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# Minimizing Contact Points and Using Overlap on Two Layers

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## Abstract

We study solutions for 2-layer models that minimize the number of contact points and the effect of overlap on the channel width. All known algorithms for the 2-layer model without overlap use $\Theta(dn)$ contact points. We present an algorithm for a restricted class of channel routing problems that uses $2d-1$ tracks and $O(n)$ contact points. While channel routing problems of this restricted class were successfully used to prove lower bounds on the channel width, any proof that $2d-1$ tracks and $O(n)$ contact points cannot be achieved simultaneously in general in this model must make use of some special properties not present in the restricted channel routing problem. We supply insight into those properties and into some of the difficulties that must be overcome by an algorithm that uses $O(n)$ contact points. For the 2-layer model with overlap we present an algorithm that solves any channel routing problem on a channel of $2d-1$ tracks using at most $3n$ contact points and vertical overlap of length 1. We also present a lower bound on the channel width for the $k$-layer model with $k$-fold overlap, $k \geq 2$.

## Key Words

Channel routing, two-terminal nets, density, contact points, overlap, wire layout, wiring.

## 1. Introduction

The *Channel Routing Problem* (CRP) is a wiring problem in integrated circuit design that has received much attention recently ([BB], [D], [DKSSU], [PL], [RBM]). An important and extensively studied measure in channel routing is the *channel width*, but, besides minimizing the channel width, an efficient algorithm should also minimize the number of *contact points*. Since the technological obstacles for using more than two layers and/or a small amount of overlap has decreased, it is furthermore important to understand the relationship between the number of layers and the channel width, and how overlap affects channel width and the number of contact points. In this paper we present answers to these questions for the 2-terminal CRP on 2-layer square-grid models.

The (infinite) channel of width $t$ consists of the grid points $(i,j)$, $0 \leq i \leq t+1$, $-\infty < j < \infty$, where $i$ is the track number and $j$ is the column number, and the edges connecting adjacent grid points. Grid points on track 0 and $t+1$ are called terminals. A wire is a path connecting adjacent grid points and a wire can switch from one layer to the other by using a grid point as a *contact point*. In the CRP we are given $n$ (two-terminal) nets $(p_i, q_i)$, where $p_i$ is a column number on track 0 and $q_i$ is a column number on track $t+1$, $1 \leq i \leq n$, and no two nets share a common terminal. A solution to the CRP consists of the channel width $t$ and the wires connecting the two terminals of each net.

In our models each layer can be used to run horizontal and vertical wires. In the *2-layer model* wires on different layers can cross or share a corner (i.e., form a knock-knee), but are only allowed to overlap in the overlap model. Solutions presented for the 2-layer model without overlap in [BB] and [RBM] achieve a channel width of $2d-1$ and use $O(dn)$ contact points when solving an $n$ net CRP of density $d$. (The density $d$ is the maximum over all $x$ of the number of nets $(p,q)$ for which $p<x<q$ or $q<x<p$). Leighton has shown that there exist

CRPs of density $d$ that require $2d-1$ tracks [L].

We first describe an algorithm for one-sided CRPs in the 2-layer model without overlap that uses $2d-1$ tracks and at most $4n$ contact points. A one-sided CRP consists either of right and trivial nets (right CRP), or of left and trivial nets (left CRP). A net $(p_i, q_i)$ is a *right* net if $p_i<q_i$, a *left* net if $p_i>q_i$, and a *trivial* net if $p_i=q_i$. Lower Bounds on the channel width for this and for another, more restrictive, model have been obtained by considering only right (or left) CRPs ([BR], [L]). While it is not known whether $2d-1$ tracks and $O(n)$ contact points can be achieved simultaneously, our result thus says that right (or left) CRPs alone are not powerful enough to prove it cannot be done. We furthermore give some insight into the difficulties that must be overcome by an algorithm that uses $O(n)$ contact points. We then show that any CRP can be solved on $2d-1$ tracks using $O(n)$ contact points when vertical overlap of length 1 is allowed between two nets. We thus trade knock-knees and $O(dn)$ contact points for slightly more overlap and $O(n)$ contact points. An algorithm given in [RBM] uses vertical overlap of length $O(d)$ to achieve $O(n)$ contact points.

We will follow the definitions introduced in [PL] and distinguish between the *wire layout* and the *wiring* of a CRP. Informally, the wire layout describes the path of the wires in the channel without considering their assignment to the layers, while the wiring gives the assignment of wires to the layers. See Fig. 1.1, where (*ii*) shows the wiring of the wire layout given in (*i*). Wires running on the top (bottom) layer are called T- (B-)wires.

In section 3 we consider the $k$-layer model when $k$-fold overlap is allowed. The trivial lower bound on the channel width for this model is $\lceil d/k \rceil$. Using a counting argument we show that for a large class of CRPs at least $\lceil d/(k-1) \rceil$ tracks are needed even when horizontal and vertical overlap of arbitrary length are allowed.

## 2. Two Layers and no Overlap

We describe a channel routing algorithm for the 2-layer model without overlap that solves right CRPs on a channel of width $2d-1$ by using at most $4n$ contact points. The algorithm for left CRPs is analogous. We assume that the right CRP does not contain any trivial nets (trivial nets can easily be added after the algorithm), and that the CRP is *full*; i.e., each terminal (excluding the ones on the right and left end of the channel) is either the starting or the ending point of a net. Each right CRP of density $d$ uniquely decomposes into $d$ *right runs*. A right run is a maximal sequence of nets $(p_1,q_1)$, $(p_2,q_2)$, ..., $(p_{i_k},q_{i_k})$ such that $p_j < q_j$ and $q_j = p_{j+1}$.

The obvious wire layout for right CRP is to assign each run to one of $d$ tracks and to produce a knock-knee when a net ends and a new one starts up; see Fig. 1.1 ($a$). If the wiring is then obtained by using $d-1$ intermediate tracks to accommodate necessary contact points, up to $dn$ contact points are needed ([BB], [RBM]). An even stronger result holds: there exist right CRPs that require $\Omega(dn)$ contact points when wired on a channel of width $2d-1$ with all nets having the 'simple' shape (i.e., a vertical, a horizontal, and a vertical wire). Thus, when minimizing the number of contact points the shape of the wired nets has to be *different* from the simple shape.

