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Abstract

We develop algorithms for the two-terminal n-net channel routing problem on the L-layer model when k-fold overlap is allowed, $k \geq 2$. For $k \leq \lceil L/2 \rceil - 2$ we show how to solve the channel routing problem using $d/k + 1$ tracks, which is only one track more than the optimal channel width. For $\lceil L/2 \rceil - 1 \leq k \leq \lceil L/2 \rceil + 1$ the width used by our algorithms differs from the optimal one by a small constant factor. We also present algorithms for the 3- and 4-layer model with double overlap that use fewer tracks than the general channel routing algorithm. All algorithms use $O(n)$ contact points and run in time $O(n)$.

Key Words

Channel routing, two-terminal nets, overlap, channel width, contact points.

1. Introduction

One of the most common channel routing problems is the two-terminal channel routing problem (CRP), where the two terminals to be connected lie on opposite sides of the channel. Solutions to this problem that minimize the channel width and the number of contact points have been developed for 2- and 3-layer wiring models ([BB], [D], [H], [PL], [RBM], [RF]). Routing in 3- or more layer models in which wires are allowed to overlap appears to be feasible and can be viewed as a first step towards three-dimensional channel routing. It is thus important to study the relationship between the number of layers, the amount of overlap, the channel width, and the number of contact points. In this paper we develop efficient algorithms for the L-layer model with k-fold overlap, $k \geq 2$.

In our model each of the L layers can be used to run horizontal and vertical wires. If k-fold overlap is allowed, up to k wires can run in the same column or on the same track of the channel as long as they are on different layers. For $k \leq \lceil L/2 \rceil - 2$ our algorithm uses $d/k + 1$ tracks, which is only one track more than the optimal channel width. (The density d is the maximum over all x of the number of pairs (p,q) for which $p<x<q$ or $p>x>q$, and p must to be connected with q.) For $k = \lceil L/2 \rceil - 1$, $\lceil L/2 \rceil$, and $\lceil L/2 \rceil + 1$ the width differs from the optimal one by a small constant factor. We also present algorithms for the 3- and 4-layer model with double overlap that use $3d/4 + 3$ and $2d/3 + 3$ tracks, respectively. All algorithms presented in this paper use $O(n)$ contact points and run in time $O(n)$, where n is the number of terminals to be connected.

A channel of width w consists of the grid points (i,j), $0 \leq i \leq w+1$, $-\infty < j < \infty$, where i is the track number and j is the column number, and the edges connecting adjacent grid points. Grid points on track 0 and $w+1$ are called *terminals*. A wire is a path connecting adjacent grid points, and a wire can switch from one layer to the another by using a grid point as a *contact point*. If a wire switches from layer l to layer l', $l<l'$, the *contact point condition* must be satisfied: layers $l+1$, ..., $l'-1$ cannot be used at that grid point. The two-terminal CRP consists of n nets (p_i, q_i), where p_i is a column number on track 0 and q_i is a column number on track $w+1$, $1 \le i \le n$, and no two nets share a common terminal.

A number of papers have considered the 2-layer model in which wires on different layers can cross or share a corner (i.e., form a knock-knee). In this model a CRP can be wired on $2d-1$ tracks using $O(dn)$ contact points ([BB], [RBM]). The channel width of $2d-1$ is optimal for some CRPs [L]. When the same wiring rules are used in the 3-layer model, a CRP can be wired on d tracks using $O(n)$ contact points ([PL]). In order to achieve a channel width less than d horizontal overlap has to be used. Horizontal overlap is not as powerful as it may first appear. In [H] we have shown that a large class of CRP's requires $d/(L-1)$ tracks on an L-layer model with L-fold overlap.

We consider models with $L \ge 3$ and $k \le L-1$. In section 2 we describe our general channel routing algorithm. An increase in k results in a decrease in the channel width as long as $k \le \lceil L/2 \rceil + 1$. We discuss why allowing the algorithm to use more than $(\lceil L/2 \rceil + 1)$-fold overlap does not decrease the width any further. In section 3 and 4 we present improved algorithms for the 3- and 4-layer model with double overlap, respectively.

2. The L-Layer Model with k-fold Overlap

In this section we present our general channel routing algorithm for the L-layer model with k-fold overlap, $L \geq 3$. We first give an algorithm for $k \leq \lceil L/2 \rceil - 2$ that is within one track of the optimal channel width. We then describe the modifications necessary for $\lceil L/2 \rceil - 1 \leq k \leq \lceil L/2 \rceil + 1$. Our algorithms determine the wiring by scanning the channel from left to right, column by column. Since each column containing a terminal of a net is processed in constant time, and no work is done in columns containing no terminal, our algorithms run in $O(n)$ time.

A net (p_i, q_i) is called a *right* net if $p_i < q_i$, a *left* net if $p_i > q_i$, and a *trivial* net if $p_i = q_i$. A *right run* is a maximum sequence of nets (p_1', q_1'), (p_2', q_2'), ..., (p_{i_k}', q_{i_k}') such that $p_j' < q_j'$ and $q_j' = = p_{j+1}'$; a *left run* is defined analogous. Right and left nets are grouped together in the channel similar to the approach used in [RBM], the topmost tracks of the channel contain the left nets, and the bottommost tracks contain the right nets. In our algorithms a constant number of empty tracks are between the two groups of nets, and no empty tracks are within a group. All tracks in the channel, except the topmost one in the group of the right nets, the bottommost one in the group of left nets, and the empty tracks in between, contain exactly k wires. The length of the horizontal overlap is determined by the coordinates of the terminals of the nets, and we use at most double vertical overlap of length $O(d)$.

A track in the channel is of type 1, type 2, or empty (i.e., it contains no horizontal wires). Let T_1 be the set containing the first $\lceil L/2 \rceil - 1$ layers, i.e., $T_1 = \{1, 2, ..., \lceil L/2 \rceil - 1\}$, and let T_2 be the set containing the last $\lfloor L/2 \rfloor$ layers, i.e., $T_2 = \{\lceil L/2 \rceil + 1, ..., L\}$. A *type 1* track contains wires of layers in T_1, and a *type 2* track contains wires of layers in T_2. Within the group of right a left nets we alternate tracks of type 1 and type 2. Unless otherwise stated layer $\lceil L/2 \rceil$, which does not appear in T_1 or T_2, is used to run the vertical connections. We refer to a wire on layer l as an l-wire.

In our algorithms we show how to continue a right run, i.e., how to wire when a right net ends and a new right net starts in the same column, and we describe how to wire when a right and a left net start in the same column. The wiring of the other situations follows from these two cases. When continuing a right run we switch the ending net into a v-wire and put the new net onto a track without violating the contact point and overlap condition. For $k \le \lceil L/2 \rceil - 2$ this can be done quite easily. Since $|T_i| \le k+1$, there is always one layer in T_i that is currently not used on a track, $i=1, 2$. As k increases, the wiring gets more difficult and the wiring rules get more involved.

