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Electrical Modeling of Pressure Pulsation Dampers

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INTRODUCTION

In large and middle reciprocating compressor systems the levels of pressure pulsation are always higher than those considered permissible. Therefore it is often necessary to install a suitable pulsation damper. The damping effect of any such device depends greatly on its location and on the characteristics /impedance/ of any elements or fittings built into the pipe-line beyond the damper. Due consideration must be given to the influence of these characteristics when designing a suitable pulsation damper for a particular installation. The damper may be regarded as a concentrated element in a homogeneous section of pipeline: its effect both on pulsations before and beyond it are of importance.

Pressure pulsation in reciprocating compressor systems can be damped by many types of dampers. The selection of the optimum dimensions and the location of such dampers should be made during the design of an installation and the task of making the correct choice can be solved by means of the electroacoustic analogy and the appropriate analogue machine.

ANALOGY CRITERIA

Pulsating gas flow in a straight pipe of constant cross section is described by partial differential equations derived after making a number of simplifying assumptions. The equations of conservation of mass, momentum, and an equation of state may be combined to yield the system of equations /1/ [3], [6], [7]. In this system the gas density is assumed to be constant and has a value equal to its mean temporal value. In accordance with assumptions made when deriving acoustic equations a linearised frictional resistance term has been included.

\[
\begin{align*}
\frac{\partial p}{\partial x_a} &= M_{oa} \frac{\partial Q}{\partial t} + R_{oa} Q \\
\frac{\partial Q}{\partial x_a} &= C_{oa} \frac{\partial p}{\partial t}
\end{align*}
\]

/1/

In which

\[ M_{oa} = \frac{\gamma_0}{A} ; C_{oa} = \frac{A}{\gamma_0 a^2} ; R_{oa} = \frac{\rho_0 b}{A} ; b = \frac{3U}{cD} \]

The differential equations describing the propagation of current I and voltage V in an electrical transmission line having distributed parameters are presented below. Each element of the line of length dx e has

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resistance $R_{oe}$, inductance $L_{oe}$, capacitance $C_{oe}$ and leakage conductance $G_{oe}$ [5], [9], [10]. If the leakage conductance $G_{oe}$ is neglected, as is commonly the practice with low voltage lines, then
\[ \frac{\partial V}{\partial t} = L_{oe} \frac{\partial^2 V}{\partial x^2} + R_{oe} V \] \[ \frac{\partial I}{\partial t} = C_{oe} \frac{\partial V}{\partial x} \] Equations /2/ are analogous to equations /1/ which describe pulsating gas flow in a pipe-line. Identity of the differential equations describing two different physical phenomena is not a sufficient condition to establish their complete similarity. Mathematical modelling allows a solution to a given problem to be obtained provided that the parameters appearing in the equations are univocal and that corresponding boundary conditions are maintained. When values for the physical parameters appearing in equations /1/ and /2/ have been obtained and providing that corresponding boundary conditions are employed, satisfaction of the following analogy criteria ensures dynamic similarity for phenomena occurring in an electrical transmission line and a pipe-line.
\[ \pi_1 = L_{oe} C_{oe} = M_{oa} C_{oa} = \text{const} \]
\[ \pi_2 = \frac{L_{oe}}{V C_{oe}} = \frac{M_{oa}}{V C_{oa}} = \text{const} \]
\[ \pi_3 = \frac{L_{oe} \omega_e}{R_{oe}} = \frac{M_{oa} \omega_a}{R_{oa}} = \text{const} \]
\[ \pi_4 = \omega_e t = \omega_a t = \text{const} \]
Providing that numerical equality of the corresponding similarity parameters $\pi_1$, $\pi_2$, $\pi_3$, $\pi_4$ is achieved there will be dynamic similarity between pipe-line and electrical transmission line phenomena.

**CONSTRUCTION OF AN ELECTRICAL TRANSMISSION LINE TO MODEL A PIPE LINE**

An electrical transmission line which will model a given pipe-line can be made in the form of a finite length of co-axial cable. This approach has a number of disadvantages among which are: difficulty in making a cable with prescribed electrical parameters, the large dimensions of such a cable and its cost [2].

A more practical model of a pipe-line is achieved by constructing an equivalent electrical ladder network of the electrical transmission line using distributed parameters. The network consists of an appropriate number of meshes connected in series. The greater the number of meshes used in the equivalent ladder the better the model of a given transmission line might appear to be. However, the choice of the number of meshes to be used depends on the answer to questions raised as a result of investigations of the behaviour of this type of model [9]. The analogy criteria /3/ must be satisfied by every mesh of the ladder network and it is thus essential that each mesh is associated with the correct values of inductance, capacitance and resistance reckoned on a basis of unit length of line.

