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ANALYSIS OF BENDING STRESSES IN CANTILEVER TYPE SUCTION VALVE REEDS

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ABSTRACT

Design techniques are described which allow prediction of bending stresses in reed valves from theoretical and experimental data. Their application is to aid the development of more durable valves.

A relatively simple analysis is described which permits valve stresses and impact velocities to be estimated from the initial pressure drop across a valve reed. (Conversely, experimental measurement of valve stresses allowed prediction of the initial pressure drop at valve opening and hence impact velocities). The simplifying assumptions in the analysis are enumerated; these simplifications provided a model which was economical of computer resources.

The finite element method was then applied to check the simple model and predict both static and dynamic reed displacements. The resultant finite element model yielded static and dynamic stresses, together with natural frequencies and mode shapes of reeds. It was found that dynamic stress predictions allowed estimates to be made of the valve reed life in relation to the fatigue limit of the valve reed.

Comparison between the results from the simple model and the more detailed finite element models justified the use of the simpler analysis. Hence the shape of a valve reed could be economically studied with a view to reducing bending stresses and impact velocities.

INTRODUCTION

The design and development of modern high speed refrigeration compressors has increased the emphasis on the design of the self-acting suction and discharge valves. Reliability has always been the hallmark of the refrigeration compressor and the valves are recognised as being among the most highly stressed components and as such require advanced engineering techniques in their development. The demand for improved performance from compressors results in an increase in the duty placed on the valves and the need to optimise for both performance and reliability.

In the past there have been a number of attempts to analyse valve behaviour which predict the performance of the valves and relate performance to failure mechanisms. Most of these techniques are expensive of computer time and therefore difficult to use in design optimisation. Also all models predict valve behaviour within the limitations and constraints imposed by the model and the assumptions used during the analysis. Most models allow reasonable predictions to be made of compressor performance but are less reliable in predicting valve life (1).

The present paper describes a semi-empirical approach to the subject where a simple mathematical model is used in conjunction with test results to predict the performance and reliability of cantilever suction valves in a reciprocating compressor. The validity of the model has also been checked by comparing the resultant stress distributions with those obtained by finite element analysis. The finite element method techniques have also produced interesting confirmation of the dynamic behaviour of the valve as observed experimentally.

ENERGY TRANSFER MODEL

The Model and Assumptions

The simple model is based on the assumption that the work done by the gas in opening the valve is absorbed by the valve either as strain or kinetic energy which allows a full definition of the behaviour of the valve to be made from the resultant system equation. In general,

\[ \int_{1}^{2} F dx = \Delta KE_{2} + \Delta SE_{2} \]  

Assumptions have been made that allow the solution of equation 1 without recourse to a full compressor simulation model or full analysis of the dynamic equations of the valve reed. These assumptions are based on simplified modelling of the behaviour of the valve and can be stated as follows:

a) The analysis is concerned with the first cycle performance of the valve and therefore conditions in the cylinder and across the valve
can be considered to be steady. For this reason the pressure drop at valve opening applies during the whole of the first cycle and the force acting on the valve during its opening cycle is constant. This assumption appears to be valid when pressure-volume diagrams from the compressor are studied.

b) The reed is perfectly clamped and behaves as a cantilever which is simply supported by the reed stops when the tip displacement tends to be greater than the permitted lift.

c) The kinetic energy of the reed can be calculated by assuming that the velocity at any point in the reed is proportional to its displacement and can be calculated from,

\[ KE = \int \frac{1}{2} m \dot{r}^2 \, v \, dt = K \dot{r}^2 \]

2.

d) The gas load on the valve is taken as point loads acting on the centre line of each valve port and the strain energy calculated from the resultant deflection curves for the valve.

Outline of the Solution

Fig. 1 diagrammatically represents the energy relationship for the valve model described above and several interesting points can be made.

The total work done by the gas in taking the valve to its maximum displacement point is given by the area oabeo and this energy transfer level must be equal to the total strain energy in the valve at maximum displacement as given by the area ocbdefo.