(a) *Algorithm for* $k \le \lceil L/2 \rceil - 2$

When $k \le \lceil L/2 \rceil - 2$, a type 1 (2) track contains k wires that are on layers in T_1 (T_2). One spare track, track e, is between the two groups of nets. Assume that a right run continues, and that the right net ending in column j is on track i and in layer l, and that track i is of type 1. Then, $1 \le l \le \lceil L/2 \rceil - 1$ and there is another layer l' in T_1 not currently used in track i. The new right net continues on track i in layer l', and the wiring is done as follows. Since tracks $i-1$ and $i+1$ are of type 2, use the contact point $(i-1, j)$ to change the v-wire of the new net into an l'-wire, and use $(i+1, j)$ to change the l-wire of the ending net into a v-wire. See Fig. 2.1. The continuation of a left run is analogous. Whenever a right (left) net on track i ends, and and no new right (left) net starts up, we continue another right (left) net of the topmost (bottommost) track on track i.

The empty track e is used in 2 situations. When the topmost track of the right nets and the bottommost track of the left nets are of the same type and a run on either track continues, we use track e to accommodate the contact point for the net starting up. Track e is also needed in the wiring of a double startup which is described below.

A *double startup* occurs when a right and a left net start in column j. We then produce a vertical double overlap of length $O(d)$. Assume the left net is to be put on track t_l and the right net on track t_r. Let r_1 and r_2, $r_1<r_2$, be two layers not used on track t_r, and l_1 and l_2, $l_1<l_2$, be two layers not used on track t_l. If t_r and t_l are different type tracks, we use the contact points (t_r,j) and (t_l,j) to change the v-wires of the starting nets into r_1- and l_1-wires, respectively. See Fig. 2.2 (i).

Thus, w.l.o.g. let t_r and t_l be tracks of type 1. Assume that r_1, r_2 and l_1, l_2 are pairwise disjoint (if they are not, the wiring is actually easier) and that $l_1<r_2$ (if not, use the mirror-image wiring). The v-wire of the left net starting in column j uses the contact point (t_r+1,j) to change into an r_1-wire, which crosses track t_r, and uses the contact point (e,j) to change into an l_1-wire. The right net starting in column j uses a contact point (e,j) to change from a v-wire into an r_2-wire. See Fig. 2.2 (ii). Since $v>r_2>l_1$ and $r_2>r_1$, the contact point condition is satisfied. The double contact point at (e,j) can be avoided for the cost of a second empty track. This concludes the algorithm when $k \leq \lceil L/2 \rceil - 2$.

(b) *Algorithm for* $k = \lceil L/2 \rceil - 1$

In this algorithm we distinguish whether or not L, the number of layers available, is even or odd. If L is even, i.e., $L=2m$ and $k=m-1$, the algorithm is similar to the previous one. Recall that $|T_1| = m-1$ and $|T_2| = m$ in this case. A type 1 track contains now $k-1 = m-2$ wires on layers in T_1, and a type 2 track contains now $k = m-1$ wires on layers in T_2. We again have on each type of track one layer not currently used, and can thus use the algorithm given above. Since we alternate tracks with k-fold and $(k-1)$-fold overlap, we use a total of $2d/(L-3) + 1$ tracks. The optimal channel width is $2d/(L-2)$.

When L is odd, i.e., $L=2m+1$ and $k=m$, we let a type 1 track contain $k = m$ wires on layers in T_1, and a type 2 track contain $k-1 = m-1$ wires on layers in T_2. Since $|T_1| = |T_2| = m$, we need new wiring rules for the continuation of a run on a type 1 track. The continuation of a run on a type 2 track is done as before. On type 1 tracks we no longer have a spare layer that can be assigned to the new net. We can put a new l-wire on track i not before column $j+1$. Our new wiring rule changes the v-wire of the starting net into an l-wire on track $i-1$, runs this l-wire on track $i-1$ to column $j+1$ creating an m-fold overlap on track $i-1$, and runs it down to track i in column $j+1$. The wiring is shown in Fig. 2.3 (i). The vertical segment put into column $j+1$ causes a conflict in two situations.

The first problem occurs when the net starting in column j is of length one, i.e., it ends in column $j+1$. We then run the net of length one entirely on layer v, and put the horizontal unit segment on track i. The net starting in column $j+1$ can be placed on track i in column $j+1$.

The second conflict occurs when another net on track i and in layer l', $l'<l$, ends in column $j+1$. The v-wire of the net coming down column $j+1$ cannot use the contact point $(i-1,j+1)$ for changing into an l'-wire. But we can switch from layer l to layer v on a type 2 track when layer l' is present (since $l'<l$). Fig. 2.3 (ii) shows the wiring for this case. The l'-wire of the ending net 'slips' onto track $i+1$ in column j, the l'-wire on track $i-2$ slips onto track i in column $j+1$, and the new net starting in column $j+1$ continues on track $i-2$ as an l'-wire.

We also need additional wiring rules for the double startup. Assume that a right and a left net start in column j and that t_r and t_l, the tracks the nets have to be put on, are of type 1. The algorithm may have to place the right net in layer r and the left net in layer l, or both nets in layer l. In this case none of the 'freedom' used in the double startup of algorithm (a) is available. If both nets have to be placed in layer l we wire as shown in Fig. 2.4. (The wiring of the nets when they are put on different layers is easier and is omitted.) The wiring of Fig. 2.4 requires 3 empty tracks between the group of right and left nets, and puts a vertical segment of length 2 into column $j-1$ and $j+1$.

The vertical segment put into column $j-1$ can at most cause a double overlap with a v-wire. This is shown as follows. If column $j-1$ has a right and a left net starting, it contains already a double overlap. But in this case track t_l has the layers l_1 and l, and track t_r has the layers r_1 and l available in column $j-1$. The startup wiring in column $j-1$ uses either layers l and l_1 or layers l and r_1, which leaves different layers for column j. Since we can wire the double startup for different layers in column j without placing a vertical segment into column $j-1$, the above statement is true.

If column $j+1$ has a right and a left net starting, they are the first wires on a type 2 track. Applying the wiring of Fig. 2.2(ii) produces a 3-fold overlap in column $j+1$. The 3-fold overlap is avoided by wiring column j and $j+1$ as shown in Fig. 2.5, where l' is an arbitrary layer in T_2. No problems arise when column $j+2$ has a right and a left net starting, which have to be put on track t_r and t_j, respectively. In this case column $j+1$ contains then an ending right net and an ending left net and thus no vertical wire segment between track t_l and t_r. The case when the nets starting in column j end in column $j+1$ is slightly different, but can be handled easily. Hence, our algorithm uses $2d/(L-2)$ + 3 tracks, and the optimal channel width is $2d/(L-1)$.

