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The Dynamic Behavior of Valve Reeds in Reciprocating Gas Compressors - Analytical Study

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THE DYNAMIC BEHAVIOUR OF VALVE REEDS IN RECIPROCATING GAS COMPRESSORS

ANALYTICAL STUDY

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ABSTRACT

Aspects of the static and dynamic behaviour of valve reeds were considered. Initially static deformation and stress in a cantilever suction valve reed were calculated using the finite element method. Some useful conclusions were drawn despite the neglect of dynamic effects.

The free vibration of a cantilever reed was derived using the same method. The computer program used, DRST4, was examined in relation to convergence of solution. The results were compared with those obtained by other (Rayleigh) methods and with experimental values obtained by laser holography.

The finite element method was then applied to predict the dynamic motion and stress patterns in a cantilever suction valve reed in response to the pressure-time history in a compressor during the suction phase of the cycle. Another program, RESP95ST, was used in conjunction with DRST4. The predicted values under dynamic conditions appeared to be reasonable and logical: predicted displacements are compared with experimental values in a sequential paper.

INTRODUCTION

The automatic valves are probably the most vulnerable components in a reciprocating compressor. During the last decade there has been a rapid development of mathematical models, for use with a computer, to simulate compressors and so provide an aid to the design of these components. Such models include an analysis of the motion of the moving element of the valve: often this motion is considered to be that of a single degree-of-freedom single mass-spring system. This simple approach is generally adequate when the primary purpose of the simulation model is to predict overall compressor performance. However, this approach yields only rudimentary information of use in the detailed design of a valve.

Gatecliff et al (1) and later Kuijpers et al (1a) considered the mass to be distributed as several lumped discrete elements instead of a single mass. The forced vibration of a variable width cantilever reed suction valve was then computed. Moaveni et al (2) considered the forced motion to be a summation of the free vibration mode shapes. These mode shapes may be determined experimentally, or analytically in the case of simple geometries.

In general the geometry of the moving element of a compressor valve is not simple and the conditions at the boundaries are complex. The method of finite elements despite its high demands in computer core capacity and time has advantages in the analysis of such structures and has been used in studies of the static deformation and free vibration of reeds (3,4,5,6). In this paper the finite element method has been applied to study the static, free and forced vibration behaviour of a cantilever suction valve reed.

DESCRIPTION OF PROGRAMS

A program (DRST4, Direct Stiffness Program) was used to analyse the static deformation and free vibration of a suction valve reed.

In DRST4 an attempt is made to minimise input data and to be efficient in the use of computer core capacity, both in the method of solution and in the structure of the program. DRST4 consists of seven separate but interdependent programs. The major operations are: evaluation of element property matrices, assembling them into the overall stiffness matrix, solution of the stiffness equation and finally evaluation of stresses and accumulated loads. By using discrete programs for these operations economy of core capacity is achieved.

For computation reasons in the eigenvalue economiser version of the program, consistent formulation of the mass matrix was preferred (14). The alternative of using lumped masses which results in a diagonal mass matrix normally requires a larger number of elements to achieve a reasonable representation of mass and increases the computer storage required (but will decrease computer time, especially for the fundamental mode shape and frequency). Consistent formulation of the mass matrix is preferred when high natural frequencies and mode shapes are required and incompatible elements are used (10). The Cholesky decomposition method is used in the solution of the stiffness equation. It retains the banded form of the stiffness matrix so reducing the necessary computer core capacity, sometimes at the expense of computer time (10). Restriction in the number of permissible d.o.f. were imposed because the inverse of a banded matrix is fully populated. In the dynamic analysis of free vibrations where more d.o.f. are needed, the eigenvalue economiser was used especially when the process of evaluating the mode shapes became excessively expensive of computer capacity and time for systems with many d.o.f. When the eigenvalue economiser was employed, a larger number of d.o.f. could be permitted but there was a significant loss of accuracy at high natural frequencies. In general the predicted frequencies were higher than those obtained when the full number of d.o.f. was used. Unacceptable errors also start to appear for natural frequencies and mode shapes greater than about $\frac{1}{3}$ - $\frac{1}{4}$ of the total number of d.o.f. being assumed to describe the problem, even when the eigenvalue economiser is not used.

The Givens-Householder inverse iteration technique was used to retrieve the eigenvectors so reducing the computer storage and time requirements. The method is efficient when many eigenvalues and eigenvectors are required and when the associated matrices are full or have a large halfband-width. However, the method does not take advantage of the banded form of stiffness and mass matrices and its accuracy is less when the stiffness matrix is ill-conditioned.

A program, RESP95ST (Response, 95 max d.o.f., stress) based on the direct integration (step-by-step) method was used to analyse the dynamic behaviour of a valve reed subjected to a pressure-time history.

The non-linear nature of conditions at the reed boundaries favoured the use of a direct integration method although the mode super-position method needs less computer core capacity and time. Moreover, when the boundary conditions change at reed impact many modes of vibration are excited which are better handled by the direct integration method.

The derivatives of the ordinary second order differential equation which describes the motion of the system can be approximated in different ways, resulting in different degrees of accuracy.

The general difference equations used in the program RESP95ST are represented by

$$\dot{X}_{t+\tau} = \dot{X}_t + (1-\gamma)\tau\ddot{X}_t + \gamma\tau\ddot{X}_{t+\tau} \quad \ldots\ldots\ (1)$$

$$X_{t+\tau} = X_t + \tau\dot{X}_t + (\tfrac{1}{2}-\beta)\tau^2\ddot{X}_t + \beta\tau^2\ddot{X}_{t+\tau} \quad \ldots\ (2)$$

where X, $\dot{X}$ and $\ddot{X}$ are the displacement, velocity and acceleration matrices respectively, τ is a time interval ($\tau = \Theta\Delta t$). The scalar factor Θ relates the calculation time interval Δt with the time interval τ over which a certain form of acceleration is assumed to pertain. The parameter γ indicates how much of the acceleration has entered into the relation for velocity during the time interval τ. β is a parameter for acceleration and the subscript t is a counter denoting the number of times each matrix is evaluated.