In our algorithm $d$ tracks are used to run horizontal wires and $d-1$ intermediate tracks are used for the contact points. The intermediate track between track $i$ and $i+1$ is called track $m_i$, $1 \le i \le d-1$. The wiring is determined column by column, from left to right. It tries to run all horizontal wires as T-wires and all vertical wires as B-wires. This is not always possible: when a net reaches its final column as a T-wire, the wire immediately continuing on this track has to start off as a B-wire. The algorithm removes such a B-wire from the track within the next two columns by continuing it on another track as either a T-wire, or as a B-wire (if it again participates in a knock-knee). Between any two columns in the channel there will be at most 3 B-wires. Furthermore, a horizontal T-wire changes into a B-wire only when its corresponding net reaches the final column (in some cases it will be the column before the final one).

In the $j$-th step of the algorithm we determine the vertical wires in column $j$, and the horizontal wires between column $j$ and column $j+1$. All horizontal T-wires - except the one corresponding to the net ending in column $j$, and possibly the one ending in column $j+1$ - continue as T-wires. We only consider the horizontal B-wires, which we try to change into T-wires, and the net starting in column $j$, the net ending in column $j$, and, in some cases, the net ending in column $j+1$. We first describe the two routines used when processing one column. The slipping routine, which changes B-wires into T-wires by letting them 'slip' onto other tracks, and the take-down routine, which runs vertical wires from one track to another track.

The *slipping routine* has one argument $i$, a track number, and causes each B-wire on track 1 to $i-1$ to 'slip' onto a higher numbered track (less than $i$) and change into a T-wire. Thus, after the slipping routine tracks 1 to $i-1$ are guaranteed to contain only T-wires, and track $i$ contains a T- or a B-wire.

Let $b_1, b_2, \ldots, b_c$ be the tracks containing a B-wire between column $j-1$ and $j$, $b_l < b_{l+1}$, $1 \le l < c$, $b_c \le i-1$. The net starting in column $j$ runs down column $j$ as a B-wire, switches into a T-wire on track $m_{b_1-1}$, and continues as a T-wire on track $b_1$. The B-wire in track $b_l$ behaves in the analogous way, $2 \le l < c$. The wire on track $b_c$ runs down column $j$ as a B-wire and continues on track $i$ as either a B- or a T-wire, depending on the type of the wire on track $i$. See Fig. 2.1. The number of contact points needed in the slipping process is equal to the number of B-wires on track 1 to $i$.

The *take-down routine* has two arguments, $i_1$ and $i_2$, both of which are track numbers, $i_1 \leq i_2$. We run the wire currently on track $i_1$ down column $j$ until it reaches track $i_2$ and use contact points as needed. If $i_2 = d+1$, the wire ends at the terminal in column $j$; in all other cases it continues on track $i_2$ (its type is determined by the wire previously on track $i_2$).

Assume that the net ending in column $j$ is currently on track $i$, and that a net starts in column $j$. If the wire on track $i$ belonging to the ending net is a *B-wire*, we perform the slipping routine down to track $i$ and the take-down routine from track $i$ to $d+1$. Column $j+1$ contains then only T-wires on track 1 to $i$ (a more careful analysis of the algorithm shows that actually *all* horizontal wires are T-wires).

If the wire on track $i$ belonging to ending net is a *T-wire*, we are forced to put a B-wire on track $i$ between column $j$ and $j+1$. In order to have a T-wire on track $i$ and to continue the horizontal B-wire currently on track $i$ as a horizontal T-wire as soon as possible, look at the net ending in column $j+1$. We distinguish between two cases. First assume that the net ending in column $j+1$ is on a track below track $i$. When the B-wire now on track $i$ participates in the slipping routine for the net ending in column $j+1$, a T-wire is put on track $i$ in column $j+1$. The B-wire slips from track $i$ onto a higher numbered track, where it could still run as a B-wire. See Fig. 2.2($i$).

In the second case the net ending in column $j+1$ is on track $i'$, which is above track $i$. We perform the slipping routine from track 1 to $i'$, take the wire previously on track $i'$ down to track $i$, and take the T-wire on track $i$ belonging to net ending in column $j$ from track $i$ down to $d+1$. See Fig. 2.2($ii$). Track $i$ contains now a B-wire that ends in column $j+1$. Thus all horizontal B-wires between track $i'$ and track $i$ (which were crossed over in take-down($i'$,$i$)) will be changed into T-wires when processing column $j+1$. In the worst case we can have 3 horizontal B-wires between two columns. This occurs when a T-wire ends in column $j-1$ and the first case holds, and a T-wire ends in column $j$ and the second case holds, as shown in Fig. 2.2($ii$). (Note that between column $j+1$ and $j+2$ we have only T-wires.)

The wiring algorithm produces nets of the shapes shown in Fig. 2.3. Each wired net consists of 3 parts: the *slipping part*, which consists only of B-wires and contains $l$ *slips*, $0 \le l \le d$. The first $l-1$ slips can only occur when the horizontal B-wire is the last wire participating in the slipping routine and the wire of the net ending in the corresponding column is a T-wire. The horizontal B-wire following the last slip can be of length 2 (when the situation described in Fig. 2.2($ii$) occurs), all other horizontal B-wires have length 1. The *horz-part* of the wired net consists of a horizontal T-wire ending in the column in which the net ends (Fig. 2.3($i$)), or one column before (Fig. 2.3($ii$) and ($iii$)). The *take-down* part of the wired net consists either of a straight B-wire, or a B-wire with one slip and possibly two contact points before the slip. Thus, each wired net contains at most 4 contact points, and the total number of contact points is at most $4n$. Since we only have to consider the nets ending in column $j$ (and possibly $j+1$), the net starting in column $j$, and the B-wires entering column $j$, one column is processed in constant time. Hence, the wiring algorithm wires a right CRP in time $O(n)$ and uses $O(n)$ contact points.