(c) *Algorithm for* k = $\lfloor L/2 \rfloor$ and $\lfloor L/2 \rfloor + 1$

Consider first the algorithm for $k = \lfloor L/2 \rfloor$. For even L let a type 1 (2) track contain $m-1 = k-1$ wires on layers in T_1 (T_2). The continuation of a type 1 track is done as the continuation of a type 2 track for odd L given in algorithm (b). The horizontal unit segment put on the type 2 track above (below) creates an m-fold overlap. The continuation of a type 2 track in done as in algorithm (a). Hence, the number of tracks needed is $2d/(L-2) + 3 = d/(m-1) + 3$.

For odd L a type 1 (2) track contains all $m = k-1$ wires on layers in T_1 (T_2). The continuation of a right run on a type 1 and type 2 track is done as the continuation of a type 1 track in algorithm (b). It produces an $(m+1)$-fold overlap on a track for one horizontal unit. Since the startup procedure needs 3 empty tracks, we need a total of $2d/(L-1) + 3 = d/m + 3$ tracks.

One further reduction in the channel width is possible for $L=2m$ and $k = \lceil L/2 \rceil + 1 = m+1$. Let each type track contain all the layers possible, i.e., a type 1 track contains $m-1$ wires and a type 2 track contains m wires. The algorithm is like algorithm (c) for odd L and uses $2d/L-1 + 3$ tracks.

We briefly discuss the shape of the nets our algorithms produce. All wired nets (p,q), except trivial nets and nets of length one, have exactly two contact points, which are in column p and q, respectively. The horizontal wire segment can contain slips of length 1 or 2. Slips of length 2 are made when the wiring rule shown in Fig. 2.3(ii) is applied.

None of the algorithms allows a track to use layers 'below' and 'above' layer v. This would be necessary for reducing the channel width further when $k \geq \lceil L/2 \rceil + 2$ while still having one layer reserved for vertical connections. For $L=3$ and 4 we can design better algorithms by allowing a track to contain wires in layers below and above layer v. This is possible for small L, since the layers are 'close' enough to layer v for placing the contact points.

3. The 3-Layer Model

The algorithm presented in section 2 uses d tracks on the 3-layer model when double overlap is used, and d tracks can be achieved without the use of overlap [PL]. We show how to wire a CRP on 3 layers using $w = 3d/4 + 3$ tracks and double overlap. The topmost tracks in the channel will again contain the left nets, the bottommost tracks contain the right nets, and the three empty tracks are in the middle.

We refer to the 3 layers as the t- (top-), v- (vertical-), and b- (bottom-) layer, and vertical wires run on the v-layer. We distinguish between two types of tracks. A *double track* is a track containing a t- and a b-wire, and a *single track* is a track containing a t- or a b-wire. Within the group of left (right) nets, the nets are assigned to the tracks according to the *single-single-double schema*: track 1 (w) is a single track, track 2 ($w-1$) is a double track, tracks 3, 4 ($w-2$, $w-3$) are single tracks, etc. In general, track $2+3i$ ($w-1-3j$) is a double track, and tracks $2+3i+1$, $2+3i+2$ ($w-2-3j$, $w-3-3j$) are single tracks.

We first outline how to continue a right run and again omit symmetric cases. When a right run currently on track i continues in column j, the wiring depends on whether i is an lower single, upper single, or a double track. Within the group of right nets, track i is a lower single track if track $i+1$ is a double track. Hence, track $i-1$ is an upper single track.

Before giving the wiring rules for each type of track, we describe the routine *cont*. It has three arguments, two track numbers, $i1$ and $i2$, $i1<i2$, and one column number, j. Cont($i1$, $i2$, j) takes the wire currently on track $i2$ down column j, and continues the v-wire coming down column j on track $i1$ or $i2$, depending on the type of wire currently on track $i1$. See Fig. 3.1 for the two possible cases.

If the net ending in column j is on a *lower single track*, use the contact point (i,j) to let the wire on track i switch into a v-wire. Since track $i-1$ is a single track, call cont$(i-1, i, j)$, and the new net is continued on track $i-1$ or i.

Next consider the situation where the net ending in column j is the t-wire of a *double track* (the case for the b-wire is analogous). Fig. 3.2 shows the general solution. The vertical segment in column $j+1$ can cause a problem when a net on track i, $i-1$, or $i-2$ ends in column $j+1$. We thus have to consider 4 special cases.

case 1: *The net starting in column j ends in column j+1.*

Keep the net of length one on layer v and run the horizontal unit segment on track $i-1$. If the situation shown in Fig. 3.2(i) occurred, call cont$(i-2, i-1, j+1)$ to place the net starting in column $j+1$; if the situation shown in Fig. 3.2(ii) occurred, call cont$(i-2, i, j+1)$.

case 2: *The wire that is on track i-1 before column j ends in column j+1.*

Whether it is a t-wire (and slips onto track i in the general solution) or a b-wire (and stays on track $i-1$), the wiring rules given so far cannot be applied in column $j+1$.

Fig. 3.3(i) shows the solution when a t-wire is on track $i-1$ before column j; Fig. 3.3(ii) the one for the b-wire. In both cases the net starting in column j runs on the track determined by cont$(i-2, i-1, j)$. No vertical wire segments are placed into column $j+2$, which is processed next.

case 3: *The b-wire on track i ends in column j+1.*

If track $i+1$ contains a t-wire, or tracks $i+1$ and $i+2$ contain both a b-wire, perform the wiring shown in Fig. 3.4(i) and (ii), respectively. In both situations the net ending in column $j+1$ slips onto track $i+1$ in column j. The new nets coming down column j and $j+1$ can then be wired easily.

The hard case occurs when track $i+1$ contains a t-wire and $i+2$ a b-wire. We then look at the wiring in column $j-1$: Column $j-1$ can be empty, contain a v-wire with no contact point on track i, or can contain a contact point on track $i+1$, which was established by cont$(i+1, i, j-1)$. (Note that there can be no contact point on track i.) In the first two cases we let the t-wire on track i slip onto $i+1$ in column $j-1$ and complete the wiring as shown in Fig. 3.5(i). The new nets are wired according to cont$(i-1, i, j)$ and cont$(i-1, i, j+1)$.

In the last case we undo the wiring in column $j-1$, and wire the nets that start and end in columns $j-1$, j, and $j+1$ together - without putting any vertical segments into column $j+2$. Fig. 3.5(ii) shows the wiring when a continuation as in Fig. 3.1 (i) had occurred in column $j-1$; Fig. 3.5(iii) when a continuation as in Fig. 3.1(ii) had occurred.

case 4: The wire on track i-2 ends in column j+1.

Track $i-2$ is an upper single track, and the wiring rules for a net on an upper single track are given below. These rules cannot be applied when track $i-1$ and $i-2$ contain a different type wire between column $j+1$ and $j+2$. The reader might find it useful to look at the solution for the upper single track in order to see why this case is needed. Fig. 3.6 gives the wiring for the 2 possible cases.