If γ is taken as zero a negative damping results which will involve a self-excited vibration arising solely from the numerical procedure. If γ is greater than $\frac{1}{2}$ a positive damping is introduced which will increase the response. The value of mathematical damping is always proportional to $\gamma - \frac{1}{2}$ and therefore if $\gamma = \frac{1}{2}$ there will be zero mathematical damping (15). In the program RESP95ST it is assumed that $\gamma = \frac{1}{2}$.

The value of β often lies between 0 and $\frac{1}{4}$ (16): choice depends upon the physical characteristics of the system, the accuracy desired and the stability of the procedure. Making $\beta = \frac{1}{4}$ corresponds to a constant acceleration over the time interval equal to the mean of the initial and final accelerations: $\beta = 1/6$ corresponds to linear variation of acceleration in each time interval: $\beta = \frac{1}{8}$ corresponds to a step function, with a constant value equal to the initial acceleration for the first half of the time interval and constant value equal to the final acceleration for the second half. For structures with very low damping characteristics $\beta = \frac{1}{12}$ is recommended (16) although Newmark (15) claims that studies of damping yield better results when values of β used are in the range from $1/6$ to $\frac{1}{4}$ rather than in a range below $\frac{1}{8}$. In the present study $\beta = 1/6$ was used successfully.

The method is unconditionally stable if $\gamma = \frac{1}{2}$, $\beta \geq \frac{1}{4}$ and $\Theta = 1.0$ although round off errors may affect accuracy. These values are suitable for systems with as many as 40 d.o.f. (16).

If $\gamma = \frac{1}{2}$, $\beta = 1/6$ the method reduces to the Wilson Θ-method. The acceleration vector varies linearly during each time interval τ. The method is also unconditionally stable when $\Theta > 1.37$ and in most cases a Θ-value of 1.4 appeared to give good results (9).

In program RESP95ST it is assumed that $\Theta = 1$ which results in a linear variation of acceleration within the time interval Δt. If there was a parabolic variation of acceleration within Δt the accuracy of the method might increase (8) and allow an increase in the time increment without incurring instability: however, the number of computer operations per time interval would increase.

In unconditionally stable systems the time interval Δt can be large provided that the necessary high frequencies of the system can be reproduced adequately. Otherwise a form of mathematical damping proportional to Δt is often introduced (17).

The method used in RESP95ST was conditionally stable: often instability occurred unless the time step Δt was equal or less than $\sqrt{3}\,\Delta t_{cr}$. The critical value Δt_{cr} is defined:

$$\Delta t_{cr} = \frac{T_n}{\pi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (3)$$

where T_n is the smallest period (i.e. highest frequency) of a finite element model employing n d.o.f. With an elemental length of 5 - 6 mm and a reed thickness of 0.142 mm a critical time step is about 2 μs. Thus in a compressor cycle (piston frequency e.g. 10 Hz) about 12000 time steps have to be made for the discharge valve. When smaller elements are used in the finite element model the necessary number of time steps increases with an increase in the accuracy achieved.

The effect of physical damping was investigated by assuming viscous damping and replacing the damping matrix D by

$$D = cK \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

where c is a constant determined from damping ratios (9) and K is the assembled stiffness matrix for the model of the valve reed. As a consequence of equation (4), which is the simplest case of Caughey series (9), there is a linear relationship between damping ratio and frequency, damping ratios being greater at higher modes. The advantages of this approach are its simplicity and the banded form of D. The ratio of damping to its critical value for the ith mode shape C_i', is given by the formula (17)

$$C_i' = \frac{c\, f_i}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

where c is the damping constant and f_i the ith natural frequency.

The pressure time history was predicted by a simulation model (7) applied to the particular compressor and operating conditions. Relatively few elements were used for reasons of economy of computer capacity and time. By monitoring the displacement at the reed tip the instant at which the reed first touched the stop was found. By monitoring the displacement also at a point close to the reed tip the instant at which the reed left the stop was known. The program halted each time the reed touched or left the stop, i.e. when the boundary conditions changed. Then the problem was analysed with the initial values from the previous run accounting for the new boundary conditions.

The original version of the suite of programs DRST4 and RESP95ST was provided by Soper (10) and Heyes (11), who had applied it previously to the free and forced vibration of plates with several boundary conditions.

A flat triangular flexural element was used which had 3 nodes and linear variation of internal stress or strain. The element satisfied internal but not boundary compatibility. A flat rectangular flexural element with 4 modes was also used. The internal stress and strain relationships were predominantly linear but did involve terms as high as cubic. This element satisfied internal compatibility but normal slope incompatibilities existed. Neither type of element satisfied internal nor boundary equilibrium. Each type had 3 d.o.f. per node, one displacement and two rotations. Transverse shear energy was neglected. Convergence was correct, rapid but not monotonic when rectangular elements were used: convergence criteria were applied. Test problems (10) indicated that correct convergence was achieved with the triangular elements. It was assumed that deflection within the triangular element could be described by a third degree polynomial: a six degree polynomial was used with the rectangular element. Neither of the polynomials was complete (10).

