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ANALYSIS OF FLOW THROUGH ROOTS BLOWER SYSTEMS

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ABSTRACT

Kinematic analysis of a Roots blower with involute flanks and circular root and tip sections is explained. The thermo-fluid model for the blower is described. Two mathematical models have been used to model the blower and its piping system. In the first model flow is assumed steady in the pipes, whereas in the second the unsteady flow in the pipes is taken into consideration. Leakage from the pressure side to the suction side of the blower is treated with a simple model. The change of volumetric efficiency at different operating conditions is given. It is possible to investigate the property variations with time at different parts of the blower system with the computer program written. Qualitative comparisons of the results with experimental ones are satisfactory.

INTRODUCTION

American engineer Roots developed this version of positive displacement machines and showed some examples in 1867. Since then, because of their inherent simplicity Roots machines become widely used in many industrial applications. The capacity of these machines can go up to 20.000 lit/s. Roots blowers may be used in two stages and their compression ratios can go up to 2.5. Their rotational speed range is 600 - 3000 rpm. Due to the displacement type of operation of the blower, pulsating flow takes place in the suction and delivery pipes.

Various investigations have been carried out on Roots blowers. Their suitability as superchargers is investigated by Ryde [1]. The estimation of volumetric efficiency and leakage in Roots blower is an important factor which has been investigated by various researchers [2-7]. This investigation is on the modelling and solution of the flow through a Roots blower considering the unsteadiness of flow in all parts of a blower system. A computer program is developed for this purpose which could simulate Roots blower systems. Only qualitative comparisons was made possible due to the lack of well documented test cases in the open literature during the time of investigation.

KINEMATICS OF ROOTS BLOWER

The Roots blower is a positive displacement type compressor consisting two rotors rotating in opposite directions within a casing as shown in figure 1. The relative positions of the rotors are

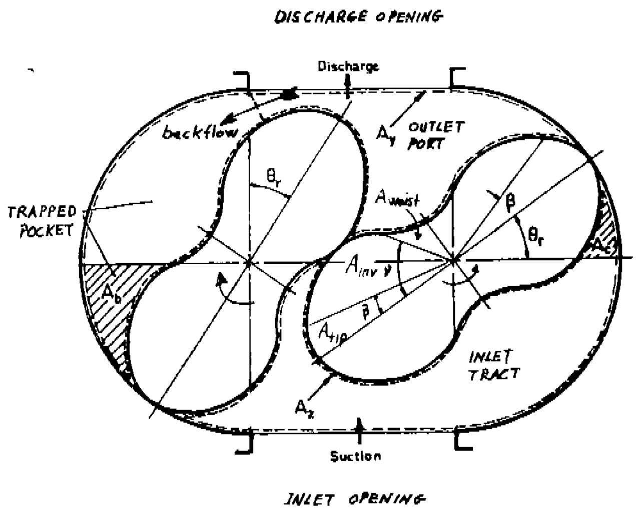

Figure 1. Roots Blower Geometry

maintained by a pair of gears in such a way that a fine clearance is present between rotors and casing. As the rotors rotate, the air is drawn into the space between the rotor and casing. Then, inlet air is trapped between the rotor and casing as the tip of the rotor passes the edge of the inlet opening. As the rotation continues the opposite tip of the rotor passes the edge of the outlet opening and the trapped air is pushed through the outlet opening. The space between the rotor and casing where gas is trapped is referred as "trapped pocket", whereas the space at suction and discharge sides of the blower are called "inlet tract" and "outlet port" respectively. The backflow from the outlet port to the trapped pocket is an important factor in the operation of the machine.

The general requirements of geometry are; that the machine should have a maximum possible displacement volume, no interference between the lopes to provide rotation, and split cylindrical form for easy machining. These requirements are met by a two lobe rotor of involute or cycloid flank profiles for single point of contact and no interference. In this investigation involute flank profile is chosen. The root and tip of the rotor is assumed to be in a circular form.