The last type of track to be discussed is the *upper single track*. If track i is an upper single track and the wires on track i and $i+1$ are on different layers, we perform the wiring shown in Fig. 3.7(i). If the wires on track i and $i+1$ are on the same layer, we wire as shown in Fig. 3.7(ii). The problems caused by the vertical segment in column $j+1$ are, with the exception of case 3, identical to the ones for an ending double track. When the b-wire on track $i-1$ ends in column $j+1$, we wire as shown in Fig. 3.7(iii). Because of case 4, it is now guaranteed that tracks i and $i+1$ contain wires on different layers. This eliminates all complicated conflicts that could be caused by the vertical segment now in column $j+2$.

In order to maintain the single-single-double structure within the group of right (left) nets, we again take down (up) the right (left) net on the topmost (bottommost) track of the group whenever a right (left) net ends and no new one starts in the same column. In the wiring of a double startup (i.e., a right and a left net start in the same column) we put, whenever possible, the two starting nets on different layers. In this case the wiring is similar to the one described in section 2. If both starting nets are the second wires on a double track and need to be placed on the same layer, we wire as shown in Fig. 2.4. Again, the vertical segment put into column $j-1$ does not create a problem. The vertical segment put into column $j+1$ causes now a problem when a right and a left net start in column $j+1$. We can put them on different layers, but the wiring is done as shown in Fig. 3.8.

We thus can solve a CRP in the 3-layer model with double overlap on $3/4d + 3$ tracks using $3n$ contact points. The lower bound on the channel width in the 3-layer model when 3-fold or double overlap are allowed is $d/2$.

4. The 4-Layer Model

The algorithm presented in section 2 requires $d + 3$ tracks on 4 layers when double overlap is used and $2d/3 + 3$ tracks when triple overlap is used. Recall that triple overlap occurs on the tracks for only one horizontal unit. In this section we describe an algorithm that achieves $2d/3 + 3$ tracks by using only double overlap. We refer to the 4 layers as the t- (top-), v- (vertical-), m- (middle), and b- (bottom-) layer. Let a *double track* contain a wire on the t- and m-layer, and let a *single track* contain a wire on the t- or b-layer. Within each group of right and left nets we alternate double and single tracks.

The continuation of a right run, when the net ending on track i and column j is on a *single track*, is done as follows. Make a contact point at (i,j) to let the ending net switch from the t- or b-layer into the v-layer. If the ending net is on the t-layer, let i' be the track containing a b-wire so that $i' < i$ and no b-wire is between track i' and track i. Take the b-wire from track i' down to track i, and continue the new net on track i' as a t-wire. See Fig. 4.1(i). If the ending net is on the b-layer, take the t-wire from track $i-1$ down to track i, and continue the new net on track $i-1$ as a t-wire. The new net is thus treated like a new net in the continuation of a double track, which is discussed below. Fig. 4.1(ii) shows the wiring when track $i-2$ contains a b-wire.

Consider now the continuation of a run currently on a *double track*. If a t-wire on a double track ends, make a contact point at (i,j) for the ending net. Take either the t-wire from track $i-1$ or the t-wire belonging to the new net down to track i in column $j+1$. Fig. 4.1(i) and (ii) show the 2 possible cases. If an m-wire on a double track ends, we make a contact point at $(i+1,j)$ for the ending net. The v-wire coming down column j changes into an m-wire at $(i-1,j)$. This switch can be done independent of the type of wire on track $i-1$. See Fig. 4.2(iii).

The wiring rules given in Fig. 4.2 can produce conflicts similar to the ones in the 3-layer algorithm when a net on track $i-1$ or i ends in column j. We consider 3 cases.

case 1: *The net starting in column j ends in column j+1.*

> We then keep the net of length 1 entirely on the v-layer. If the situation shown in Fig. 4.2(i) and (ii) had occurred, the horizontal unit segment is put on track $i-1$, and we complete the wiring as shown in Fig. 4.3. In Fig. 4.3(ii) we treat the net starting in column $j+1$ like one continuing on a double track at a t-wire.
>
> If the wire ending in column j was on the m-layer, the horizontal unit segment in put on track i. A new m-wire is put on track i in column $j+1$ by letting the net starting in column $j+1$ change into an m-wire at $(i-1, j+1)$.

case 2: *The wire that is on track i-1 before column j ends in column j+1.*

> When the wiring was done as in Fig. 4.2(iii) and track $i-1$ contains a t-wire, we can apply rule 4.1(i) in column $j+1$.
>
> For the three remaining cases we do need new wiring rules. Let the net ending in column j end as before, and use the rule given in 4.1 to switch the wire on track $i-1$ into a v-wire and to wire the net starting in column j. The v-wire runs on track i or $i-1$ to column $j+1$ where it ends. Column $j+1$ looks now like column $j+1$ of case 1; i.e., the net ending in column $j+1$ appears to be a net of length one continuing the run on the double track. We thus wire column $j+1$ as described above. Fig 4.4 shows the resulting wiring when an m-wire ends in column j and the b-wire above is ends in column $j+1$.

case 3: *The second net currently on double track i ends in column j+1.*

> If the second net is a t-wire (i.e., the m-wire did end in column j) and track $i-1$ contains a t-wire, we cannot apply the wiring rule of Fig. 4.2(i) in column $j+1$ without violating the contact point condition. We then wire as shown in Fig. 4.5.

The wiring of a double startup is similar the one described in section 3. We thus use $2d/3 + 3$ tracks when routing in the 4-layer model with double overlap.

5. Remarks

We have developed channel routing algorithms for the L-layer model when overlap can be used. For $k \leq \lceil L/2 \rceil$ our algorithms use fewer tracks when allowing $(k+1)$-fold overlap instead of k-fold overlap. More than $(\lceil L/2 \rceil + 1)$-fold overlap does not reduce the channel width any further. This motivates a number of interesting questions.

(*i*) Can our algorithms be improved and/or extended?

(*ii*) We have shown that a number of CRP's require $d/(L-1)$ tracks when L-fold overlap is allowed ([11]). The best achievable upper bound is $\frac{2d}{L-1} + 3$. Note that in the 3-layer model we can achieve $3d/4 + 3$ tracks, while $d/2$ is the lower bound. Thus, we can do better than a factor of 2 off from the optimal channel width. Can we prove better lower bounds by incorporating the contact point condition into the lower bound argument?

Acknowledgments

I would like to thank Donna Brown for helpful discussions and comments.