The finite element models employed had various mesh configurations and d.o.f. Two typical configurations are shown in Figure 1. When arranging the mesh geometries for the static and free vibration problem, cognisance was taken of line of symmetry. This reduced the size of each problem but increased the number of problems to be solved, since displacement could be symmetric or antisymmetric about each line of symmetry.

The cost effectiveness of a finite element solution depends on many factors including the mesh generation technique, the type of elements, the polynomials used, the mesh subdivisions and the matrix calculation algorithm: cost-effective computing is enhanced by using higher degree polynomials and single branch instead of multi-branch logical statements. Reduction of the running time and storage required may thus be achieved.

The computer time to predict the forced dynamic behaviour of the reed was on average 25 - 30 times longer than the time necessary for a free vibration analysis, which in turn was double the time necessary for a static analysis. To predict forced dynamic behaviour the computer core required was about 75 K. The suite of programs used for free vibrations and static situations was considered to be efficient: it is hoped to improve the program efficiency for the case of forced vibration. The computer used was a Honeywell 6060 with 256 K main stores and 400 M filestores.

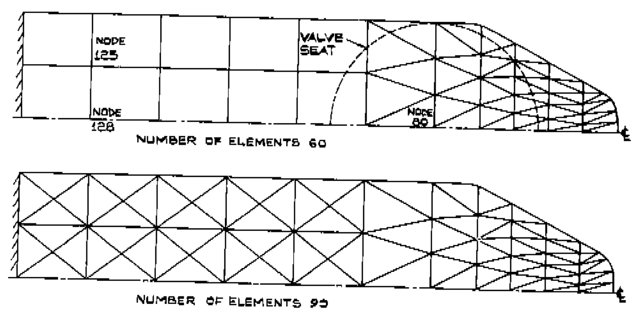


FIG 1 FINITE ELEMENT MODEL FOR HALF OF SUCTION VALVE REED WITH 135 DEGREES OF FREEDOM

STATIC ANALYSIS

The static displacements and resulting stresses for a suction valve reed were computed. It was assumed that the gas drag force due to the pressure difference across the reed over the valve port could be concentrated at the centre of the port. Figure 2 illustrates the three dimensional displacement of the reed when just touching the stop and also after contact with the stop when a pressure difference of 10 bar existed across the valve.

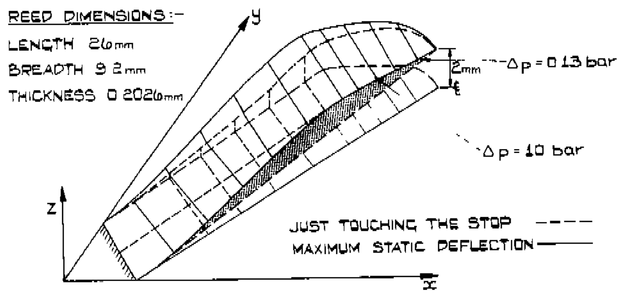


FIG. 2. STATIC DEFORMATION OF HALF SUCTION VALVE REED

At this impractically large pressure difference, the static deflection at the port was about the same as the permitted lift at the tip. Figure 3 shows the longitudinal and transverse static stress* pattern when the gas pressure difference across the valve was 2 bar, still a larger pressure difference than would be expected in the dynamic situation during compressor operation.

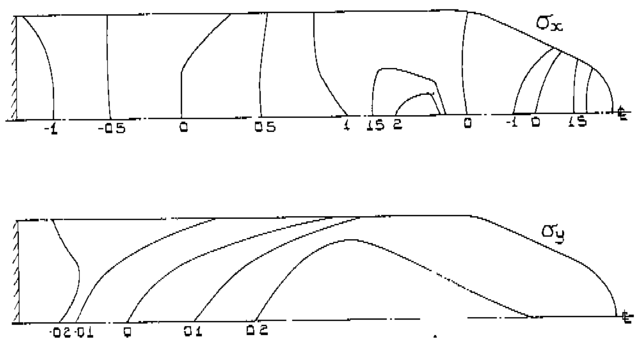


FIG.3 STATIC FIELD STRESS (N/mm^2) FOR HALF SUCTION VALVE REED AFTER TOUCHING THE STOP AT REED TIP PRESSURE LOADING 2 bar.

Examination of a range of conditions showed that stress levels increased with increase in diameter of the port and with increase in permitted lift at the reed tip (4). Figure 4 shows the effect of the point stop on static displacement and stress. The valve reed became considerably stiffer after contact. While stresses increased after contact this increase was small compared to the increase of stress which would occur with increasing lift in absence of a stop.

While the overall stress pattern and intensities appeared to be rational, the stresses in component elements of the model were not always so. This phenomenon was less pronounced when the number of elements was increased. While an exact solution cannot be obtained from such model, possibly triangular elements with six nodes would give improved accuracy.