The number of slips in the slipping part of a wired net can be bounded for the cost of two additional contact points (they will occur in the first vertical wire of the slipping part of a net). Assume no more than $s$ slips are wanted in a net, $s \ge 1$. Let the net ending in column $j$ be on track $i$, and let $w$ be the wire that would have to slip again in slip($i$). Instead of making a slip in wire $w$, run $w$ horizontally and let the net starting in column $j$ slip onto track $i$. Wire $w$ will then change into a T-wire in column $j+1$ or $j+2$.

The wiring algorithm can be modified to solve *weak* CRPs on $2d-1$ tracks using $O(n)$ contact points. A weak CRP is a CRP that contains right and left nets with the following restriction: in all columns where one terminal belongs to a right and one to a left net, we either have *always* a right net end and a left net begin, or a right net begin and a left net end. Thus, the two situations shown in Fig. 2.4 cannot occur together. (In a wire layout of a weak CRP all nets can have the simple shape.) If situation 2.4($i$) occurs the algorithm processes the channel left to right; if situation 2.4($ii$) occurs right to left. During the algorithm right nets and left nets are handled separately; i.e., a vertical wire of a right net passes over a horizontal B-wire of a left net and vice-versa. Hence the algorithm contains a right and a left slipping routine and the number of contact points needed is still $O(n)$.

If we remove the restriction of Fig. 2.4 the idea used in the previous algorithms fails. The $d$-track wire layout might have to contain some nets of non-simple shape. Preparata and Lipski show that nets of the shapes shown in Fig. 3.1 are sufficient, [PL]. The ordering of the wires in the channel is now crucial, and the use of a slipping routine to remove horizontal B-wires as done in our algorithm destroys the ordering. At the present, it is not known whether $2d-1$ tracks and $O(n)$ contact points can be achieved simultaneously in the 2-layer model without overlap. A proof that it cannot be achieved for all CRPs might use a 'worst-case' CRP that contains nets of the form shown in Fig. 2.4, and nets that require a U-shape in the $d$-track wire layout. In the next section we show that allowing a constant amount of vertical overlap reduces the number of contact points to $O(n)$ for all CRPs.

## 3. Two Layers and Vertical Overlap

While it is not known how to achieve $O(n)$ contact points and $2d-1$ tracks simultaneously in the 2-layer model without overlap, we show how to wire any CRP in a channel of $2d-1$ tracks using at most $3n$ contact points and a vertical overlap of length 1. We thus trade knock-knees and $O(dn)$ contact points for slightly more overlap and $O(n)$ contact points. Our algorithm consists of a wire layout and a wiring step. The wire layout of the CRP on $d$ tracks is found by using the algorithm given in [PL]. We then interleave the $d$ tracks with $d-1$ intermediate tracks. The intermediate track between track $i$ and $i+1$ is again called $m_i$, $1 \le i \le d-1$. The wiring changes the shape of the nets determined by the wire layout algorithm only at the knock-knees, where it, in general, replaces the knock-knee by a slip and a vertical unit overlap in one of the nets involved. We run all horizontal wires as T-wires, and all vertical wires, except the ones participating in the unit overlap, as B-wires.

The wiring proceeds column by column, and, since the wire layout algorithm produces nets of the shapes shown in Fig. 3.1, one of the following situations (or a subset of one) occurs in column $j$:

- $(i)$ a right (left) run continues and hence column $j$ contains a right (left) knock-knee
- $(ii)$ a right run and a left run start (end); in this case column $j$ can contain the vertical segments of U-shaped nets; see Fig.3.2

Each right knock-knee on track $i$ in column $j$ is wired according to the rule shown in Fig. 3.3($i$); left knock-knees are wired according to Fig. 3.3($ii$). The vertical unit segment put into column $j+1$ conflicts with the wiring to be done in column $j+1$ when the new wire leaves track $i$ in column $j+1$, when the contact point $(m_{i-1}, j+1)$ is needed in the case of a right knock-knee in column $j$, or when the contact point $(m_i, j+1)$ is needed in the case of a left knock-knee in column $j$. We thus have to look into column $j+1$ before wiring the knock-knees in column $j$.

The contact point $(m_{i-1}, j+1)$ is needed if column $j+1$ contains a left knock-knee on track $i-1$. We solve this conflict by letting either the wire on track $i$ slip onto track $m_i$ in column $j-1$ (Fig. 3.4($i$)), or by letting the wire on $i-1$ slip onto track $m_{i-2}$ in column $j$ (Fig. 3.4($ii$)). The slips are not possible if column $j-1$ contains a left knock-knee on track $i-2$ *and* a right knock-knee on $i+1$. This occurs when the layout contains wires of the shape shown in Fig. 3.5, which is not possible in the PL-layout. Thus one of the slips can always be done. If the contact point $(m_i, j+1)$ is needed, column $j+1$ contains a right knock-knee on track $i+1$, and letting one of the wires slip as before resolves the conflict.

Another set of wiring rules is needed when track $i$ contains a sequence of wires of length 1. In the case of left knock-knees, each wire of length 1 belongs to a net of length 1, and in the case of right knock-knees the wire can also be part of a U-shaped net. If the sequence of length 1 wires is even, we wire as shown in Fig. 3.6. If it is odd, we wire either as shown in Fig. 3.7($i$), or, when the contact point cannot be used, as shown in Fig. 3.7($ii$). Again, one of the two solutions is always possible.

Since U-shaped nets require at most 4 contact points, all the other nets at most 2, and the wire layout contains no more than $\lfloor n/2 \rfloor$ U-shaped nets, we need no more than $3n$ contact points. Using the result of [GT] we can show that the running time of the algorithm is $O(n)$.

## 4. A Lower Bound for the Overlap Model

In the $k$-layer model with $k$-fold horizontal overlap $\lceil \frac{d}{k} \rceil$ is the trivial lower bound on the channel width. We improve this bound and show that for a large class of CRPs $\lceil \frac{d}{k-1} \rceil$ tracks are needed even when horizontal and vertical overlap of arbitrary length is allowed. The resulting lower bound of $d$ for the 2-layer model has been stated without a proof in [L]. Since no wires can cross a $k$-fold horizontal or vertical overlap, and it seems intuitive that in some situations the ability to use a $k$-fold overlap might not help. For example, if a trivial net is wired as a straight vertical wire, at most a $(k-1)$-fold horizontal overlap can occur on a track in this column, and if a trivial net is wired any other way we increase the density of the CRP.