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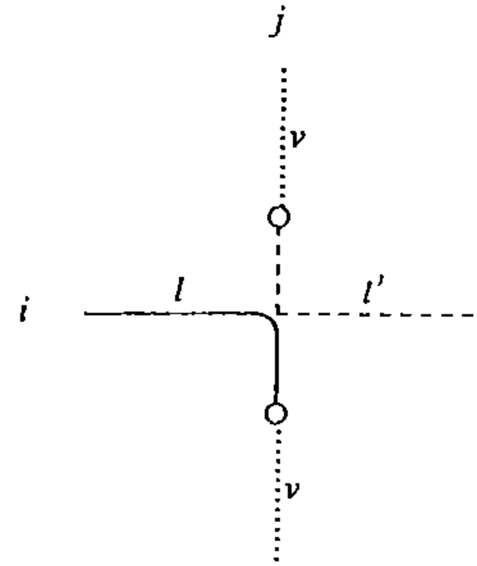
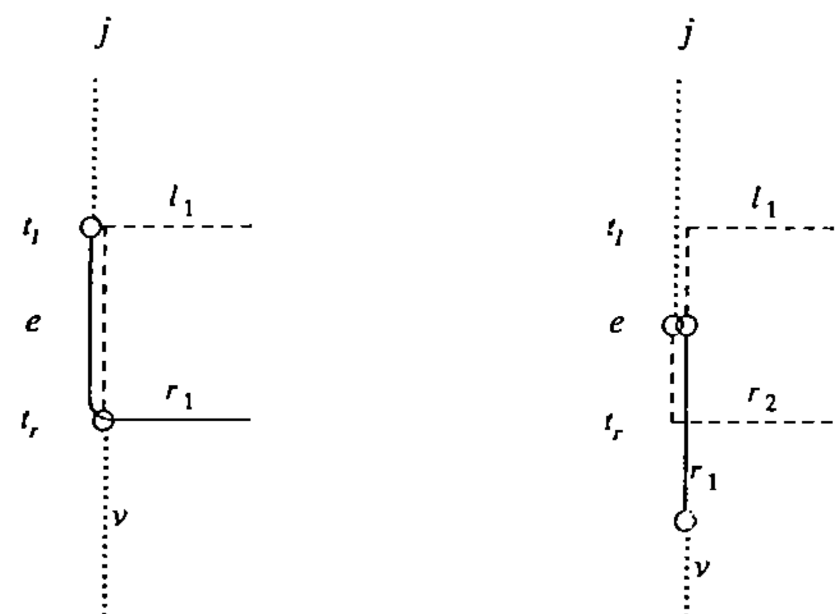


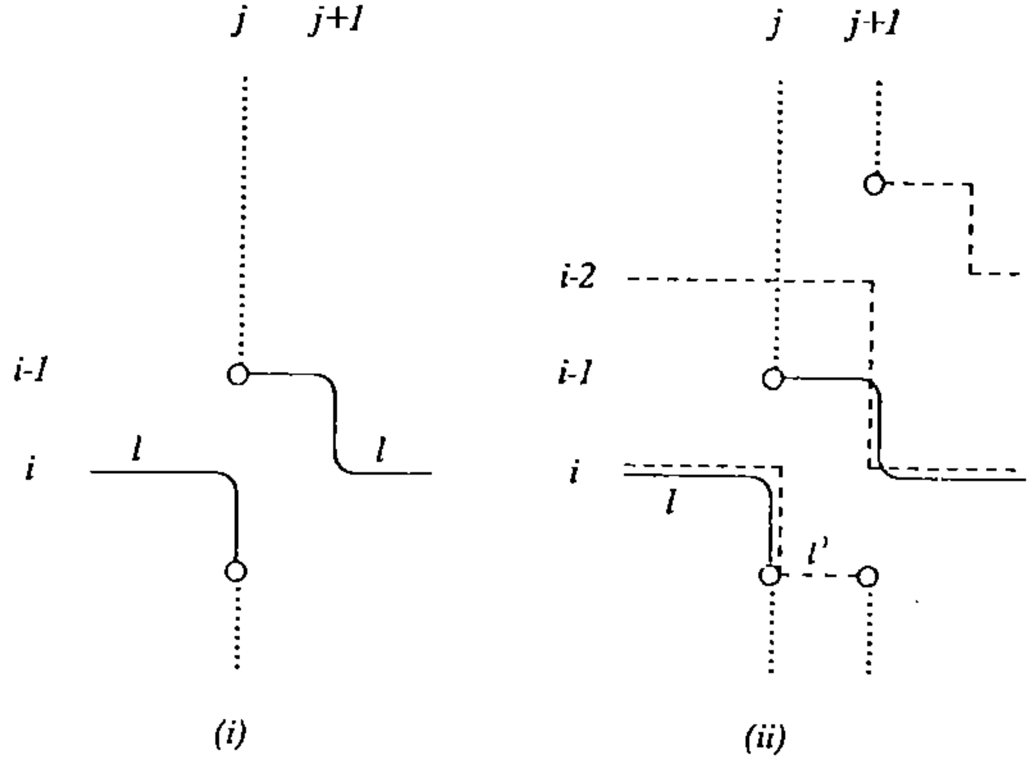
Continuation when $k \leq \lceil L/2 \rceil - 2$
Fig. 2.1



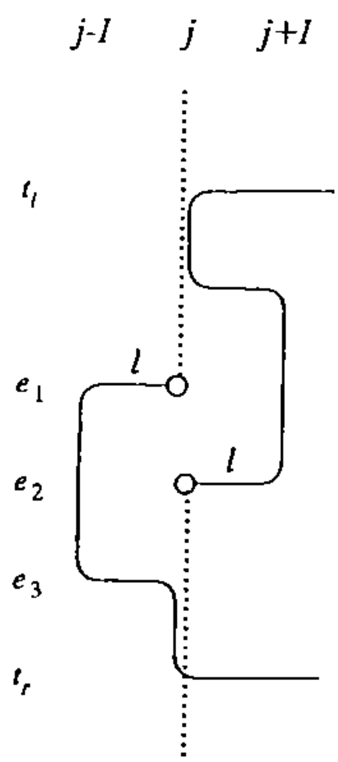
t_l and t_r are of different type
(i)

t_l and t_r are of type 1
(ii)