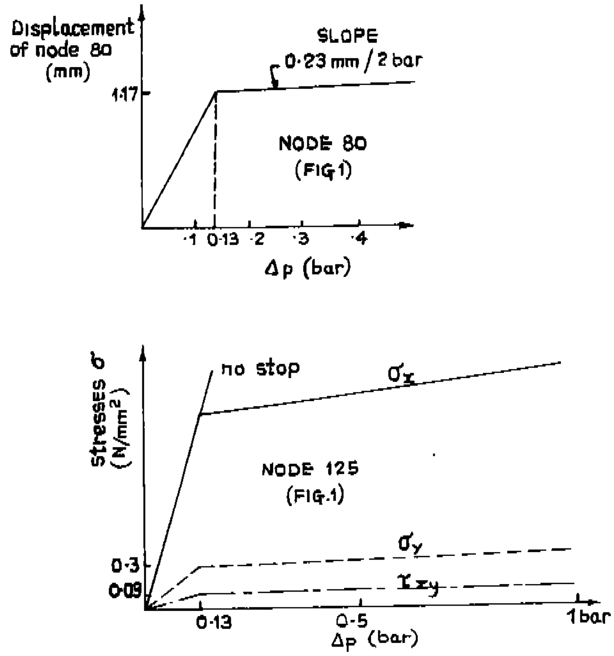


FIG. 4 EFFECT OF POINT STOP ON STATIC DISPLACEMENT AND STRESSES FOR CANTILEVER SUCTION VALVE REED

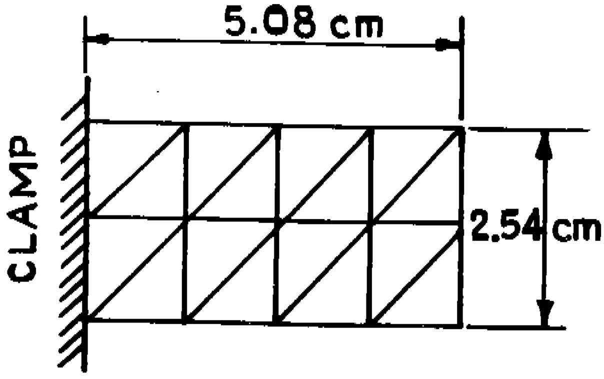
FREE VIBRATIONS

To assess the suitability of the program, DRST4, to apply the method of finite elements limits of error were estimated by (a) comparison between calculations using this program with results from other analytical methods and with values obtained by experiment for similar cases, (b) a study of convergence using progressively smaller elements.
For the rectangular plate illustrated in Figure 5, Table 1 shows that there was good agreement between the frequencies at the first four modes computed by the program and values quoted by Zienkiewitz (12).

Table 1

Mode	1st	2nd	3rd	4th
Program DRST4 (Hz)	838	3638	5283	11875
Zienkiewitz (Ref. 12)	846	3638	5266	11870
Discrepancy %	-1	0	0.3	0

* Values of stress are the usual bending and twisting moments per unit length expressed per mm^2 (10, 17). Maximum actual stresses on the surface of the reed are found by multiplying the values quoted by $6/h^2$ where h is the thickness of the reed.



thickness 0.254 cm
Youngs Modulus 20.69E6 N/cm^2
Poisson Ratio 0.3
density 0.00783 kg/cm^3

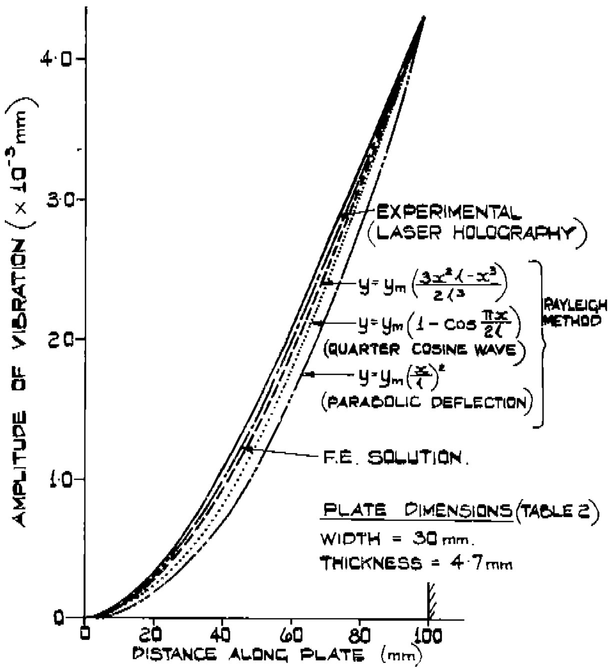
FIG 5 FINITE ELEMENT MODEL
FOR CANTILEVER PLATE

Table 2 contains values of the frequency of a similar rectangular clamped plate in its fundamental mode using the program (line 1), experimental values (line 2) using laser holography and values using the Rayleigh method (lines 3, 4, 5). The Rayleigh method yielded upper limits of natural frequency and was sensitive to the assumed nature of the deflection

Table 2

Line	Method	1st Mode (Hz)	Discrepancy % w.r.t. experiment
1	Program DRST4	399	7
2	Experiment	373	-
3	$y = y_m \left(\frac{x}{l}\right)^2$	506	36
4	$y = y_m \left(1 - \cos\left(\frac{\pi x}{2l}\right)\right)$	415	11
5	$y = y_m \left(\frac{2x^2 l - x^3}{2l^3}\right)$	404	8

The amplitude of vibration along the plate in the first mode is plotted, using these methods, in Figure 6. (The experimental hologram is included in Figure 6 by courtesy of Dr. A.J. Waddell). Good agreement is shown between the displacements at the fundamental mode predicted by the method of finite elements, the Rayleigh method and measured by laser holography.



STROBOSCOPIC HOLOGRAM OF THE RECTANGULAR CANTILEVER PLATE WITH THE FRINGES MARKED

FIG. 6. AMPLITUDE OF VIBRATION VS DISTANCE FROM CLAMPED ROOT FOR THE FIRST MODE SHAPE-RECTANGULAR CANTILEVER PLATE.

Values of frequency of the suction valve reed vibrating freely in its first few modes, both symmetric and antisymmetric are plotted against number of finite elements in Figures 7(a) and 7(b). The convergence of the values of the computed natural frequencies was good, even when the number of elements was relatively small. This rapid convergence, even with few elements, was expected. The assumed displacement state yields an element stiffness which is too large because the polynomials describing the internal displacement state were not complete (10). However, compatibility properties of the elements used result in an increase of flexibility. The two effects nearly cancel and fast, but non-monotonic convergence is a consequence of a differing ratio of change of these two effects over the range of mesh sizes used. Agreement with values calculated by the Rayleigh method (broken lines) was satisfactory.