Consider full right (or full left) CRPs of density $d$, $d>k$, that consist of $n+d$ nets and satisfy the following conditions:

(*i*) $n \geq kd^2/(k-1)^3$

(*ii*) the density between column $j_1$ and $j_1+1$ and between column $j_2$ and $j_2+1$ is $j_1$, $1 \leq j_1 \leq d-1$, $j_2 = n+2d-j_1$;

(*iii*) the density between column $j$ and $j+1$ is $d$, $d \leq j \leq n+d$.

We thus can divide the channel into the *startup window*, where density $d$ is achieved as fast as possible, the *main window*, where density $d$ is maintained, and the *finishing window*, where the last $d$ nets end. We show that such CRPs require $\lceil d/(k-1) \rceil$ tracks in the $k$-fold horizontal and vertical overlap model.

Assume we solve a CRP of the described class on a channel consisting of $t$ tracks, $\lceil \frac{d}{k} \rceil \leq t \leq \lceil d/(k-1) \rceil - 1$. At least $n-d$ of the $n$ nets starting in the main window have their corresponding final terminal also in the main window. Each wired net contains at least $t+1$ vertical unit segments. (Note that the main window contains $k(t+1)n$ vertical unit segments.) We obtain the lower bound on the channel width by computing two quantities: $V_{\min}$, the *minimum number of verti-*

*cal unit segments needed* in the main window in order to wire the $n$ nets starting in the main window, and, $V_{max}$, the *maximum number of free vertical unit segments* in the main window. Each free vertical unit segment can contain one of the at least $t+1$ vertical segments needed for each net that is completely wired in the main window. Since $V_{max} \geq V_{min}$ is needed, we obtain the bound on $t$.

We first compute $V_{min}$. Some of the nets starting in the main window can *escape* into the startup or the finishing window to run their vertical wire segments. On the left side of the main window $d$ nets enter and thus $kt-d$ of the $kt$ horizontal tracks are available for escaping nets. Since each escaping net has to return into the main window, at most $(kt-d)/2$ nets can escape on this side. Analogous, $d$ nets leave the main window on the right side, and at most $(kt-d)/2$ nets can escape the on the right side. Hence, at least $n-(kt-d)-d$ = $n-kt$ nets have to be wired completely within the main window, and $V_{min}$ = $(t+1)(n-kt)$. Our lower bound does not include the vertical segments needed in the main window by the escaping nets, the nets coming from the startup window and ending in the main window, and the nets starting in the main window and ending in the finishing window. We furthermore do not consider the effect of contact points on vertical segments.

When computing $V_{max}$ we consider how horizontal overlap affects the availability of vertical unit segments. We first show that column $j$, $d+1 \leq j \leq n+d$, contains at most $kt-d+1$ free initial vertical unit segments. An initial vertical unit segment is a vertical unit segment that can be counted towards the $t+1$ vertical unit segments needed for each net wired within the main window. We have the following lemma.

**Lemma.** Assume track $i$ contains an $m$-fold overlap between column $j-1$ and $j$, $0 \leq m \leq k$, $1 \leq i \leq t$. Then column $j$ can contain at most $k-m$ initial vertical unit segments between track $i-1$ and $i$.

**Proof**: For each one of the $m$ horizontal wires on track $i$ between column $j-1$ and $j$ we consider 2 cases.

*Case 1:* The wire continues on track $i$ to column $j+1$, or it continues in column $j$ towards track $i+1$. In both situations no wire of the same type can be placed between track $i-1$ and $i$, and the number of initial vertical unit segments is reduced by 1.

*Case 2:* The wire continues in column $j$ towards track $i-1$. Such a wire can belong to a net that "goes upwards", as shown Fig. 4.1. The vertical segment put into column $j$ is not an initial vertical unit segment and its presence reduces the number of initial vertical unit segments by 1.

The wire can also belong to a net that contains a U-shape, as shown in Fig. 4.2. The vertical unit segment between track $i$ and $i-1$ is actually an initial one, but at the same time we increase the density between column $j-1$ and $j$. Let $i'$ be the track number of the corresponding second wire of the net between column $j-1$ and $j$. The horizontal wire on $i'$ disallows a vertical unit segment between track $i'$ and $i'-1$. We account for the loss of this segment when looking at the wire on track $i$, and reduce the number of initial vertical unit segments by 1. ▪

We now proceed to prove that column $j$ contains at most $kt-d+1$ free vertical unit segments. For each net having more than one horizontal wire on a track between column $j-1$ and $j$ consider only the wire segment on the lowest numbered track and ignore the others. We thus consider exactly $d$ wire segments. Let $p_m^j$ be the number of tracks between column $j-1$ and $j$ that contain an $m$-fold overlap, $0 \le m \le k$, and let $free_j$ be the number of free vertical unit segments in column $j$. Then,

$$\sum_{m=0}^{k} p_m^j = t, \qquad \sum_{m=0}^{k} m p_m^j = d, \text{ and} \tag{1}$$

$$\sum_{m=0}^{k} (k-m)\, p_m^j = free_j. \tag{2}$$

Assume the $p_m^j$'s, $0 \le m \le k-2$, have been fixed. Then $t - \sum_{m=0}^{k-2} p_m^j$ tracks have to accommodate $d - \sum_{m=1}^{k-2} m p_m^j$ wires by using either $(k-1)$- or $k$-fold overlap. (Since $t < d/(k-1)$, there have to be some tracks with an $k$-fold overlap.) It follows, that

$$p_k^j = d - (k-1)t - \sum_{m=1}^{k-2} m p_m^j + (k-1) \sum_{m=0}^{k-2} p_m^j \quad \text{and}$$

$$p_{k-1}^j = kt - d - k \sum_{m=0}^{k-2} p_m^j + \sum_{m=1}^{k-2} m p_m^j .$$

Using (2), we get $free_j = kt - d$. Since we did not take into account the vertical segment between track $t$ and track $t+1$ (which contains the exit terminals), the number of free vertical segments in column $j$ is at most $kt-d+1$, and $V_{max} = n(kt-d+1)$.

