1980

Simulation of a Reciprocating Compressor Accounting for Interaction between Valve Movement and Plenum Chamber Pressure

A. B. Tramschek

J. F. T. MacLaren

Follow this and additional works at: https://docs.lib.purdue.edu/icec

https://docs.lib.purdue.edu/icec/356

This document has been made available through Purdue e-Pubs, a service of the Purdue University Libraries. Please contact epubs@purdue.edu for additional information. Complete proceedings may be acquired in print and on CD-ROM directly from the Ray W. Herrick Laboratories at https://engineering.purdue.edu/Herrick/Events/orderlit.html
ABSTRACT

A mathematical model is described which will simulate a reciprocating compressor on a digital computer. Initially it is assumed in the model that the pressures in the valve plenum chambers remain constant. A sufficiently converged solution is obtained in about three compressor cycles. The inherently intermittent and unsteady flow is then calculated by integrating the solution to the simultaneous differential equations which describe the flow through the suction and discharge valves and the valve movement. A Fourier analysis is performed on this predicted flow through each valve and the resulting Fourier coefficients are used in an acoustic analysis of the compressor suction and discharge systems in order to predict the pressure variation in the appropriate plenum chamber. These pressure variations are then used to revise the initial flow rates and the mutually interdependent movement of the valves. An iterative process requiring about four cycles of computation was generally successful in obtaining a cyclically repeatable solution. In some instances resonance occurred between natural frequencies of the gas vibration in the suction or discharge systems and a harmonic of the compressor speed. Under these conditions it became necessary to introduce mathematical damping to simulate the viscous damping in the real system. It was difficult to select a value of damping coefficient appropriate to achieving adequate agreement between analytical and experimental plenum pressure records. Application of the model resulted in little change in compressor performance compared with values predicted by a simpler model in which the plenum chamber pressures are assumed constants. However the interaction between valve movement and non constant plenum chamber pressures modified the valve action; hence the new model is of use when making a detailed analysis of valve behaviour.

INTRODUCTION

In a compressor fitted with automatic valves, the valve movement during the suction and discharge process is a function of the pressure difference across the valve. Both the valve movement and the pressure difference are dependent on compressor speed, compressor pressure ratio, valve inertia, valve spring stiffness, fluid properties, etc. In early models, which assume that the valve plenum chamber pressures remain constant, (1, 2, 3,) two simultaneous differential equations, which relate the more important relevant variables, provide a mathematical model to describe the suction and discharge events in the cycle. One is a non-linear equation which expresses the flow through the valve as a function of the pressure difference across the valve and the displacement of its moving element. The other is an equation which describes the valve displacement in terms of the relevant gas and spring forces acting upon it. The valve is considered to be a single-degree-of-freedom spring mass system. Both equations are expressed as function of time and hence of crankangle.

These equations, solved either by graphical, analog, or digital computer by numerical methods, yield the displacement of each valve and the pressure difference across it. The pressure in the cylinder is obtained by subtracting (during suction) or adding (during discharge) the pressure difference across the valve to the appropriate plenum chamber (constant) pressure. Integration of the flow and pressure variations over a cycle permits the evaluation of volumetric efficiency and power consumption. The assumption that the suction and discharge plenum chamber pressures remain constant is a simplification which provides a relatively simple model which is economical to use. Such a model is still of use as an aid to design despite the availability of more complete, but more complex, models.

Since the mass flow of gas through positive displacement compressors is inherently intermittent, a fluctuation of pressure must occur in the finite volume plenum chambers and associated piping. This fluctuation is coupled with the valve action and each can have a significant effect on the other and on compressor performance. Simulation models which account for these complex pulsation phenomena have been developed for use on analogue computers (5) and hybrid (analogue/digital) computers (6). Most models, however, have been used with digital computers. References (7, 8, 9, 10) are examples of papers presented which contain descriptions of these more complete models at the four previous Compressor Technology Conferences at Purdue University.
Improved numerical methods have been developed (11) to solve the partial differential equations which describe the gas flow in a compressor system as functions of space and time. If the compressor system is already in existence a technique has been developed (12) by which experimentally obtained records of pressure time histories in the plenum chambers can be superimposed on the basic simulation (1,2,3), so providing a hybrid (analytical/experimental) model.