Geometry of the Roots Blower

For the geometry chosen, the machine can be described by 4 variables these are:

1. Pressure angle (ψ)
2. Distance between the centers
3. Rotor length (L)
4. Outlet and Suction opening dimensions.

Figure 2 shows the detailes of the involute rotor flanks. The pitch circle diameter of the timing gears is equal to the distance between the centers of the lobes. By definition, the locus of the center of curvature of involute curve is a circle, which is referred as base circle. The normal to the

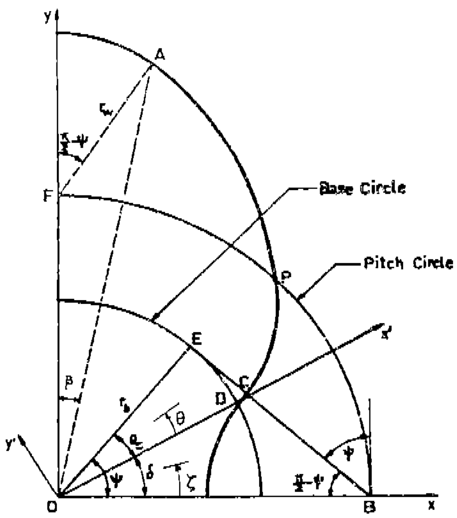

Figure 2. Involute Profile Rotor.

involute curve is tangent to the base circle at all times. The involute portion of the lobe profile may start from the base circle or from any point outside it. If the involute portion starts from the base circle the pressure angle is said to be critical.

During the operation, the involute flanks of the two lopes run in contact at all times as the pitch circles roll in contact without slipping. Using this condition, it is possible to write the following equations, for radius of circular root and tip portions r_w, base circle radius r_b and rotor diameter D.

$$r_b = r_p \, \mathrm{Cos}\psi \qquad (1)$$

$$r_w = r_b \frac{\pi}{4} \mathrm{Cos}\psi \qquad (2)$$

$$D = 2(r_p + r_w) \qquad (3)$$

Critical pressure angle becomes $\psi_{crit} = \tan^{-1} \pi/4$. The location of the starting point of involute profile relative to the rotor major and minor axis (x,y) is given by the following equations

$$x_c = r_b \, [(1/\mathrm{Cos}\psi) - \pi/4 \, \mathrm{Sin}\psi] \qquad (4)$$

$$y_c = r_b \, \pi/4 \, \mathrm{Cos}\psi \qquad (5)$$

The involute geometry angles θ_c and θ_A are obtained from

$$\theta_c = \tan\psi - \frac{\pi}{4} \qquad (6)$$

$$\theta_A = \tan\psi + \frac{\pi}{4} \qquad (7)$$

θ_c is shown in figure 2.

The area of the rotor lobe is calculated from the following equation

$$A_r = 4(A_{tip} + A_{inv} + A_{waist}) \qquad (8)$$

The differential areas of quarter lobe is shown in figure 1. From the geometry of blower following relations may be written between $\nu, \beta, \theta_A, \theta_C$ and pressure angle

$$\nu = \frac{\pi}{4} + \tan\psi - \psi - \theta_c + \tan^{-1}\theta_c \qquad (9)$$

$$\beta = \frac{\pi}{4} + \tan\psi - \psi - \theta_A + \tan^{-1}\theta_A \qquad (10)$$

The areas of the various portions of the lobe may be calculated using the following integrals

$$A_{inv} = \int_\theta \frac{1}{2} r_b \, \theta^2 \, d\theta \qquad (11)$$

$$A_{tip} = \int_\zeta \left(r_p \, \mathrm{Cos}\zeta + \sqrt{r_w^2 + r_b^2 \, \mathrm{Sin}^2\zeta}\right) d\zeta \qquad (12)$$

$$A_{waist} = \int_\zeta \left(r_p \, \mathrm{Cos}\zeta - \sqrt{r_w^2 - r_b^2 \, \mathrm{Sin}^2\zeta}\right) d\zeta \qquad (13)$$

where θ is measured from involute axis x', and ζ from rotor axis .