We need $V_{max} = n(kt-d+1) \ge (t+1)(n-kt) = V_{min}$. Let $t = \lceil d/(k-1) \rceil - 1$, then $n \le dk(d-k+1)/(k-1)^3$ is necessary for $V_{max} \ge V_{min}$. Since $n \ge kd^2/(k-1)^3$ and $d > k$ for all CRPs in the class considered, the condition cannot be satisfied and $\lceil d/(k-1) \rceil$ tracks are needed.

The 2-layer model is the most extensively studied model, and the immediate question is whether or not the lower bound of $d$ is tight for this model. Horizontal overlap alone does not help: Leighton's lower bound argument [L] can be modified to show that $2d-1$ tracks are still needed for some CRP. Brown [B] gives an algorithm that uses vertical overlap of length $O(d)$ and horizontal overlap of constant length and $\frac{7}{4}d + c$ tracks, for some constant $c$. We conjecture that $d$ is not achievable in the 2-layer overlap model and it would be interesting to narrow the gap between $d$ and $\frac{7}{4}d$. Algorithms for the $k$-layer model with double overlap have been studied in [H], and general upper bounds for the $k$-

layer model with $k$- or $(k-1)$-fold overlap remain to be developed.

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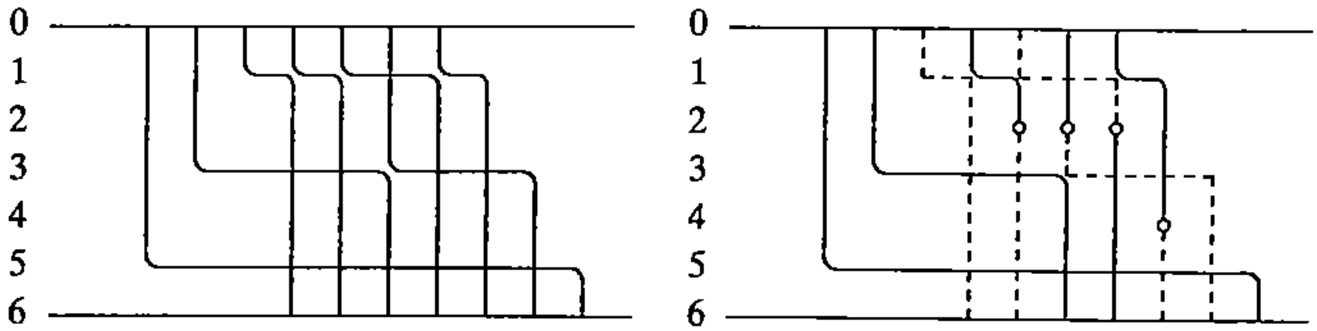
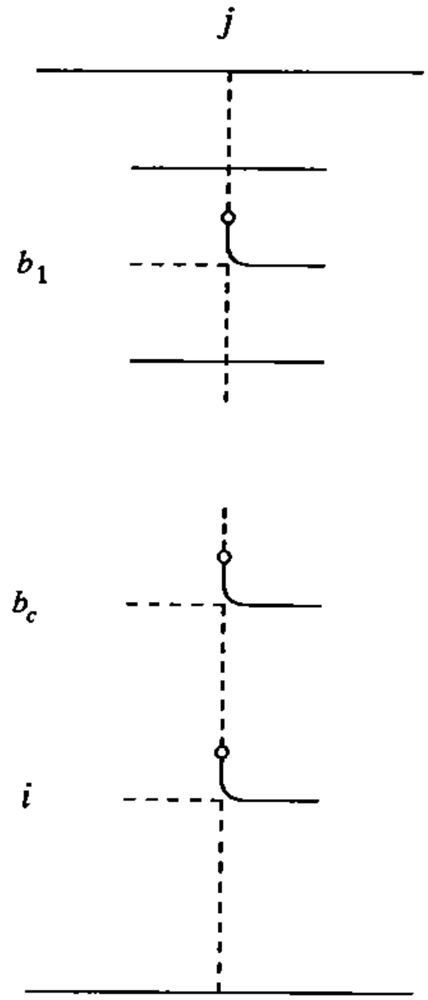
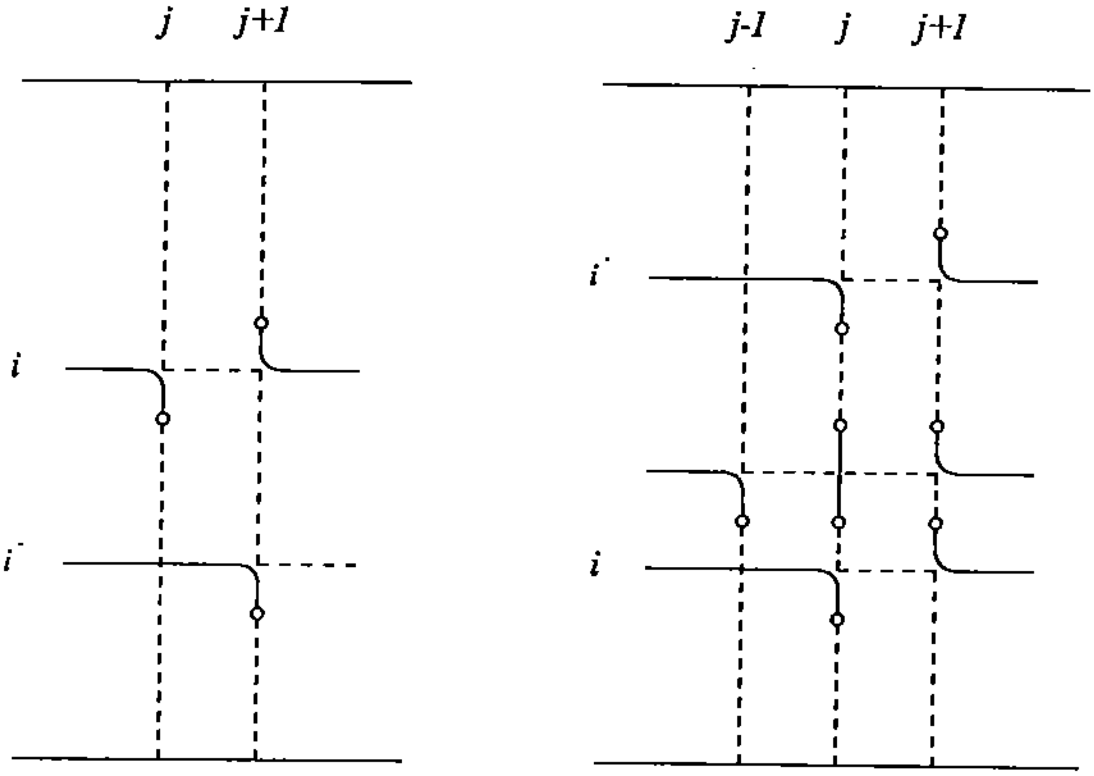