The present paper describes a model which may be used to predict plenum chamber pressures during a cycle and account for their interaction with valve displacement. The investigation extends that by Elson and Soedel (13) by including a description of the suction side of the compressor in the simulation model. While the amplitude of the pressure fluctuations in the suction plenum is small in absolute terms it is often about the same percentage of the mean plenum chamber pressure as on the discharge side. The suction valve is relatively weakly sprung and its motion may be unstable, particularly during the valve closure phase around piston reversal at outer dead centre. Hence plenum chamber pressure fluctuations can have a marked effect on suction valve displacement and the instant at which that valve closes finally. It has been shown (14) that the phasing of such fluctuations may have more effect than their amplitude.

OUTLINE OF PROCEDURE

To initiate the procedure a relatively simple simulation model of a compressor is employed. In the model, of the type reviewed in reference (4), the two simultaneous non-linear differential equations referred to in the Introduction are integrated numerically to evaluate the pressure difference across a valve and the valve displacement. From this information pressure - crankangle and pressure - volume diagrams may be plotted (Fig 1), the assumption being made that the pressures in the suction and discharge plenum chambers (P₁ and P₄) remain constant throughout the cycle. The solution is commenced at point 0 (Fig 1) where the assumption is made for this first cycle that the discharge valve is closed, the pressure difference across it is zero and the piston is at inner dead centre. The gas in the clearance volume is expanded polytropically until the suction valve begins to open at point 1, or slightly later if valve spring preloading and/or oil stiction at the valve seat have been accounted for. Then the two equations are solved numerically until the suction valve is finally closed (point 2), at or soon after piston reversal at outer dead centre. Polytropic compression occurs from point 2 to point 3 and a similar pair of equations are solved during the discharge process from point 3 to point 0'. The conditions at point 0' are compared to those assumed initially for point 0: if conditions are significantly different the cycle is repeated with 0' as a revised starting point. Usually the solution converges adequately after three compressor cycles.

The predicted unsteady flow rate through the valves during the third cycle, expressed by a Fourier series, is then utilized to predict the pressure variation during the cycle in the respective finite volume plenum chambers. These, now varying, values of P₁ and P₄ are used as revised conditions in the simulation model of the compressor and a revised prediction made of flow rate and the interrelated valve displacements. The procedure is repeated several times until a cycle is obtained where no significant differences are observed in the successive evaluation of cylinder and plenum pressures or of valve displacements. The results from each step of the iterative process during the last cycle of the simulation model are integrated to evaluate the volumetric efficiency, power consumption etc. of the compressor.

PRESSURE VARIATION IN A PLENUM CHAMBER

The suction and discharge sides of a compression system may be represented approximately by a plenum chamber of finite volume V connected to a pipe of length L and cross-sectional area A (Fig 2).

Since the volume flow through a compressor valve is periodic then a Fourier analysis of the flow as a function of time (crankangle) allows the flow to be expressed as the sum of a series, i.e.

\[ Q = Q₀ + Q₁ \cos wt + Q₂ \cos 2wt + Q₃ \cos 3wt - - - \]

\[ Qₙ \cos nwt + Q₁' \sin wt + Q₂' \sin 2wt + Q₃' \sin 3wt - - - Qₙ' \sin nwt \]
First prediction of pressures, valve displacements and mass flows (3rd cycle of simulation model)

Fourth prediction of pressures, valve displacements and mass flows (3rd cycle of simulation model)
First prediction of pressures, valve displacements and mass flows. (3rd cycle of simulation model)

Second prediction of pressures, valve displacements and mass flows (3rd cycle of simulation model)

Fourier terms (n) | Suction | Discharge
---|---|---
15 | 30 | 30 | 60
FIG. 7 INTERSECTION OF FUNCTION (w) FOR SUCTION AND DISCHARGE SYSTEMS WITH MULTIPLES OF COMPRESSOR SPEED
As a result of this flow a periodic pressure variation will be generated in the pipe and the plenum chamber. By using the acoustic wave equation to describe the pressure variation in the pipe and by applying appropriate boundary conditions a solution may be derived for the resulting pressure variation.