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Meshes may be represented as four terminal \( \Pi, L \) or \( T \) networks as shown in Figures 1a, 1b, 1c depending on the location of the various electrical elements. The complete ladder network is a series connection of individual meshes and constitutes a multisectional filter. A ladder network consisting of \( \Pi \) shape four terminal meshes has been found to be the most useful for pipeline modelling purposes.

Fig. 2. The general view of meshes with various magnifications.

On the photograph /Fig. 2/ the constructive solution of meshes shape \( \Pi \) used by the Author is presented. This photograph shows two meshes having various magnifications.

ELECTRICAL MODEL OF A DAMPING CHAMBER

Figure 3a illustrated a damping chamber which is frequently used as an independent damper and may be a component of the more complex dampers as well. A damping chamber is modelled as a section of a pipe-line with the diameter \( D_t \) and the length \( l_t \). The number of meshes modelling a damping chamber depends on its length /nothing less than two meshes it is necessary to use/. In the process of modelling it is necessary to regard the local resistances owing to sudden modifications of the cross-section in the planes I and II. The acoustic resistance levels in these planes are calculated by means of dependence /4/

\[
R_a = \zeta \frac{8Q}{2A}
\]

The electrical model of the damping chamber is shown in Figure 3b.

CONSTRUCTION AND BEHAVIOUR OF A RESONANCE TYPE DAMPER

Figure 4a illustrates a single resonant chamber pressure pulsation damper. The damper consists of a cylindrical chamber – the resonator, and a central pipe connected to it by a series of orifices. When the orifices perforate the central pipe in a single plane normal to the axis of the pipe the arrangement may be regarded as a Helmholtz resonator.

The behaviour of this type of damper can
be explained by considering the behaviour of a dynamic vibration absorber and the analogy between the behaviour of mechanical and acoustic systems /Fig. 4b/, [4]. Gas filling the central line is regarded as the main mass $m_1$ and its compliance resulting from its compressibility is represented by a spring of stiffness $k_1$. Gas in the orifices behaves as a series of small masses which may be considered as the mass of the vibration absorber $m_2$. The compliance of the gas contained in the resonator chamber, which results from the compressibility of this gas, is represented by a spring of stiffness $k_2$. Providing frictional effects and the throttling of the gas in the orifices are neglected then the behaviour of the damper can be expressed in mechanical terms. The model comprises two masses and two springs. Mass $m_1$ is acted upon by a periodic exciting force $F_0 \sin \omega t$ corresponding to the periodic processes occurring in compressor suction and discharge lines.

Analysis of the behaviour of the model shows that if the natural frequency of vibration of the auxiliary system made up of mass $m_2$ and the spring of stiffness $k_2$, is equal to the frequency of the exciting force then the amplitude of vibration of the mass $m_1$ is zero. Since the equivalent of the auxiliary system of the mechanical model in the acoustic system is the resonant chamber plus the mass of gas located in the orifices, the problem may be reduced to one of calculating the natural frequency of vibration of the resonator system.

The natural frequency of vibration of such a damper can be calculated using a formula derived for the Helmholtz resonator /Fig. 5a/.

$$f_{oa} = \frac{a}{2\pi} \sqrt{\frac{\beta_0}{V_t}}$$

where $C_o = \frac{n_0 A_0}{l_o}$

and $l_o = l_o + \Delta l$

Fig. 5. Scheme of the Helmholtz resonator /a/, and its electrical model /b/.
The correction $\Delta l$ takes into account the mass of gas near orifice which participates in the vibration together with the mass of gas in the orifice and the distance between the orifices. In the case of a damper having orifices in a single cross section the value of $\Delta l$ is $0.8d_o$ and is usually referred to as the Rayleigh correction. When the central pipe is perforated over the entire length of the resonant chamber $\Delta l$ is calculated as follows

$$\Delta l = \frac{d_o}{4F(d_o/D_u)}$$

The Fock function $F\left(\frac{d_o}{D_u}\right)$ which appears in equation /8/ takes into account the influence of the distance between orifices. Here $D_u$ denotes the diameter of a circle which has an area equal to the area of the central pipe associated with one orifice [1]. The diagram of the Fock function is shown in Figure 6. The natural frequency of vibration of the electrical model of a Helmholtz resonator /Fig. 5b/ is calculated as

$$f_{oe} = \frac{1}{2\pi} \frac{1}{\sqrt{L_tC_t}}$$

![Fig. 6. Fock function.](image)