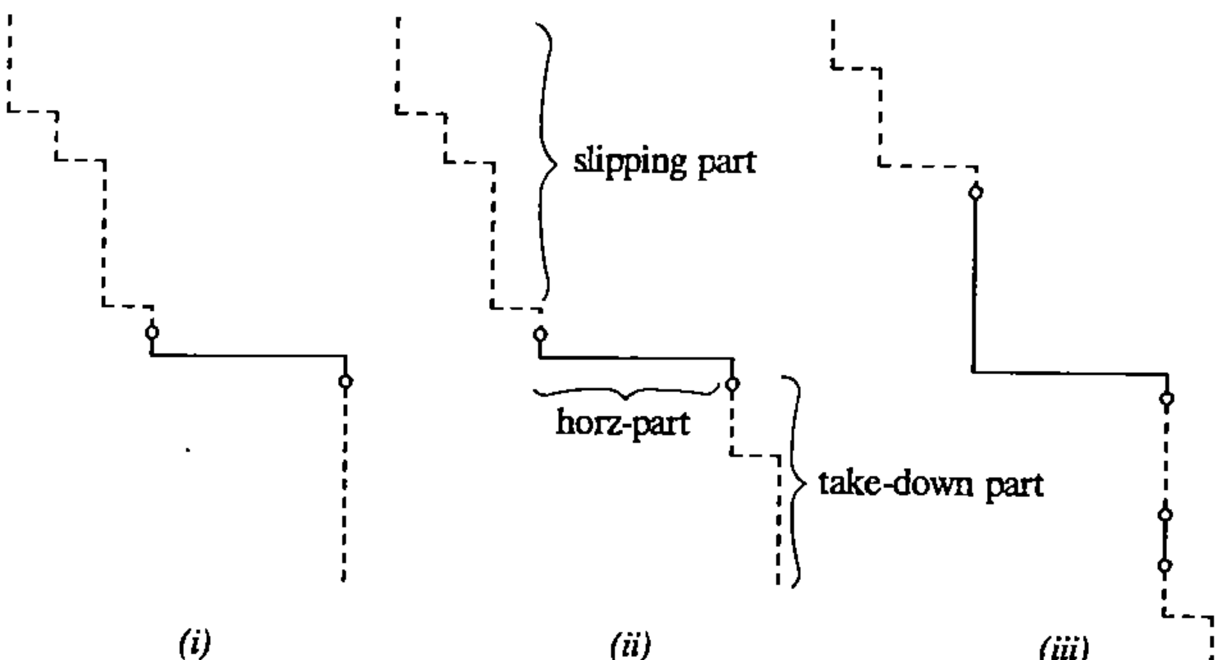

*(i)* wire layout *(ii)* wiring
of a CRP of density 3 on 5 tracks
(— T-wire, - - - B-wire)
Fig. 1.1



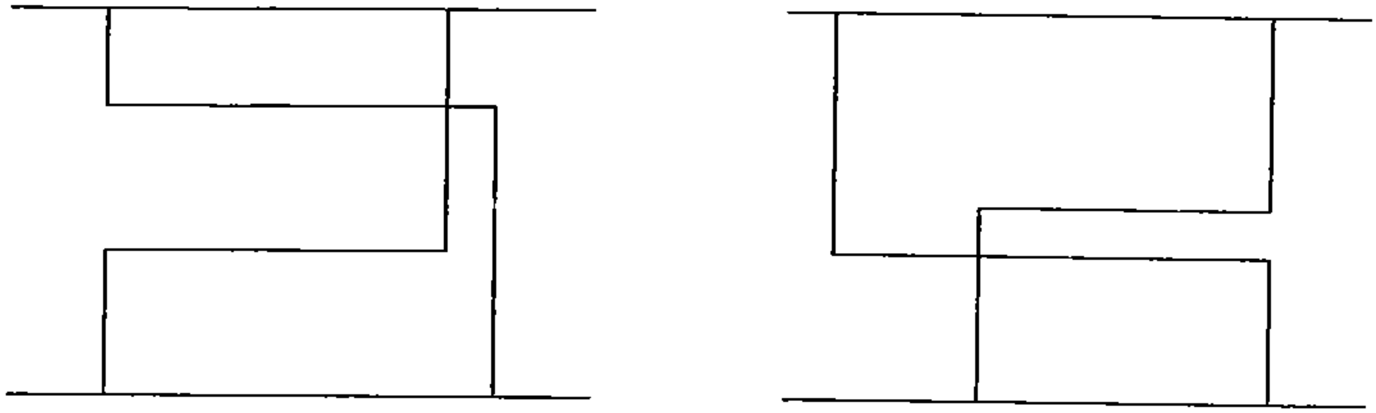

Slipping Routine
— T-wire, - - - B-wire
Fig. 2.1




Net ending in column $j+1$ is
*(i)* below track $i$ *(ii)* above track $i$
Fig. 2.2