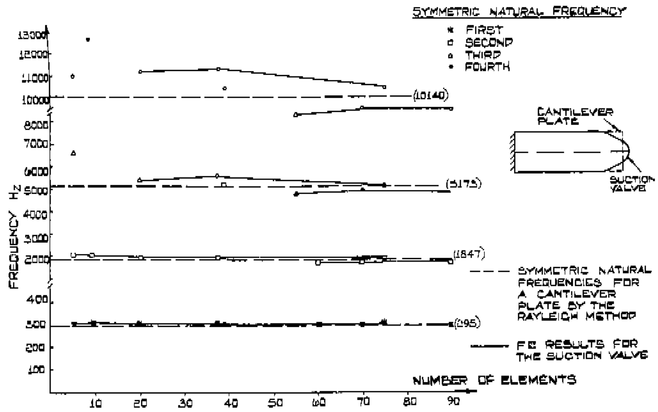


FIG. 7(a) CONVERGENCE AND COMPARISON OF THE PREDICTED SYMMETRIC NATURAL FREQUENCIES OF SUCTION VALVE REED WITH THE SYMMETRIC NATURAL FREQUENCIES OF CORRESPONDING RECTANGULAR CANTILEVER PLATE

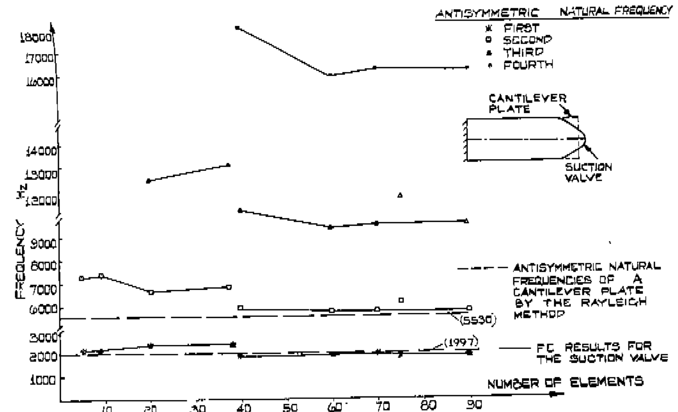


FIG. 7(b) CONVERGENCE AND COMPARISON OF THE PREDICTED ANTISYMMETRIC NATURAL FREQUENCIES OF SUCTION VALVE REED WITH THE ANTISYMMETRIC NATURAL FREQUENCIES OF A TYPICAL CANTILEVER PLATE

Values of symmetric frequency of the suction valve vibrating freely while its tip is in touch with the point stop are plotted against the number of finite elements used in Figure 7(c). The effect of clamping is examined by considering two degree of clamping is examined by considering two extreme cases. In one extreme the valve is fully clamped and in the other the valve is free to rotate at its root. The degree of clamping had more effect at the lower symmetric natural frequencies. The antisymmetric natural frequencies were hardly affected by the degree of clamping and do not change when the valve touches the stop.

There was some randomness in the values of natural frequencies predicted by the method of finite elements. In general, errors increased with increase in natural frequency but "rogue" values could occur at any mesh size having either smaller or larger than expected error. The cause of this irregular accuracy was possibly due to the fact that the finite number of d.o.f. associated with a particular mesh describe some mode shapes better than others. When the mesh is refined it is not always the same modes which are best described. It was concluded that finite element models having slight antisymmetric characteristics will generally represent antisymmetric modes better than symmetric modes. Conversely non-antisymmetric (i.e. symmetric) characteristics will represent symmetric better than antisymmetric modes.

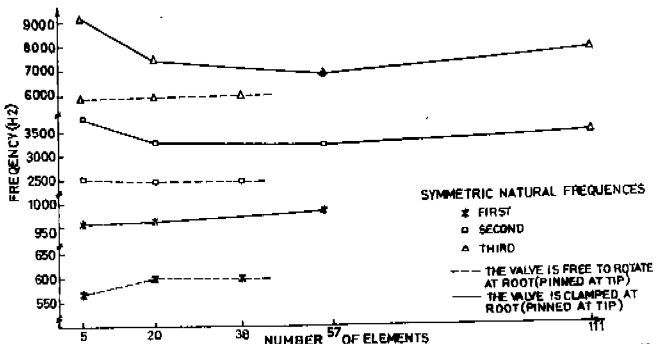


FIG. 7(c) EFFECT OF DEGREE OF CLAMPING & CONVERGENCE OF THE PREDICTED SYMMETRIC NATURAL FREQUENCIES OF THE SUCTION VALVE WITH STOP AT TIP

The natural frequencies predicted by the method of finite elements were sensitive to Young's Modulus: the effect of typical variations of Poisson Ratio and of material density was almost negligible. The degree of clamping has a significant effect (13). To investigate this effect while the valve was not in touch with the stop, extra elements were added at the line of clamping, the rigidity of these being varied (Figure 7(d)). The symmetric natural frequencies are affected with the effect more pronounced at the lower modes of vibration. Imperfect clamping lowers the lower symmetric natural frequencies by about 10%. Addition of extra elements at the reed root in the finite element model with Young's Modulus of about 8E8 - 5E9 instead of 20.69E7 kg/mms^2 makes allowance for the imperfection of clamping.