**ELECTRICAL MODELS OF A RESONANCE EFFECT PULSATION DAMPER**

If an electrical model is to be employed to simulate a pipe-line fitted with a pulsation damper then the model must represent adequately both the damper and the pipe-line in which it is fitted. The electrical model of a single chamber and perforated pipe resonance damper should consist of a part which models the section of perforated pipe and a part which models the resonance chamber together with the orifices which connect it with the main pipe-line. Perforation of the central pipe is characterised quantitatively by the orifice conductivity $C_0$. From an electrical viewpoint the damper can be modelled as a Helmholtz resonator provided that properly chosen values for the capacitance, inductance and resistance elements /Fig. 5b/ are selected. Capacitance $C_t$ is modelled from the volume of the resonance chamber $V_t$, inductance $L_t$ from the mass of gas in the orifices, resistance $R_t$ from the frictional resistance to gas flow in the orifices. The analogy criteria /3/ can be used to establish the required values of $C_t$ and $L_t$ for a given damper model. The value of $R_t$ can be defined only approximately because of the complicated character of the gas friction effects of flow in orifices.

Figure 7 shows a one-chamber resonance damper electrical model and its equivalent acoustic model. In this model the perforated central pipe is modelling by the means of three meshes which models adequately 0.25, 0.5 and 0.25 of this pipe length. The resonance chamber is connected with the central pipe in two transverse planes which are situated at a distance of 0.25 $l_t$ from the beginning and the end of the damper. It is possibility to get an electrical model of a two-chamber resonance damper by series connection of a one-cham-
EXPERIMENTAL INVESTIGATIONS

The object of the experimental investigations was to evaluate the correctness of modelling a damping chamber and a one-chamber resonance damper by means of electrical models. Experiments were carried out by the discharge pipeline of a laboratory compressed air installation and by its equivalent electrical model. Piezoelectric pressure transducer were used to record pressure pulsations at a number of locations analog the discharge pipe. The dampers were situated at a distance of 0.3 m from a reciprocating compressor and the discharge pipe was terminated by a severely throttled valve. Experimental tests were carried out with the absolute compressor discharge pressure held constant (about 3 bar) and with variable compressor rotational speed (500 - 1300 rev/min). The results of model tests were presented as the diagrams of absolute pressure pulsation levels (peak-to-peak) along a pipeline \( \Delta p_a = f(l) \) (Fig. 8, Fig. 9) adequately for a damping chamber and a resonance damper electrical models. The results of laboratory tests in the cross-sections of pipe situated 0.15 m before the damper and 0.15, 0.5 m beyond the damper were shown in these diagrams as black circles.

CONCLUSIONS

Basing oneself on the comparison of pressure pulsation absolute levels obtained from the laboratory and model tests in the same sections it is possible to note that the required accuracy of the real courses reproduction in installation model is secured by using of damping chamber and resonance damper models which are presented. By means of presented damper models it is possible to obtain the considerable compliance of the reproduction of absolute pressure pulsation levels before and beyond the damper as well. The accuracy of these courses reproduction is sufficient for evaluation of the damping effect obtained by means of discussed dampers in the projected installations.

NOMENCLATURE

\( a \) velocity of sound in the gas
Fig. 8. Diagrams of dependences $\Delta p_a = f(l)$ for the outlet pipe-line with a damping chamber; the results of laboratory tests are shown as black circles.
Fig. 9. Diagrams of dependences $\Delta p_A = f(l)$ for the outlet pipe-line with a resonance damper; the results of laboratory tests are shown as black circles.
A  cross sectional area of the pipe-line

$A_0$  orifice area

b  slope of the non linear resistance versus flow characteristic

$C_{oa}$  acoustic capacitance per unit length of pipe-line

$C_{oe}$  electrical capacitance per unit length of transmission line

$C_o$  orifice conductivity

$\mathcal{d}_o$  orifice diameter

D  internal diameter of pipe-line

$D_t$  internal diameter of chamber

$D_u$  diameter of an equivalent circular area

f  frequency

$G_{oe}$  leakage conductance per unit length of transmission line

I  current

k  spring stiffness

$l_e$  analytical length of orifice

$l_o$  geometric length of orifice

$\mathcal{l}$  orifice length correction

$L_{oe}$  electrical inductance per unit length of transmission line

m  mass

$M_{oa}$  acoustic mass /inertance/ per unit length of pipe-line

$n_o$  number of orifices in the central pipe of a damper

p  pressure

Q  volume flow rate

$R_{oa}$  acoustic resistance per unit length of pipe-line

$R_{oe}$  electrical resistance per unit length of transmission line

t  time

U  mean fluid velocity /linearizing velocity/

V  voltage

$V_t$  volume of chamber

$x_a$  distance coordinate along pipe-line

$x_e$  distance coordinate along transmission line

$\lambda$  coefficient of friction losses

$\zeta$  local drag coefficient

$\rho_0$  means gas density

$\omega_a$  circular frequency in acoustic system

$\omega_e$  circular frequency in electrical system

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