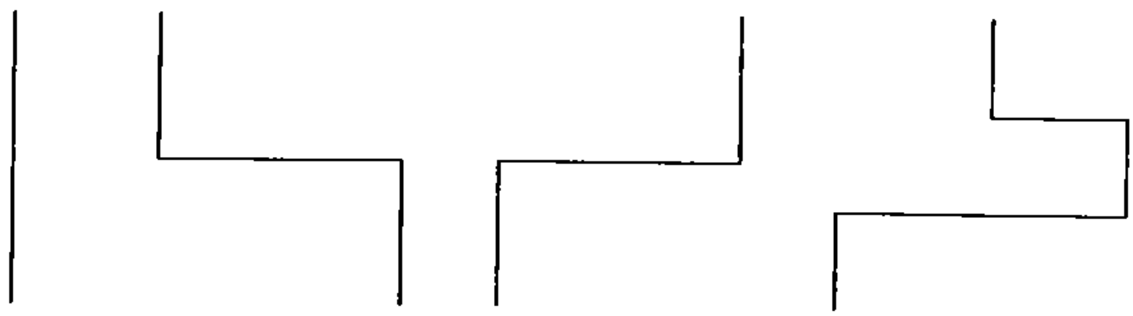



*(i)* *(ii)* *(iii)*
Shapes of the wired nets
Fig. 2.3

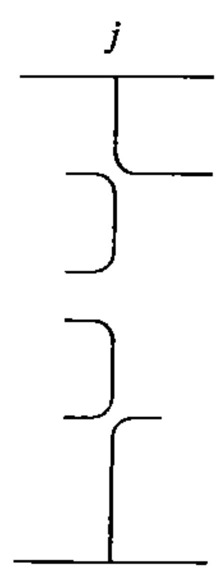


*(i)* *(ii)*

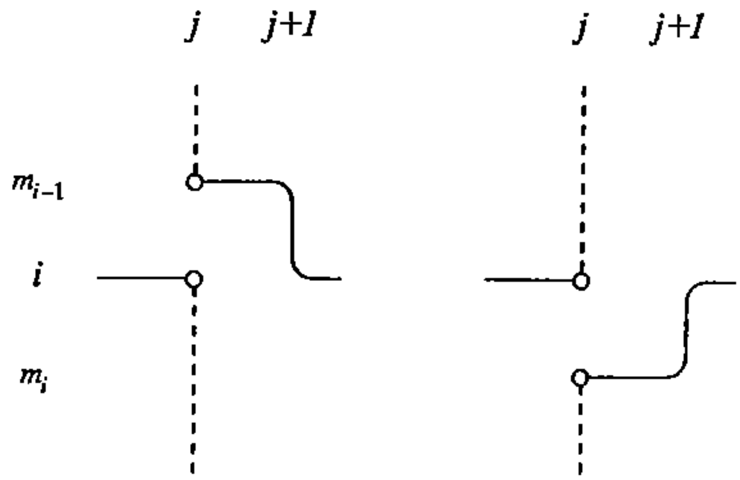
Fig. 2.4



Shape of nets in PL-layout
Fig. 3.1

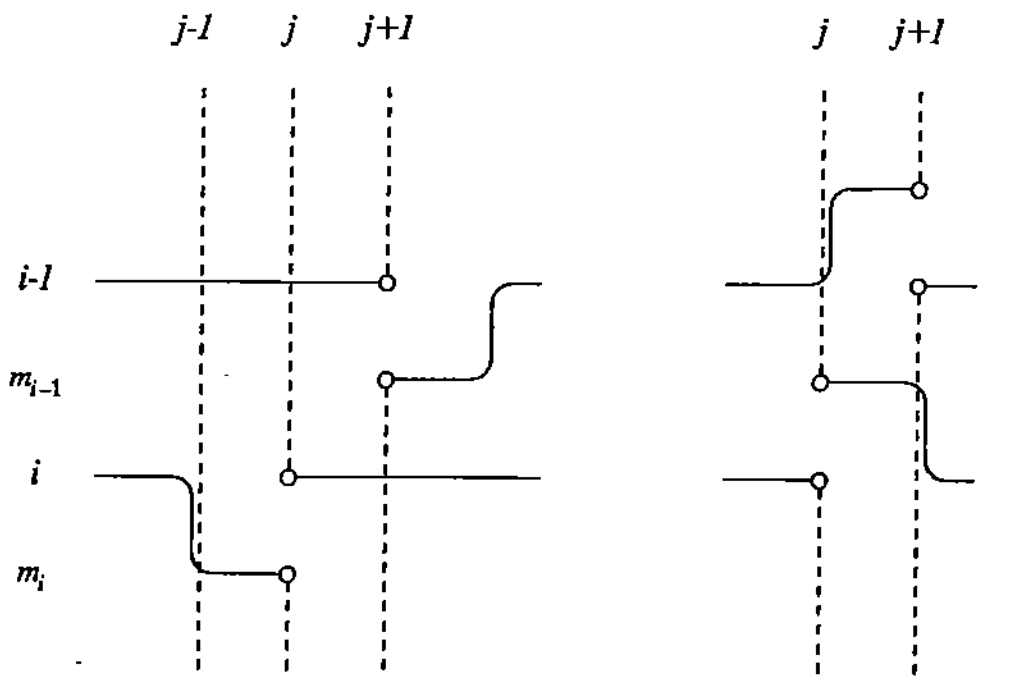



A column in the PL-layout
Fig. 3.2




*(i)* right — Wiring of a knock-knee — *(ii)* left

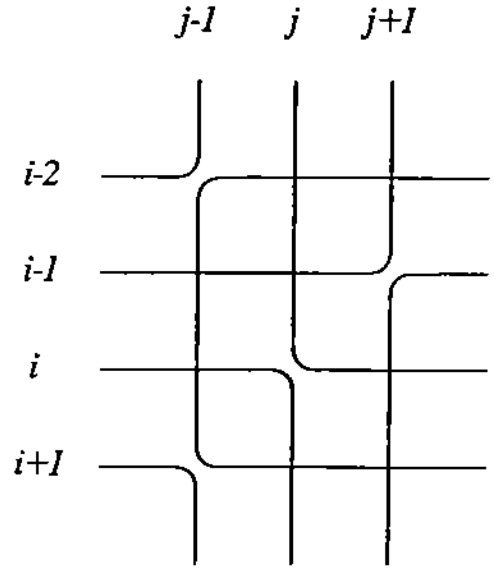
Fig. 3.3