The equations describing the sound pressure i.e. the pressure fluctuation, \( p \), about a mean value and the particle displacement \( z \), about a mean position may be written as

\[
\text{C}^2 \frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 z}{\partial t^2} \tag{1a}
\]

and

\[
\text{C}^2 \frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 p}{\partial t^2} \tag{1b}
\]

where the sound pressure is related to the particle displacement by

\[
p = -\gamma \rho_m \frac{\partial z}{\partial x} \tag{2}
\]

and \( C \) is the speed of sound at the mean conditions

\[
C = \sqrt{\gamma R T_m} = \sqrt{\frac{\gamma P_m}{\rho_m}}
\]

For the system shown in Fig 2 the following boundary conditions apply:

At \( x = 0 \), \( p = 0 \)

\[
(3)
\]

ie the pipes are connected to relatively large receivers (eg evaporator and condenser) which are at constant mean pressures \( P_m \) and \( P_m \) respectively.

At \( x = L \) or \( L' \) the pressure in the pipe was assumed to be equal to the plenum chamber pressure, so equality existed between volume strain for the chamber and the section of pipe at \( x = L \) or \( L' \).

\[
\text{P}_x = L = \left( \frac{\gamma P_m}{\rho_m} \right) \frac{Q \cos n \omega t + Q' \sin n \omega t}{1 - \frac{AC}{\rho V} \cot \frac{\pi m L}{c}}
\]

The plenum chamber pressure is given by

\[
P = P_m + P_x \cdot L
\]

If the mass flow rate through a valve, \( m \), is known then values for \( Q \) may be obtained from

\[
Q = \frac{m}{P_m}
\]

If the mass flow rate computed by the simulation model is expressed as a Fourier series

\[
m = A_0 + \sum_{n=1}^{\infty} \left[ A_n \cos (n \omega t) + B_n \sin (n \omega t) \right]
\]
where \( A_0 = \frac{1}{2\pi} \int_{0}^{2\pi} \dot{m} (\theta) \, d\theta \)
\( A_n = \frac{1}{\pi} \int_{0}^{2\pi} \dot{m} (\theta) \cos (n\theta) \, d\theta \)
\( B_n = \frac{1}{\pi} \int_{0}^{2\pi} \dot{m} (\theta) \sin (n\theta) \, d\theta \)

where \( \theta = \omega t \). Thus \( Q_n = \frac{A_n}{f_m} \) and \( Q'_n = \frac{B_n}{f_m} \) and 
the contributions of a number of sine and cosine terms may be summed to yield an estimate of the pressure fluctuation in a plenum chamber.

AN APPLICATION OF THE PROCEDURE

The procedure was applied to a single-stage, single-acting, single-cylinder air compressor 6 in (=152 mm) bore and 4.5 in (114 mm) stroke, fitted with single-ring plate valves, loaded by coil springs, at both suction and discharge sides. The nominal discharge pressure was 100 lb/in\(^2\) (69 bar) gauge and the compressor speed range was 350 to 600 rev/min. The volumes of the suction and discharge plenum chambers were each 32 in\(^3\) (524 cm\(^3\)). The pipe diameter at both suction and discharge sides was 2.125 in (54 mm). In the subsequent discussion of two examples from the results (Tests A1 and C6) the pipe length at the inlet side was 226.2 in (5.75 m) for test A1 and 117.4 in (3 m) for test C6; the pipe length on the discharge side was 158.4 in (4 m) for both of these particular tests.