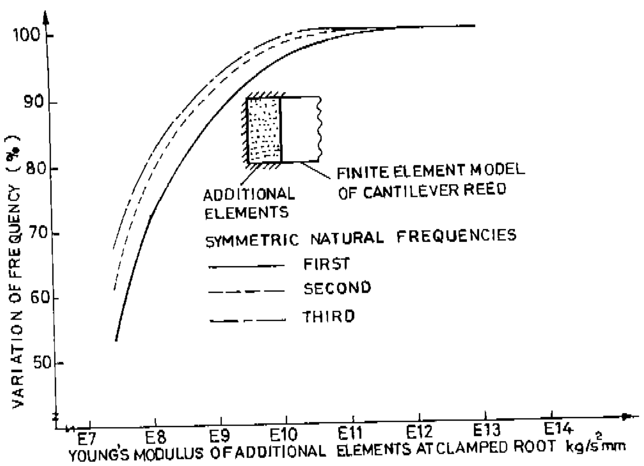


FIG. 7(d) VARIATION OF SYMMETRIC NATURAL FREQUENCIES OF THE SUCTION VALVE REED WITH DEGREE OF ROOT CLAMPING

RESPONSE TO DYNAMIC LOADING

Figure 8 shows the dynamic response of the suction valve reed, without a stop in place, as a consequence of an instantaneous gas impact load of 1 N applied at the centre of the port (Point A). The response is given for a range of values of constant c applied to the damping matrix (equation 2). The valve vibrates mainly in its first mode. Higher frequency vibrations disappear as the allowance for damping is increased.

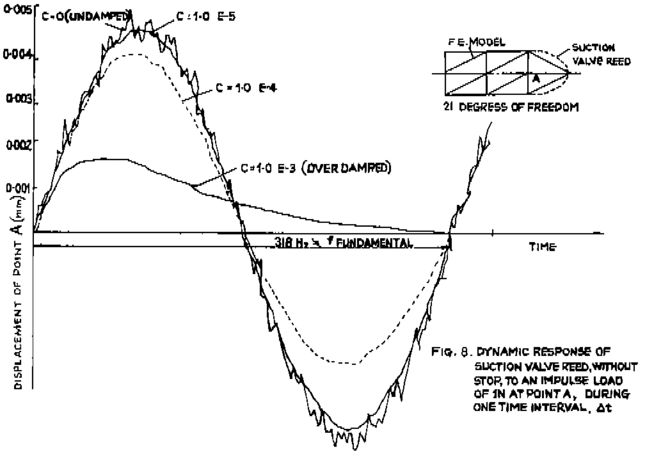


FIG. 8. DYNAMIC RESPONSE OF SUCTION VALVE REED, WITHOUT STOP, TO AN IMPULSE LOAD OF 1N AT POINT A, DURING ONE TIME INTERVAL, Δt

Figure 9 shows displacement of the suction valve reed at point A and at point B in excess of the displacement at these points when the valve tip first reached the stop. The pressure-time history across the reed during the suction process was predicted by the appropriate part of a simulation of the whole cycle in the compressor (7). Figure 9 indicates that relatively few mode shapes appear to have been excited. This might be expected because of the relatively few terms required to describe the applied gas pressure difference across a valve, particularly a suction valve.

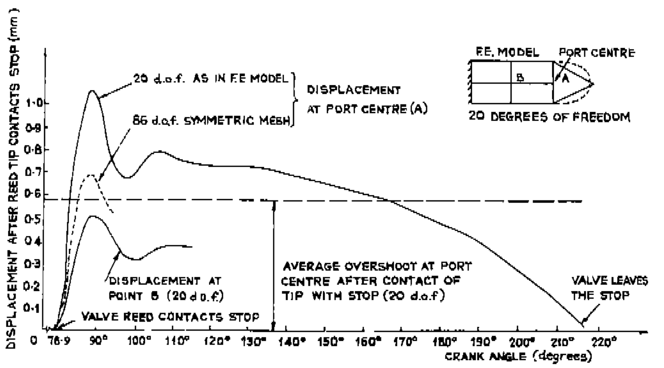


FIG. 9. SUCTION VALVE REED DISPLACEMENT AT THE CENTRE OF THE PORT AND AT POINT B AFTER THE VALVE REED TIP IS IN CONTACT WITH POINT STOP.

Figure 9 also indicates that the predicted displacements were sensitive to the number of degrees of freedom of the finite element model used. Hence a convergence study is needed to decide on an appropriate number of d.o.f. Figure 9 illustrates that the average reed displacement at the centre of the port was about twice the permitted lift at the tip. The maximum gas pressure difference across the valve during the suction process was 0.7 bar under these dynamic conditions: under static conditions the valve displacement at the valve port was only equal to the permitted lift at the tip (0.2032 mm) when a large gas pressure difference of 0.75 bar was applied across the valve. Hence the common assumption in simulation models that the lift over the port with a cantilever type suction reed equals the permitted lift at the tip is an over-estimate based on a static analysis but an under-estimate (by 60% on average in the case illustrated in Figure 9) under actual, i.e. dynamic, conditions. The suction valve tip velocity is shown in Figure 10. Any reasonable value of damping coefficient has little effect on the velocity at which the tip strikes the stop.