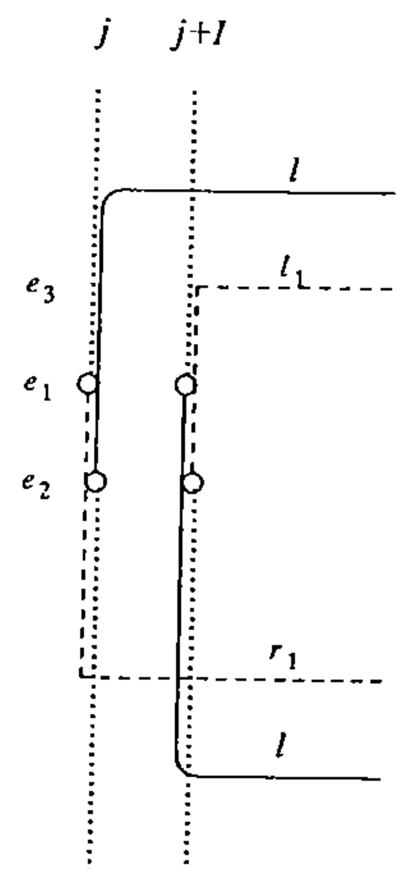
Startup Wiring
Fig. 2.2



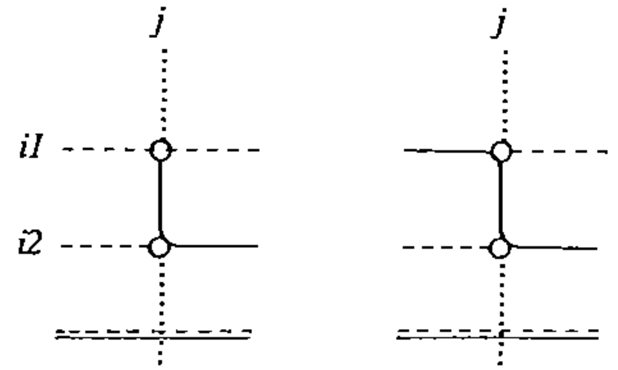
type 1 continues
Fig. 2.3



double startup on layer l
Fig. 2.4



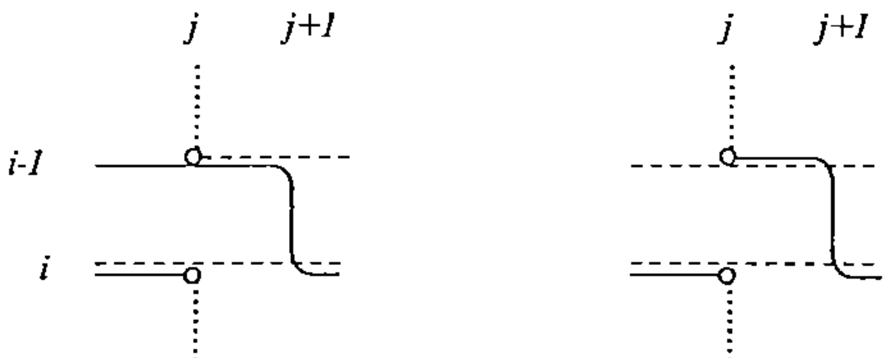
avoiding triple overlap in column $j+1$
Fig. 2.5



(i)
continuation of
the new wire on *i1*

(ii)
continuation of
the new wire on *i2*

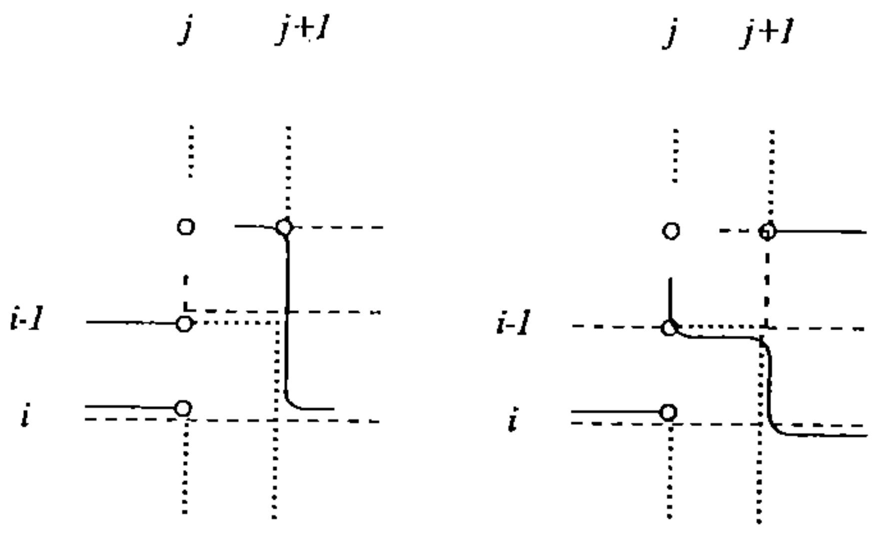
--- *t*-wire ..... *v*-wire - - - *b*-wire
Fig. 3.1



(i)

(ii)

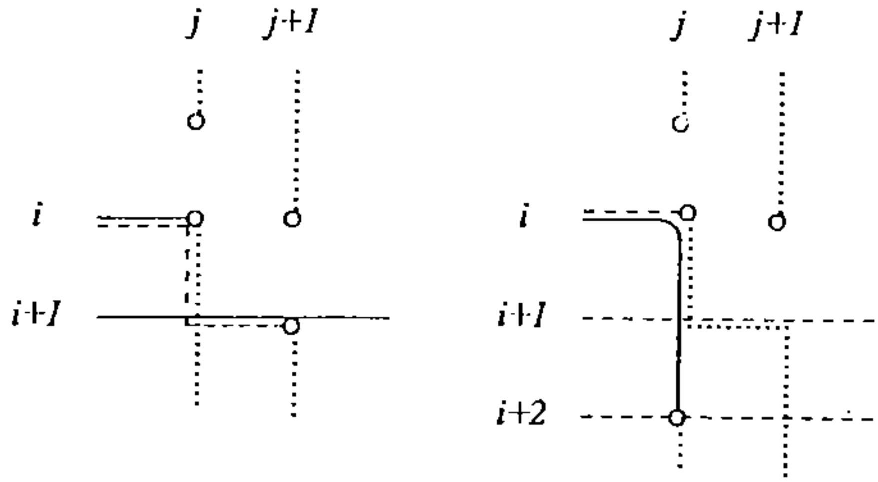
A net on a double track ends: general solution
Fig. 3.2



