
B

FIGURE 4

Closing Statement for

"A STRESS ANALYSIS OF BENDING FATIGUE SPECIMENS USING THE FINITE ELEMENT METHOD"

by

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The following is a closing statement by the authors for the paper which appeared on pages 111-118 of the Proceedings of the 1982 Purdue Compressor Technology Conference, July 21-23, 1982.

The authors claim in the original paper, that the stress state in a bending fatigue specimen is far from uniform. Furthermore it is shown that the stress in a transverse section increases considerably towards the specimen edge. Due to a mistake made by the authors, it is stated that the "stress maximum is obtained approximately 2 mm away from the specimen edge", implying that the stress decreases in the outermost region. After having checked the calculations we have found that the stress maximum is attained in the outermost gaussian point (~ 0.2 mm from the edge), no evidence for a stress decline being observed.

A consequence of this is that there is no obvious theoretical explanation to the frequent observations of initiation points ~ 0.5 mm inside the edges. However, it should be pointed out that a more precise analysis of the stress state in the edge region probably requires a much finer finite element mesh than that used in the present investigation. Moreover, the rounded shape of the edge has to be taken into account. A suggestion for future work would be a more detailed assessment of stress state in the edge region, incorporating effects of rounded corners at the specimen edge.

The convergence problems reported for 0.254 mm specimen thickness have been overcome by dividing the step size (cf table 1) of the incremental ADINA-integration by a factor of ten.