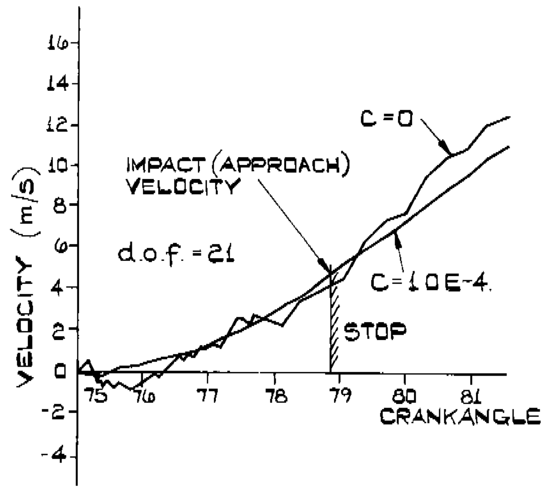


FIG.10 SUCTION VALVE TIP VELOCITY

Figure 11 shows different positions of the suction valve reed at different crankangles. This figure shows that at opening the valve tip leaves the valve plate last (position A) and strikes it first at closure (position H), predictions which should be checked by wear patterns on reed, stop and valve plate, following lift tests. In general the reed action is similar to that described by Gatecliff (1).

The theoretical stresses near the clamped root of the suction valve reed during free lift are shown in Figure 12. Longitudinal and transverse stresses increased greatly when the permitted lift was increased.

Figure 13 illustrates stress patterns during dynamic operation of the suction valve reed. The levels of stress were low. Slight under-estimate (10% to 15%) of the maximum stresses occurred if coarse meshes (d.o.f. 20) were employed. Strong violation of symmetry in the stress patterns was observed with non-symmetric meshes.

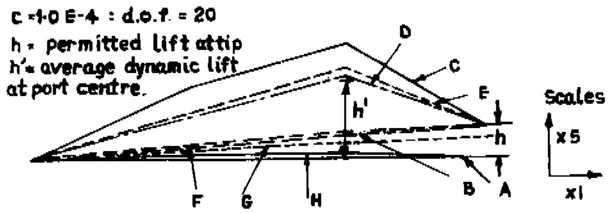


FIG.11. MOTION OF SUCTION VALVE REED

A : 75° : The pressure difference across suction valve has commenced
B : [illegible].9° : The valve tip touches stop at this crankangle (degrees after T.D.C.
C : 89° : Maximum valve displacement
D : 97° : Valve closing
E : 110° : Valve reopening
F : 227° : Valve leaves the stop
G : 239° : Valve closing
H : 242.5° : The reed tip touches the valve plate, but valve not yet fully closed (62.5 degrees after B.D.C.)

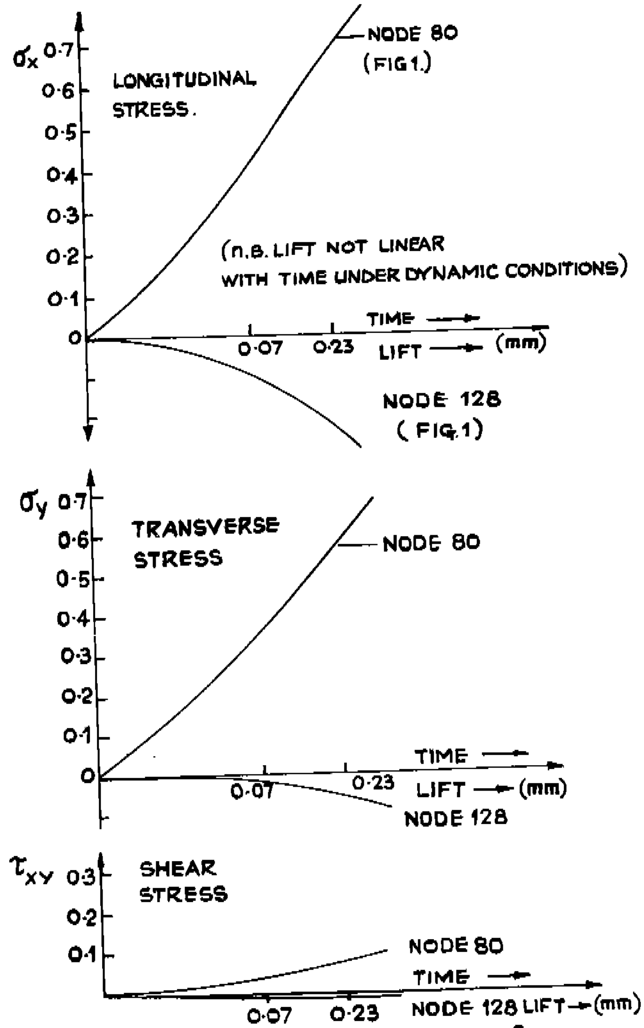


FIG.12. THEORETICAL STRESSES (N/mm²) VERSUS FREE LIFT FOR SUCTION VALVE REED

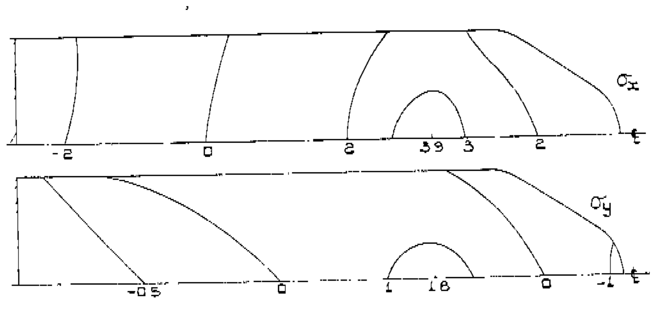


FIG. 13 MAXIMUM STRESS N/mm² FOR SUCTION VALVE REED DURING DYNAMIC OPERATION

The model predicts dynamic bending stresses that occur in the valve reed flexing under operating loads. Impact stresses are not predicted.