(i)
t-wire on track *i-1*

(ii)
b-wire on track *i-1*

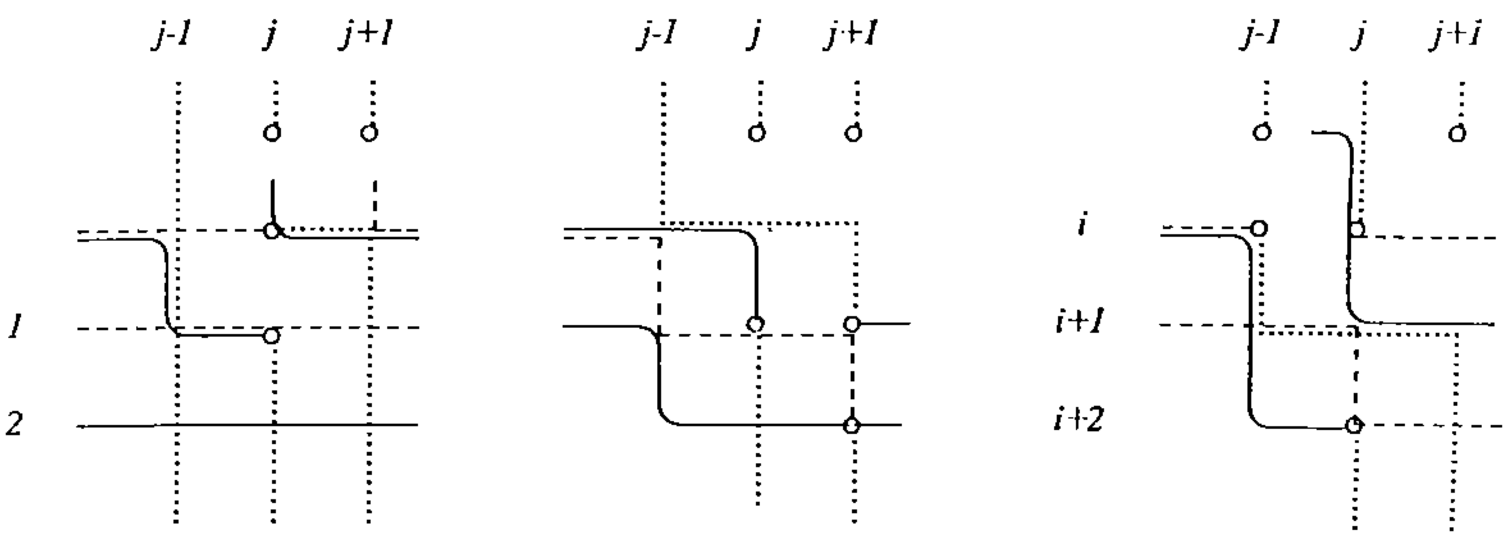
Fig. 3.3



(i)
i+1 contains a *t*-wire

(ii)
i+1 and *i+2* contain a *b*-wire

Fig. 3.4

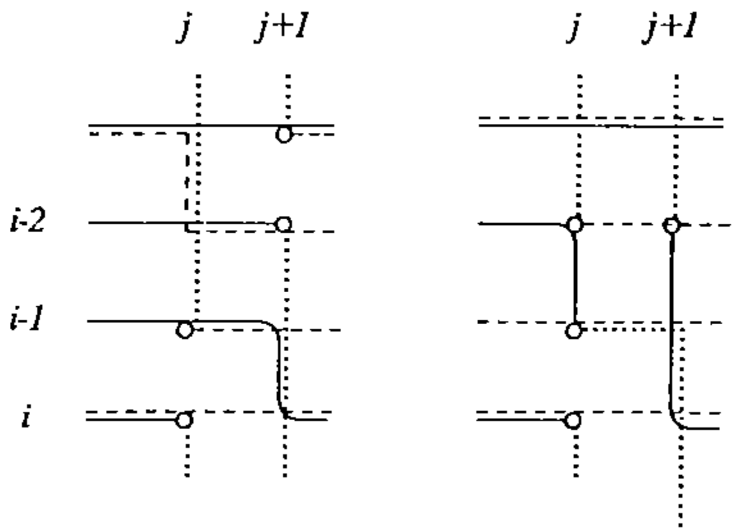


(i)
an *v*-wire in column *j-1*

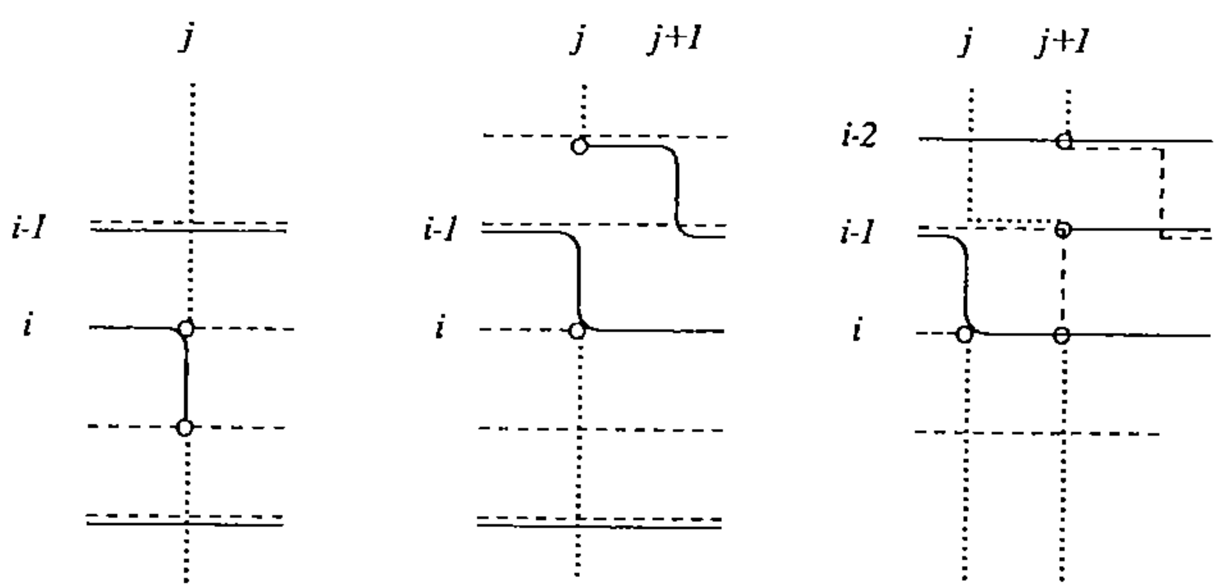
(ii)
continuation 3.3*(i)* in column *j-1*