The velocity of the valve tip as it approaches the stop can be calculated using equation 2, noting that the kinetic energy of the valve at this point is given by the area oagco.

Fig. 2 shows the deflection shape adopted by the valve during its first cycle of stress. The full dynamic deflection shape is calculated using the techniques already described while the static deflection shape is that taken up by the reed when acted upon by the gas load only. It is assumed that the magnitude of the first cycle of stress of the reed is controlled by an oscillation of the reed about the static deflection shape, the amplitude of the oscillation being determined by the full dynamic deflection and being equally spaced around the static deflection shape. This simplifying assumption allows the stress distribution and cyclic stress variation in the reed to be estimated and used to determine the best plan shape of the reed in relation to valve ports.

Pressure Force Relationships

In order to determine the first cycle stress behaviour of the valve using the methods of the
energy transfer model, an estimate of the pressure force required to open the valve must be made. By consideration of the mechanisms that control the pressure drop required to open the valve, it is reasonable to assume that the force required to open the suction valve is proportional to the inlet pressure and governed by the valve seat geometry. Fig. 3 shows a plot of suction valve pressure drop during opening for a given design of compressor obtained from pressure-volume diagrams and demonstrates the dependence of the valve opening force on suction pressure. For a given family of valves and valve seat geometries the valve opening pressure drop is reasonably invariable and can be used in the first stage of the valve optimisation design programme.

Valve Design and Verification

Fig. 4 shows the resultant optimum shape and stress distribution for a given valve design obtained using the techniques described earlier in the paper and experimentally determined values of valve pressure drop. This analysis shows the valve shape to be optimised in relation to the disposition of suction and discharge ports within constraints of fatigue life and reliability. The analysis also allows the investigation of variables such as valve reed thickness and lift to be based on an initial set of test results and optimum values of these variables selected without recourse to a large test programme.

While analytical models offer guidance and help understanding during the design task, experimentally determined results are required in order to assess the validity of the theory, bearing in mind the assumptions made.
Fig. 5 shows the dynamic stress pattern measured by strain gauges located at the root of the reed. These results compare well with the first cycle stress variation predicted by the model. Fig. 6 shows a comparison of measured root stress for a given reed compared to the stress predicted by the simple model and again the agreement is fairly good.

In optimising the plan shape of the suction reed, one objective is to maximise the plate area available for discharge ports. This can be achieved by removing metal from the suction reed in areas of low bending stress always bearing in mind that excess removal of material may reduce the torsional stiffness of the reed and lead to torsioned stability problems. The location of the point of minimum stress and the variation of stress along the length of the reed is obviously important and can be calculated, as shown by Fig. 4, bearing in mind the assumptions made in the model. Fig. 7 shows measured stress values along the length of the reed compared to predicted values and shows that the point of minimum stress was fairly well located by the simple model.

The importance of initial pressure drop required to open the valve has always been recognised because of its effect on the performance of the compressor and on the stresses in the valve. The techniques that have developed from the simple model described here allow examination of this pressure drop and means of changing it to be carried out in more detail. If the pressure drop is reduced then the stresses in the valve will reduce and the pumping efficiency of the compressor increase. By measuring the stresses at the root of the valve, estimates can be made of the initial pressure drop and the effect of valve plate modifications can be studied.

Fig. 8 shows stress levels for two different valve seat configurations which show a 15% change in stress, indicating a similar alteration in initial pressure drop. This is particularly important for large compression ratios. In the case cited, the pumping efficiency of the compressor was improved by some 20% by use of the valve plate with the lower
pressure drop at compression ratio 18:1. The improvement corresponded to that predicted by simple volumetric efficiency calculations using the reduced pressure drop.

Finally, the relationship between the first cycle stress and the kinetic energy of the reed enables estimates to be made of the impact velocity of the reed against its stop. A plot of impact velocities from experimental valves is given in Fig. 9 and compared with values determined by the computer prediction models described in Reference (1). This technique is still being developed and will form the basis for studies of valve reed impact that should yield valuable information in the future, bearing in mind the work reported in (2) and (3) that indicate a need better to understand valve impact phenomena.