Fig 3 shows results from the initial run of the simulation model of the compressor (the results having converged adequately by the third cycle of computation). Graphs are included of the two plenum chamber pressures (constant in the initial run), the cylinder pressure during the suction and discharge phases of the cycle, the displacement of the suction and discharge valves and the mass flow through each valve. The violent flutter of the suction valve at this low compressor speed may be observed, and the corresponding fluctuation of the mass flow through the valve.

Subsequently this mass flow is expressed by a Fourier series and the simulation model rerun. The pressure variation in each plenum chamber, caused by the fluctuations of the mass flow into and out of the chamber is evaluated together with the interactive effect of this pressure variation on cylinder pressure and valve movement. Fig 4 shows the corresponding results after the fourth run of the simulation model of the compressor, and includes the subsequent fourth revision of the mass flow. In each of the four runs the third compressor cycle was that recorded and used. The differences between Figs 3 and 4 illustrate, not only the change of the pressure variation in each plenum chamber, but the consequential change on cylinder pressures and valve movement.

The magnitude of the differences in some of the parameters calculated by the simulation model are listed in Table 1.

<table>
<thead>
<tr>
<th>Simulation Model run</th>
<th>1st (Fig 3)</th>
<th>4th (Fig 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volumetric Efficiency (Indicated)</td>
<td>83.4</td>
<td>81.4</td>
</tr>
<tr>
<td>Volumetric Efficiency (Actual)</td>
<td>78.6</td>
<td>78.5</td>
</tr>
<tr>
<td>Suction Valve opens (deg)</td>
<td>43.9</td>
<td>46.9</td>
</tr>
<tr>
<td>closes (deg)</td>
<td>167</td>
<td>192</td>
</tr>
<tr>
<td>Discharge Valve opens (deg)</td>
<td>318</td>
<td>316</td>
</tr>
<tr>
<td>closes (deg)</td>
<td>365</td>
<td>368</td>
</tr>
<tr>
<td>Impact Velocity (Suction) stop (ft/s)</td>
<td>5.5</td>
<td>4.9</td>
</tr>
<tr>
<td>Negligible (Discharge) stop (ft/s)</td>
<td>13.8</td>
<td>15.2</td>
</tr>
<tr>
<td>Negligible seat (ft/s)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Indicated Power (kW)</td>
<td>3.16</td>
<td>3.17</td>
</tr>
<tr>
<td>Indicated Adiabatic Power (kW)</td>
<td>3.0424</td>
<td>3.0424</td>
</tr>
<tr>
<td>Valve Resistance (Suction) (kW)</td>
<td>0.0900</td>
<td>0.0923</td>
</tr>
<tr>
<td>(Discharge) (kW)</td>
<td>0.0277</td>
<td>0.0349</td>
</tr>
</tbody>
</table>

Table 1 - Effect of Accounting for Pressure Variation in Suction and Discharge Plenum Chambers on Compressor Performance (Test A1)
From equation (5) it may be deduced that
\[
\frac{P}{Q} \propto \left( 1 - \frac{AC}{\pi C} \cot \frac{wL}{C} \right) = f(w)
\]
(7)

If the term \( \frac{AC}{\pi C} \cot \frac{wL}{C} \) tends to unity then \( p \) becomes very large and the (predicted) pressure pulsations may be excessive in the absence of damping.

Plots were made of \( f(w) \) against \( w \), expressed as multiples of the compressor speed for each test. Fig 7(a) relates to the results for Test A1 in Fig 4, with \( f(w) \) plotted for the first 50 multiples of compressor speed. The number of Fourier terms \( n \), equation 6) used was 10 on the suction side and 20 on the discharge side. With these values of integer \( n \), Fig 7(a) shows that there were only two peaks of \( f(w) \) on the suction side and two on the discharge side. At none of these four peak values did \( w \) coincide with a harmonic of the compressor speed and unstable or resonance conditions did not arise.