Blokage factor is defined as the ratio of the rotor cross sectional area to the area of a circle with diameter equal to the rotor diameter. The volume of air delivered in each cycle is proportional to the blockage factor. For maximum delivery blokage factor should be as low as possible. Blokage factor for involute flank lobes and circular arc tip and root sections may be calculated from

$$B = \frac{\cos^2\psi\ [\pi + \frac{2}{3}(\theta_A{}^3 - \theta_c{}^3)]}{\pi(1 + \frac{\pi}{4}\cos\psi)^2} \qquad (14)$$

It may be shown that blokage factor is minimum at critical pressure angle of 38.1° and increases as the pressure angle increases. At critical pressure angle blokage factor is 43%.

Backflow Geometric Data Determination

For the calculation of backflow rate between the outlet port and trapped pocket, it is required to calculate the variation of the cross-sectional area between the lobe and outlet port corner with the angular position θ_r. This cross sectional area is used as the throat area between two working volumes, trapped pocket and outlet port (See fig. 3).

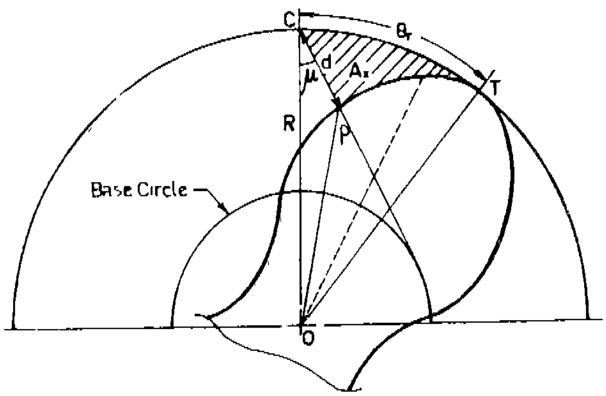

Figure 3. Interaction of Outlet Port and Trapped Pocket.

The corner distance d is obtained for the circular section from

$$d = \frac{R-(R-r_w)\cos\theta_r}{\cos\mu} - r_w \qquad (15)$$

where angle μ may be calculated from

$$\mu = \tan^{-1}\left[\frac{(R-r_w)\sin\theta_r}{R-(R-r_w)\cos\theta_r}\right] \qquad (16)$$

At the involute section the corner distance d can be obtained from

$$d = \frac{R-r'\cos\alpha}{\cos\mu} \qquad (17)$$

where $$r' = r_b\left[1 + (\nu + \frac{\pi}{2} - \mu - \theta_r)^2\right]^{1/2} \qquad (18)$$

$$\alpha = \frac{\pi}{2} - \mu - \tan^{-1}(\frac{\pi}{2} - \mu - \nu - \theta_r) \qquad (19)$$

$$\mu = \sin^{-1}(r_b/R) \qquad (20)$$

Working Volumes of the Roots Blower

In order to determine the change of porperties at the inlet and the outlet of a Roots blower it is necessary to know the crossectional areas of inlet tract, trapped pocket and outlet port, and their rate of change at each successive angular displacement. Figure 1 shows the Roots blower at a displacement angle of θ_r. Since the length of the blower L is constant, the analysis may be based on the areas between casing and lobes. For convenience, the summation of the areas of trapped pocket and outlet port is called A_y whereas the area of the inlet tract is designated by A_z. The total area between casing and the lobes is

$$A_t = \frac{\pi D^2}{4} + 2r_pD - 8(A_{tip} + A_{inv} + A_{waist}) \qquad (21)$$

From the symmetry of the blower the summation of areas A_z and A_y are constant and equal to A_t. The equation for A_y may be written as

$$A_y = [\frac{\pi D^2}{8} + Dr_p - A_r] + [A_b - A_c] \qquad (22)$$

The first part of the equation do not depend on the displacement angle θ_r. However, areas A_b and A_c as shown in the figure 1 are θ_r dependent. These areas are calculated from various combinations of areas computed using equations 11, 12, 13 and the quarter lobe area.