![Graph showing predicted tip velocity for suction reed](image)

**Figure 9**

Predicted tip velocity for suction reed

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**The Finite Element Model and Comparisons**

The energy transfer model described earlier in the paper, relies on assumptions that allow the strain and kinetic energy equations to be integrated easily with the resultant simplification of the mathematical equations and computer programmes.

Finite element techniques allow a more exact description of the energy and stress relationships in the reed, the accuracy depending on the size and therefore the number of elements used. For these studies a finite element model (4) was developed to evaluate the stresses in cantilever valves due to bending. Impact stresses were not considered as part of this study.

Figs. 10 and 11 show the grid patterns employed in the finite element model used to study both the static and dynamic behaviour of the reeds.

![Finite element model of a suction reed](image)

**Figure 10**

Number of degrees of freedom 39
Number of elements 18

![Finite element model of a suction reed](image)

**Figure 11**

Number of degrees of freedom 93
Number of elements 44

As already stated, the results of a finite element model analysis are somewhat sensitive to the number of degrees of freedom used. Because of the increase in computer time with degrees of freedom, models having 39 degrees of freedom (Fig. 10) and 93 degrees of freedom (Fig. 11) were used to establish the validity of a model using the smallest number of degrees of freedom possible. 93 degrees of freedom was considered to be the upper limit practical for the dynamic analysis using a Honeywell 6060 computer with 256 K words of central processor memory and a file store of 400 M characters.

**Static Analysis**

Fig. 12 shows predicted static displacements along the centre line of the reed with a force of 100 N (equivalent to a pressure drop of 2.5 bar across the reed) acting in the plane of the centre line of the ports. The variation in the results using the cruder grid (with 39 degrees of freedom) and the finer grid (with 93 degrees of freedom) are illustrated by:

a) the difference between 11.9 N and 8.2 N for the force necessary for the reed just to reach the stop,
b) the difference (≈ 27%) between the maximum displacements of the reed at the centre of the port.

The cruder grid did not account adequately for the stiffness of the reed (5). The static displacement at the centre of the port predicted using the finer grid, was almost equal to the permitted lift of the reed tips.

Fig. 13 shows the predicted static stress distribution along the reed. The concept of equivalent stress ($\tau_{eq}$) was used since the stress field predicted by the finite element method is two dimensional ($\tau_x$ and $\tau_y$). The criterion of equivalent displacements was employed. Computation of equivalent stresses is necessary in order to permit comparison with stress limits quoted for the reed steel. The stress distribution predicted using either grid size was almost the same, but there was a significant saving on computer resources when the relatively crude grid was employed (5). Both models predicted that the maximum static stresses occurred at the root of the reed, with another region of high stress at the centre of the port. This had already been observed using the simple energy transfer model. Transverse stresses ($\tau_y$) were only significant in the region of the valve port, as shown in Fig. 13.

Dynamic Analysis

a) Free vibrations

The analysis of reed performance is enhanced by the use of finite element method techniques because of their ability to predict the natural frequency of clamped-free and clamped-pinned cantilever reeds. These studies are important in order to avoid resonance with dominant frequencies of the pressure oscillations in the plenum chambers of the compressor (6). A knowledge of the mode shapes of the free vibration frequencies assists in locating areas of high and low stress bearing in mind the simplifying assumptions made when formulating the energy transfer model of this paper. The natural frequencies of the reed can also help to describe components in compressor noise patterns due to reed vibrations (7). The natural frequencies and mode shapes of the reed when clamped-free, as predicted by the finite element model with 93 degrees of freedom, are shown in Fig. 14a. There were only small differences (7% to 15%) between the values of the first two bending natural frequencies (146 and 1076 Hz) predicted by the finite element method and those calculated by one dimensional bending theory. After the reed tips touched the point stops, the predicted first three natural frequencies were increased from 146, 451 and 1076 Hz to 603, 914 and 2051 Hz (Fig. 14b).

The close correlation between the natural frequencies predicted by the finite element method techniques and simple bending theory, support the assumptions made about the first cycle deflection shape and rebound of the reed when discussing the simple model.
b) Forced vibrations

The finite element methods allow the behaviour of the reed under conditions of forced vibration to be studied.