Fig 5 shows results from Test C6, obtained by the first run of the simulation model. The compressor speed had been raised to 628 rev/min (from 375 rev/min in Test A1) and the length of the suction pipeline had been reduced to 117.4 in (3 m) (from 226.2 in (5.75 m in Test A1). (Fig 6 is the graphical record after only the second application of the model (ie employing the first predicted mass flow from the initial run in Fig 5). Unstable conditions were present leading to severe pressure fluctuations in the plenum chambers. The procedure aborted at the next cycle. In Test C6 (Fig 6 full lines) the integer \( n \) was 15 on the suction side and 30 on the discharge side. Fig 7(b) shows that for these conditions there are three peak values of \( f(w) \) on the suction side and without any particularly high value of \( f(w) \) at a harmonic of the compressor speed. On the discharge side there may have been resonance at the 12th harmonic of the compressor speed. The procedure was rerun for this test with \( n = 30 \) (suction side) and \( n = 60 \) (discharge side). In this larger range of \( n \) there were six peak values of \( f(w) \) on the suction side, including now a coincidence with the 18th harmonic of compressor speed, but possibly no new resonant condition on the discharge side. The results, superimposed as dotted lines on Fig 6, show that with the larger range of \( n \) the fluctuations increased, most noticeably of the pressure in the suction plenum chamber and of the predicted mass flow through the suction valve.

The larger the number of Fourier terms, \( n \), used in equation 6 the greater will be the accuracy of the representation of the mass flow through the valve. However, the number of harmonics of compressor speed involved will be correspondingly greater, ie as \( n \) is increased a resonance condition is more likely to be met. It is immaterial at which value of \( n \) the resonance occurs since no mathematical damping at any frequency has been introduced, as yet, to simulate the viscous damping in the physical system.

### NUMBER OF TERMS, \( n \)

The number of terms, \( n \), used in the Fourier series to describe the mass flow, \( \dot{m} \), was a compromise between

1. a large number needed to express accurately the mass flow predicted by the simulation model (and thereafter the pressure-time history in a plenum chamber) and
2. a small number to minimise the chances of resonance between values of \( w \) giving large values of \( f(w) \) and multiples of compressor speed (and also to economise on computer time).

The mass flow through a valve, given \( \dot{m} \), equation 6, may be rewritten as
\[
\dot{m} = \dot{m}_0 + \sum_{n=1}^{\infty} R_n \cos (n\phi + \phi)
\]
(8)

where
\[
R = \sqrt{R_n^2 + B_n^2}
\]
and \( \phi = \tan^{-1} \left( \frac{B_n}{R_n} \right) \), the phase angle.

Fig 8(a) shows a plot of \( R \) against \( n \) for test A1 (Figs 3, 4, 5a). After 10 terms \( R \) had reduced to less than 20% of its value with the first few terms, for both the suction and discharge sides. There is a possibility that the summation in equation 7 may become larger as a consequence of terms being in phase. The polar plot in Fig 8(b) shows that the higher terms were near to the pole and tend to cancel out by being out of phase, whereas the lower terms with larger magnitude were not opposed.

It was concluded that a value of \( n \) as low as 10 was probably adequate. This conclusion was based on test A1 and is not necessarily general. However test A1 was examined because it appeared that a larger number of Fourier terms might be necessary to describe the mass flow during the exceptionally severe suction valve flutter occurring under that particular test condition.