(iii)
continuation 3.3*(ii)* in column *j-1*

Fig. 3.5

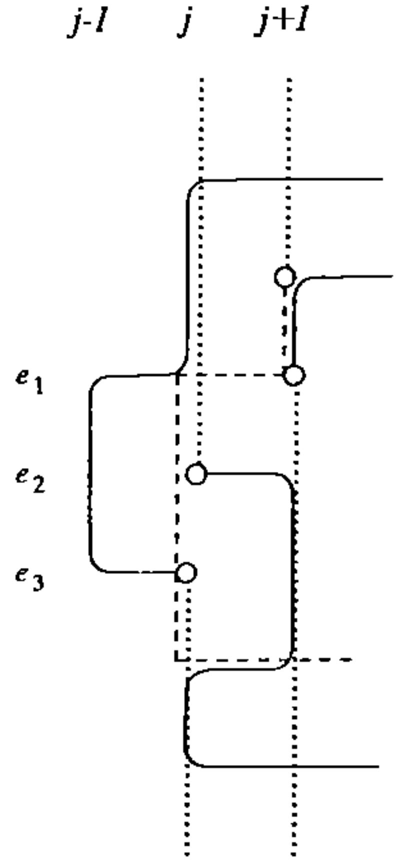


wire on track *i-2* ends in *j+1*
Fig. 3.6

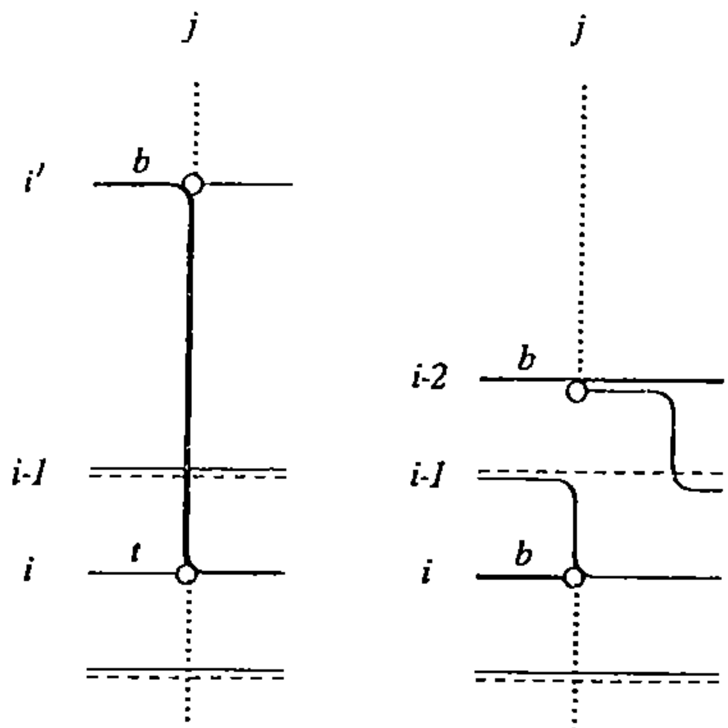


(i) *i+1* is different type
(ii) *i+1* is the same type
(iii) double track ends

Fig. 3.7



double startup in j and $j+1$
Fig. 3.8

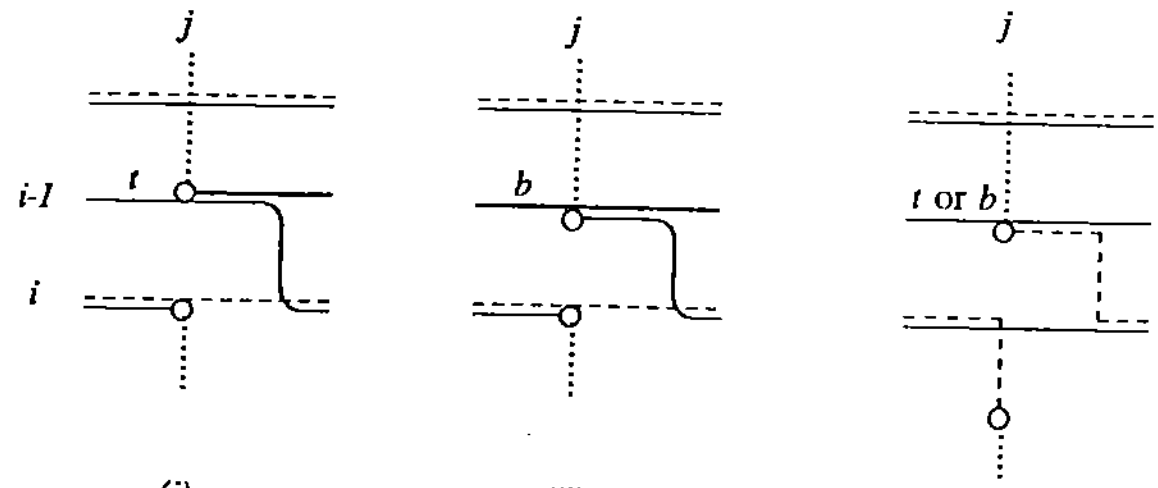


(i) single track *t*-wire ends

(ii) single track *b*-wire ends

— *t*-wire, ... *v*-wire, - - - *m*-wire, — *b*-wire

Fig. 4.1



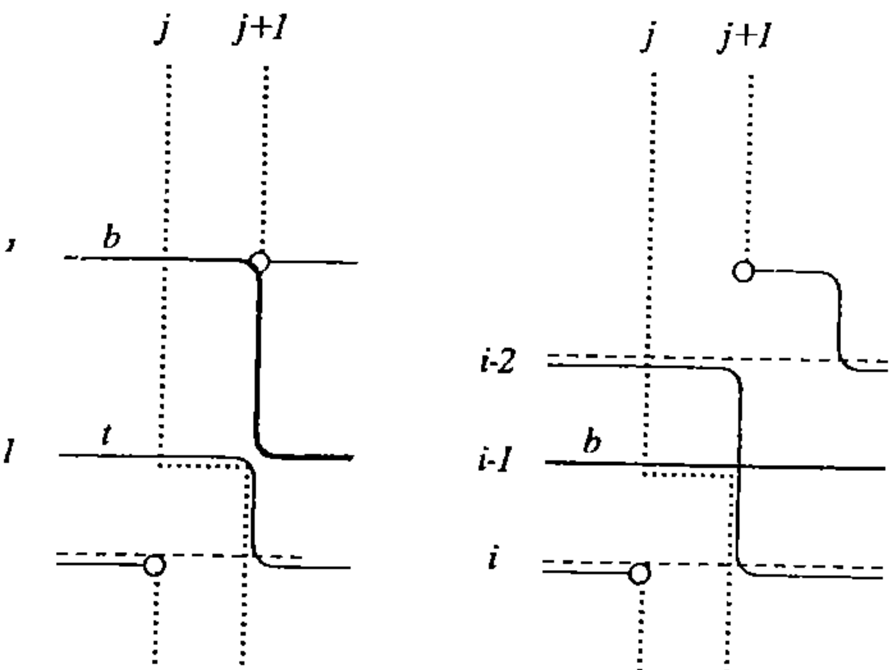
(i) (ii) (iii)

t-wire ends and above is a

t-wire b-wire

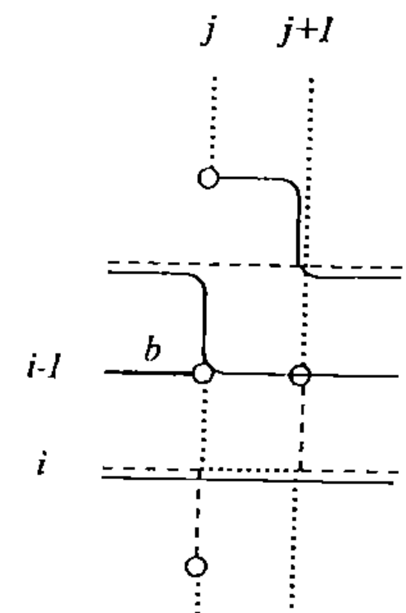
m-wire ends

Fig. 4.2



(i) (ii)

Net of length 1 starting in column j
Fig. 4.3



b-wire on i-1 ends in column $j+1$
Fig. 4.4

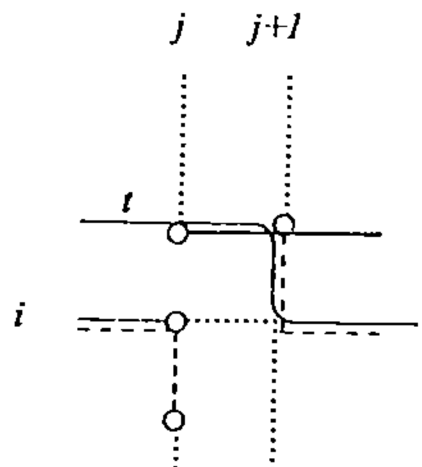


Fig. 4.5