The dynamic displacements of the reed at the centre of the port were predicted using both grid sizes (Fig. 15). The motion of the reed during the whole suction part of the compressor cycle was not completed due to the high demands on computer resources. A small internal damping ratio for the reed material of 0.005 was assumed (5). The dynamic displacement of the reed reached a maximum value soon after touching the point stops. Subsequently the reed, while still in touch with the stops, vibrated mainly at its fundamental frequency of about 600 Hz. This may be expected since the pressure difference across the reed while open did not change rapidly. The maximum dynamic displacements of the reed at the centre of the ports were about 50% greater than the permitted lift of the reed at the tips.

The distribution of maximum dynamic stress along the centre line of the reed when using each grid is illustrated in Fig. 16. The relatively cruder grid yielded values of maximum dynamic displacement at the centre of the port about 15% lower and of dynamic stresses about 27% lower than those predicted using the finer grid. Maximum dynamic stresses were about 30% greater at the reed root and 40 to 50% greater at the port centre than the stresses calculated under equivalent static conditions (with the same maximum pressure...
difference across the reed at the port. The model, using either grid size, predicted that the maximum equivalent dynamic stress occurred at the root of the reed with another area of high stress at the centre of the port.

\[
\begin{align*}
\sigma_1 & = 762 \\
\sigma_2 & = 129 \\
\sigma_{eq} & = 723
\end{align*}
\]

\[
\begin{align*}
\sigma_1 & = -1100 \\
\sigma_2 & = 100 \\
\sigma_{eq} & = -1130
\end{align*}
\]

\[
\begin{align*}
\sigma_1 & = -1280 \\
\sigma_2 & = 65 \\
\sigma_{eq} & = -1296
\end{align*}
\]

**FIG. 16** MAXIMUM DYNAMIC STRESS DISTRIBUTION

Comparison With Simple Model

The dynamic analysis was carried out using a reed opening force of 100 N and the resultant analysis for this opening force compared with the simple model using the same opening force. The stress distribution for the simple model at these conditions is shown in Fig. 16. The resulting stress distributions compare well, with the simple model predicting root stresses within 10% of those of the finer grid finite element values. The shape of the stress curve and point of minimum stress also compare well (Fig. 17). It is also interesting to note that the finite element model predicted approach velocities of 12.2 m/s when using the finer grid compared to 12.0 m/s by the simple model. An approach velocity of approximately 15 m/s had been predicted by a computer simulation (8) in which the reed is described as a single degree of freedom spring-mass system.

**FIGURE 17** PREDICTED REED STRESS DISTRIBUTION

**CONCLUSIONS**

A simple energy transfer model has been described which allows the prediction of bending stress distribution in cantilever suction reeds to be calculated and which can be used in conjunction with limited experimental tests to establish optimum reed geometries in relation to the thickness of the reed, valve lift and valve seat geometry. Once the optimum designs have been established by this method they can be checked by the finite element method to examine the natural frequency and mode shapes of the reed to verify the suitability of the design.

Finite element method techniques and experimental measurements of the stress distribution and first cycle stress variation of the valve reeds have established the accuracy of the simple model as a design tool for the geometry of reeds under consideration. The model can also provide valuable information about the approach velocity of the reed tips to the stops and the opening pressure drop across the reed when used with measured stress values.

The dynamic behaviour of this type of suction reed has been demonstrated and it is interesting to note that under full load operating conditions, the maximum dynamic displacement and stresses at the centre of the port were about 50% greater than the equivalent static values with the same pressure difference across the reed at the valve port, thus illustrating the need for a dynamic model of reed behaviour.
LIST OF SYMBOLS

F : force
\( \mathbf{v} \) : velocity
\( \mathbf{e} \) : specific mass
\( \sigma_x \) : longitudinal stresses
\( \sigma_y \) : transverse stresses
\( t \) : length of a reed
\( \sigma_{eq} \) : equivalent stresses
w : width
t : thickness
\( x, y \) : direction co-ordinates

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