### DAMPING

The predicted pressure fluctuations in the plenum chambers were compared to experimental results. Examination of the experimental results suggested that significant damping was present in the system. Accordingly the calculations to predict the pressure fluctuations were reworked using the damped wave equation for particle displacement, i.e.
\[
\frac{\partial^2 z}{\partial t^2} + F \frac{\partial z}{\partial t} + c \frac{\partial^2 z}{\partial x^2} = 0 \]
(9)
Where $F$ is a term containing a viscous damping coefficient and is defined by

$$ F = \frac{1}{d} \sqrt{\omega^2} $$

Soedel (17) found that in cases when a system was at resonance it was necessary to increase the damping factor by a multiplier $\delta$ such that

$$ F = \frac{\delta}{d} \sqrt{\omega^2} $$

Otherwise, in such cases, predicted damping factors were too low (possibly by as much as two orders of magnitude).

A solution for the pressure variation in a plenum chamber of the system shown in Figure 2, in which a flow of the form $Qe^{jwt}$ is disturbing the system, is

$$ p(L, t) = \frac{f \omega C_o}{S} \left[ \frac{\sinh (L \omega)}{S \cosh (L \omega) + \frac{S}{8} \sinh (8L \omega)} \right] Q e^{jwt} \quad \ldots \ldots \ldots (10) $$

where $\omega = \alpha + ik$

$$ \alpha = \frac{\delta}{C_o \omega} \sqrt{\frac{M_o}{S_o} \omega^2} $$

$$ k = \frac{\omega}{C_o} $$

$$ i = \sqrt{-1} $$

For a plenum chamber at $x = L$

$$ p(L, t) = \frac{f \omega C_o}{S} Q \left[ 1 + \frac{1+i \tanh (a \tan kl)}{1+i \tan kl + i \tan kl + (a+ik) \frac{V}{S}} \right] e^{jwt} \quad \ldots \ldots \ldots (11) $$

Corresponding to a forcing term of the form $Qe^{jwt}$ the pressure fluctuation is given by the real part of $Qe^{jwt}$. For a term of the form $Qe^{jwt}$ the pressure fluctuation is found using the imaginary part of equation (11). Thus the net effect of sine and cosine forcing terms may be found.

If $P(L, t)$ is written as

$$ P(L, t) = \frac{f \omega C_o}{S} Z e^{jwt} \quad \ldots \ldots (12) $$

where $Z$ is a complex quantity with $Z = |Z|e^{j\phi}$

then corresponding to a flow fluctuation, $Q$, given by

$$ Q = A \cos \omega t + B \sin \omega t $$

the pressure fluctuation, $P(L, t)$, is given by

$$ P(L, t) = \frac{f \omega C_o}{S} \left| Z \right| \left( A \cos (\omega t + \phi) + B \sin (\omega t + \phi) \right) \ldots \ldots (13) $$

Corresponding to a flow fluctuation given by

$$ Q = A \omega + \sum_{n=1}^{n=\infty} (A_n \cos n \omega t + B_n \sin n \omega t) $$

The pressure variation caused by the fluctuating terms is given by

$$ P(L, t) = \frac{f \omega C_o}{S} \sum_{n=1}^{n=\infty} \left| Z \right| \left( A_n \cos (n\omega t + \phi_n) + B_n \sin (n\omega t + \phi_n) \right) \ldots \ldots (14) $$

The modulus of the complex term $Z$ is a function of the forcing frequency $\omega$ and has maximum values at values of $\omega$ corresponding to the natural frequencies of the damped system. Soedel (17) reported that provided $\omega_n \neq \omega_{res}$ then a value of unity could be used for the multiplier $\delta$. However, should $\omega_n = \omega_{res}$ then a value of $\delta \approx 150$ could be used for systems using refrigerant $R-12$.