The same procedures were applied to a half-annular ring discharge valve in order to predict displacements and stresses under static loading, natural frequencies and mode shapes, followed by displacements and stresses under dynamic conditions. The boundary conditions as a consequence of the backing plate to such a reed makes the problem more complex. The boundary conditions and hence the matrices for the system change in a manner which is unknown a priori. It is intended to extend the program RESP95ST to account for the non-linear boundary condition for a reed with a backing plate. The use of programs such as PAFEC75 Level 3 will be considered (18).

CONCLUSIONS

Computer programs which applied the method of finite elements predicted the static and dynamic displacement and stress patterns together with the natural frequencies and mode shapes of a cantilever suction valve reed. Clamping and damping effects on the reed were allowed for. For the free vibration situation there was good agreement between the natural frequencies and mode shapes predicted by the method of finite elements, the Rayleigh method and those measured by laser holography. The small differences were a consequence of (a) imperfect clamping in the experiment if allowance for this imperfection was not made and (b) the Rayleigh method providing upper bounds to the calculated natural frequencies. Laser holography provided an accurate method for the experimental measurement of the free vibrations.

The results of the analysis under dynamic conditions when a prescribed pressure-time history in the compressor was applied to the valve reed, appeared to be sensitive to the number of degrees of freedom in the finite model used. Hence an analysis of convergence was necessary. Under dynamic conditions a range of reasonable values of damping did not have a significant effect on the velocity of impact at the stop. The valve overshot, reaching displacements which were about twice the magnitude at the port of the permitted lift at the tip. Maximum dynamic stress levels occurred just after the valve touched the stop but they were lower than the stress limits for materials normally used for valve reeds.

There was a significant difference between the reed displacement and stresses predicted by analysis based on (a) static and (b) dynamic conditions. Average displacements were twice and stresses were about 5 times larger than equivalent static values under the same maximum pressure difference across the valve at the port. Hence a dynamic analysis is justified. However, the computer time and capacity required is large and therefore expensive so that the study of a compressor and its valves should be made initially by using simpler models and the method of finite elements employed later to predict the behaviour of the valve reeds after most dimensions have been decided upon.

APPENDIX

Test Rig

The holographic interferometer used to measure the natural frequencies and mode shapes (e.g. Figure 14) consisted of:

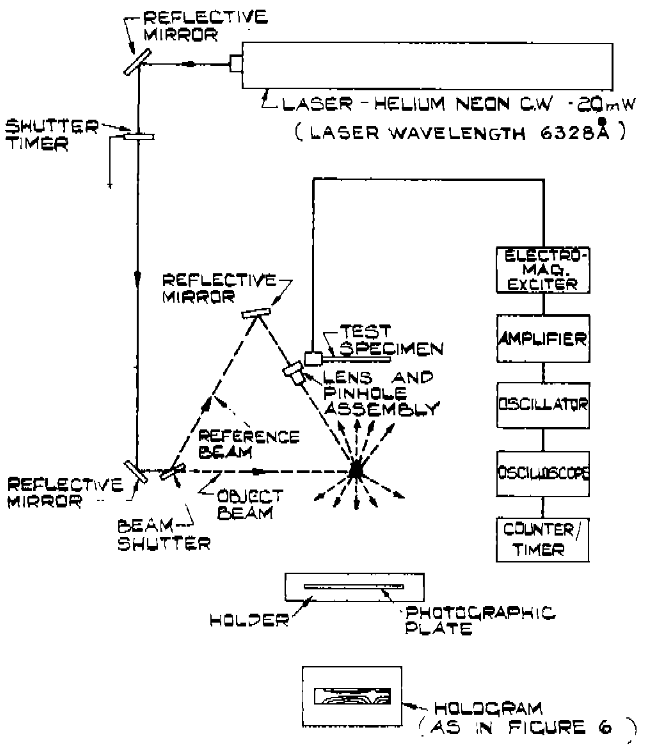


FIG 14 LAYOUT OF APPARATUS FOR HOLOGRAPHIC INTERFEROMETER

1. Laser - Helium-Neon (He - N_e) continuous wave (c.w.), power 20 mW.
2. Interferometer, comprising
   a) 25 mm diameter fully reflective coated mirrors.
   b) Shutter mechanism.
   c) Microscopic pin hole (15 micron).
   d) Real time holographic plate holder and half plate adaptor.
   e) Agfa-Gevaert Plate, Type 10E75.
   j) Automatic hologram processor containing Kodak D19 developer.
3. Excitation source, piezo-electric crystal driven by a Farnell Sine Square Oscillator, type LFM4, via a charge amplifier. A Racal Instruments Universal Counter, type 9901, monitored the excitation frequency applied to the specimen. A Telequipment, type D67 oscilloscope displayed both the input and the output from the specimen in order to identify a resonant condition.

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SYMBOLS

A, B	:	Valve points	
c	:	Damping constant	
D	:	Assembled damping matrix	
d.o.f.	:	Degrees of freedom	
E	:	Young's Modulus	(N/mm^2)
f	:	Natural frequency	(Hz)
K	:	Assembled stiffness matrix	
ℓ	:	Length	(mm)
m	:	Maximum (subscript)	
n	:	Number of degrees of freedom (subscript)	
T_n	:	Smallest period of a finite element model	(s)
x, y, z	:	Direction co-ordinates	
Δp	:	Pressure difference	(bar)
Δt	:	Time step	(s)
Δt_{cr}	:	Critical time step	(s)
ν	:	Poisson's Ratio	
ρ	:	Density	(kg/mm^3)
σ_x	:	Stress in the longitudinal (x) direction	(N/mm^2)
σ_y	:	Stress in the transverse (y) direction	(N/mm^2)
τ_{xy}	:	Shear stress in the x-y plane	(N/mm^2)

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