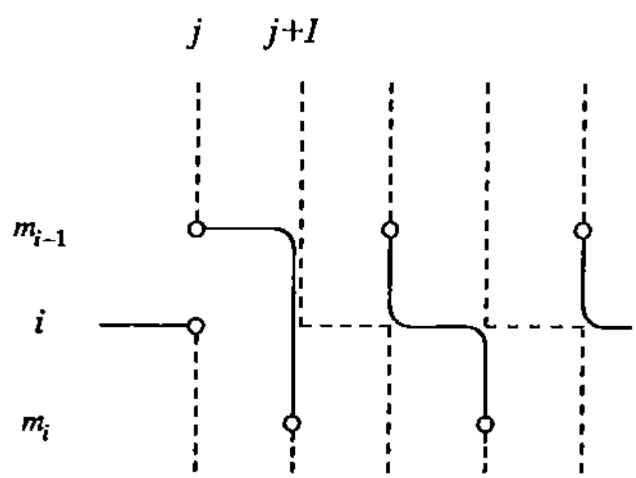

*(i)* wire on track *i* slips

*(ii)* wire on track *i-1* slips

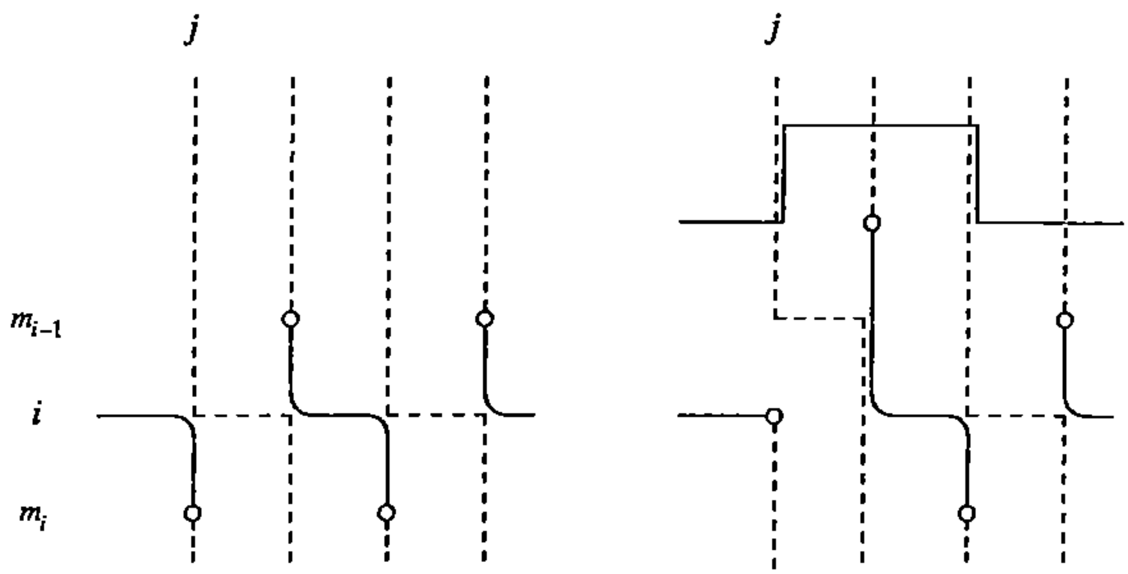
Fig. 3.4



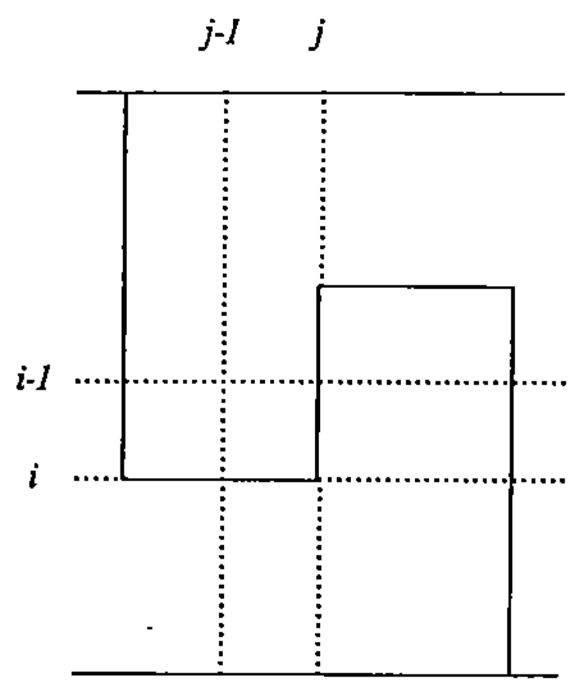

Layout not allowing any slips
Fig. 3.5



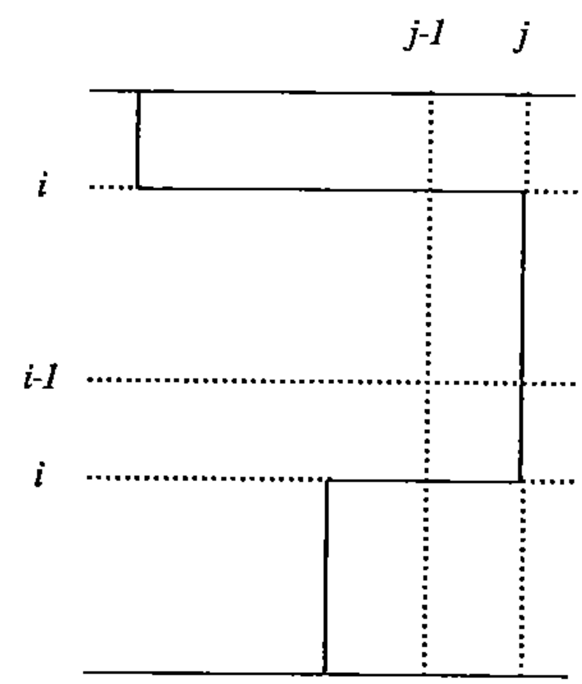

even number of length 1 wires
Fig. 3.6




*(i)* *(ii)*
odd number of length 1 wires
Fig. 3.7




net going upwards at $(i,j)$
Fig. 4.1




net with a U-shape
Fig. 4.2