Figures 9 and 10 show the variation of $|Z|$ with $\omega/\omega_n$ for the suction and discharge systems applicable to test C6 for a range of damping multipliers. Both figures show the proximity of multiples of $\omega/\omega_n$ to resonant frequencies in the range 1 $\leq \omega/\omega_n \leq 50$. For example $\omega/\omega_n = 18$ for the suction side (Figure 9) and $\omega/\omega_n = 13$ for the discharge side (Figure 10) result in low values of $I Z I$, indicating proximity to a resonant frequency. The graphs also show the effect of increasing the damping multiplier $\delta$. If $\delta$ is increased from 1.0 to 10.0 then $I Z I$ changes relatively little except in the vicinity of resonant frequencies. This finding accords with the statement by Soedel (17) that the greatest effect of damping multipliers is on harmonics close to a resonant frequency. If $\delta$ is increased to 100 then peak values of $I Z I$ are reduced further, whilst minimum values of $I Z I$ are increased slightly. With high values of the damping multiplier, resonance effects are virtually eliminated (except at low values of $\omega/\omega_n$).

Following investigation of the behaviour of $I Z I$ with both $\omega/\omega_n$ and $\delta$ the pressure variation in the plenum chambers was predicted for both the suction and discharge systems of the air compressor at the conditions in test C6. Figure 11 shows the analytical results for the third sequence of calculations for the third compressor cycle. For test C6 the computations for the suction side used 12 Fourier terms whilst calculations for the discharge side used 7 Fourier terms. In this test a damping multiplier of 10.0 was used and gave reasonable correspondence between the analytical and experimentally determined pressure variations in each plenum chamber.
CONCLUSIONS

Using the technique described it was possible to predict the pressure variation in the valve plenum chambers of the suction and discharge sides of a reciprocating air compressor system.

The investigation highlighted the necessity to determine the proximity of resonant frequencies in a suction or discharge system in relation to multiples of the forcing frequency, when the acoustic approach to pressure pulsations was adopted. Considerable care was necessary when selecting a multiplier for use with the damping term. Soedel (17) used a high value of $f = 150$ in the case of Freon 12. The present study suggests that such a value is an "overkill" and would unduly restrict the effect of high order harmonic terms.

Use of the technique resulted in little net change in parameters which specify compressor performance compared with values calculated using a simpler model wherein the plenum chamber pressures are assumed to remain constant. Such changes as were apparent concerned the action of the valves, particularly the suction valve, and use of the more complicated model is justified if valve behaviour is to be studied in detail.
REFERENCES

15. Soedel W, "Gas Pulsations in Compressor and Engine Manifolds". Notes for pre-Conference Course, Purdue University, 1978

NOMENCLATURE

\begin{itemize}
\item \(a\) - damping term, real part of complex quantity \(\mathbb{B}\).
\item \(A\) - cross sectional area.
\item \(A_0, A_n, B_n\) - Fourier series coefficients.
\item \(C\) - speed of sound.
\item \(C_0\) - reference speed of sound.
\item \(d\) - pipe diameter.
\item \(F\) - damping force per unit velocity.
\item \(i\) - \(\sqrt{-1}\).
\item \(k\) - \(\frac{C_0}{\rho}\) - (wave number).
\item \(L, L'\) - pipe length.
\item \(\dot{m}\) - mass flow rate.
\item \(n\) - harmonic number.
\item \(p\) - sound pressure (pressure fluctuation).
\item \(P_m\) - mean absolute pressure.
\item \(Q, Q'\) - volume flow rate.
\item \(\text{IRI}\) - modulus of a complex quantity (= Z).
\item \(S\) - cross sectional area.
\item \(t\) - time coordinate.
\item \(T\) - absolute temperature.
\item \(V\) - plenum chamber volume.
\item \(w\) - circular frequency.
\item \(\omega_0\) - fundamental circular frequency.
\item \(x\) - distance coordinate.
\item \(z\) - particle displacement in x coordinate direction.
\item \(Z\) - complex quantity.
\item \(Y\) - isentropic index, ratio of specific heats.
\item \(\Phi\) - complex frequency.
\item \(\phi\) - phase angle.
\item \(\delta\) - arbitrary multiplier.
\item \(\nu\) - dynamic viscosity.
\item \(\rho\) - density.
\item \(\nu\) - kinematic viscosity.
\end{itemize}