The trapped pocket has its maximum cross sectional area before it is exposed to the outlet port. The corner distance d is obtained by drawing a perpendicular to the lobe from the outlet port corner, forming a constriction between outlet port and trapped pocket. It is assumed that trapped pocket volume decreases by an amount of LA_x as the rotor rotates (See fig. 3). The volume V_x can be calculated from

$$V_x = \frac{L}{8} D^2\ \theta_r(\frac{1}{2} R\ d\ \sin\mu + A_{OTP}) \qquad (23)$$

Where A_{OTP} is the rotor segment shown in figure 3.

The variation of d and A_x during a cycle is shown in figure 4. The center distance of the blower is taken as 12.5 cm in this calculation.

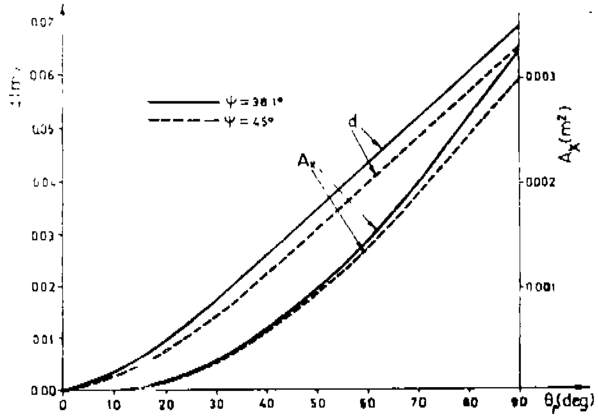

Figure 4. Variation of Backflow Distance and A_x with Displacement Angle.

MODELS FOR THERMO-FLUID ANALYSIS

The mathematical modelling of flow through the Roots blower is constructed by considering the variation of inlet tract, traped pocket and outlet port volumes with time. The increase and decrease of these volumes,and separation and integration of them,forces the fluid to be transferred from the suction to the discharge.

The modelling is done by assuming that the working fluid is perfect gas, all processes are adiabatic. Wave action is not considered in the inlet tract, outlet port and trapped pocket. Flow at the boundaries of the blower is assumed to be quasi-steady. Mixing processes are considered to proceed instantaneously to homogeneous equilibrium. It is also assumed that the flow to the throat of a constriction is isentropic. This assumption is used in calculating the backflow from the outlet port to the trapped pocket.

The time rate of change of pressure in the working volumes is obtained by using energy equation. The change of mass in these volumes is calculated through integrating the law of conservation of mass. The time rate of change of inlet tract, outlet port, and trapped pocket volumes are obtained by using numerical differentiation to the calculated volumes. The properties are integrated in time by using explicit integrating techniques. The angle (time) increment for integration is chosen to be one degree when wave action is not considered in the pipes. When unsteady flow is considered, the stability of the wave action solution dictates the time increment. The angle increment in this case was also in the order of one degree. No diverging solutions have been noted throughout the simulation runs.

The mass flow rate through any constriction is obtained from

$$\dot{m} = \frac{A\,P}{R\,T_o (P/P_o)^{\frac{k-1}{k}}} \sqrt{2c_p T_o \left[1 - \left(\frac{P}{P_o}\right)^{\frac{k-1}{k}}\right]} \qquad (24)$$

where A is the throat area P is the pressure at the constriction throat which is equal to the down stream pressure, P_o is the upstream pressure. As the leading lobe passes the outlet corner,back flow would occur which adds mass and energy into the trapped pocket This would increase the trapped pocket pressure. After a period of time the trapped pocket and outlet port pressures become same. At this condition the two volumes are considered as an integral unit.

Leakage Flow

The internal clearances of the blower form leakage paths causing the gas to flow from the delivery side back to the inlet side. Leakage reduce the volumetric efficiency of the blower. Thus it is important to estimate the leakage in a Roots blower. Three different leakage paths are present in the blower. Leakage flow may occur between the casing and lobes, between the rotors and between the rotors and casing end plates.

In modelling the leakage flow,it is assumed that the machine has no internal clearances but the leakage flow occurs from the outlet port into the inlet track through a nozzle with an equivalent area. This model of Cole et.al [7],allows for the thermal expantion of the construction. Expression used is as follows:

$$A_e = \phi\, A_o \left[1 + K(T_{op}/T_{it} - 1)\right] \qquad (25)$$

where ϕ is the contraction coefficient which is constant at all rotor positions, A_o is the total leakage area under cold conditions as found from clearance measurements. K is a coefficient which adjusts the leakage area for differential temperature effects. For leakage.flow it is assumed that the flow to the throat is isentropic. Mixing with the inlet tract gas is adiabatic, irreversible constant pressure process.

The performance parameters such as power, mass flow rate are calculated by integrating these quantities over the calculation time. The volumetric efficiency of the machine is based on the integrated mass flows through the machine and calculated from the integrated leakage flow from outlet port to the inlet tract and the net integrated mass flow through the machine.

Two mathematical models have been developed. In model A the blower is directly connected to two large receivers at the suction and delivery side. Model B allows for wave action in the suction and delivery pipes. The unsteady flow in the pipes is solved using the method of characteristics [10].

SOLUTION TECHNIQUE

In both mathematical models the suction and delivery receivers are assumed to have constant thermodynamic properties. The calculation is started from $\theta_r = 0$ position. The initial conditions in the suction pipe and inlet tract are taken to be same as the suction receiver. Initially the outlet port, traped pocket and discharge pipe conditions are taken as that of the discharge receiver.

Due to the cyclic nature of the process the calculation is extended until property variations between two cycles dissapear.

During model A calculations [8], blower suction and delivery side openings are considered as nozzles with isentropic flow through them. Since pipes are connected to the suction and delivery openings of the blower directly in case of model B, the well established open end boundary condition is utilized [9]. The boundary conditions, at the other ends of the pipes which are connected to large tanks at constant properties are also open end boundary conditions.

The computing time necessary for a model B solution of 360 degrees was found to be 80 seconds on a IBM 370/145 computer. Model A takes only 50 seconds for 360 degrees. It was observed that the computing time increases stightly with the pressure ratio.

DISCUSSIONS AND COMMENTS

Calculations are performed on a Roots blower with center distance of 12.7 cms. The length of the blower is also chosen as 12.7 cm. Critical pressure angle is used in most of the calculations. In case of model B suction and discharge pipe lengths are chosen to be 0.5 and 0.8 meters respectively. Several test runs are performed at different rotational speeds and pressure ratios (r). When leakage is introduced to the calculations, leakage area at cold conditions is taken as $A_0 = 0.000226\ m^2$. For compensating thermal expantion the constans in equation (25) are taken as $K = -0.242$ and $\phi = 0.54$.

Figure 5 shows the outlet port pressure fluctuations predicted by model A and B. The effect of leakage flow may be seen by comparing the top and the bottom figures. It must be noted that the amplitude of pressure fluctuations in the outlet port is effected by the unsteady flow in the discharge pipe. The leakage flow tends to reduce the amplitude when no wave action is considered in the discharge system.

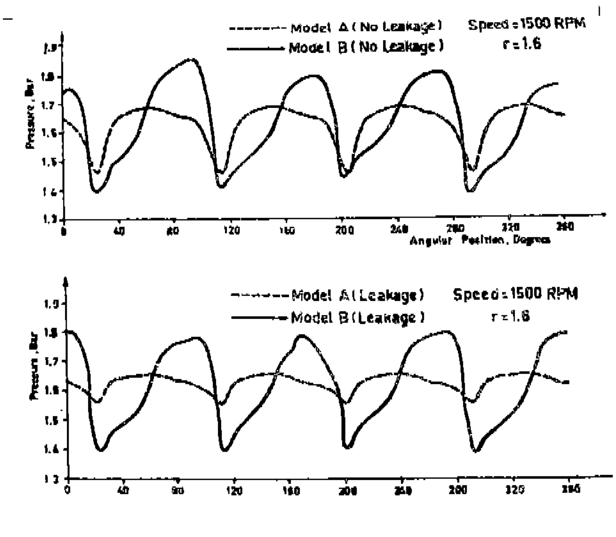

Figure 5. Variation of Outlet Port Pressure with θ_r.

Figure 6 shows the temperature fluctuations in the outlet port. Similar trend may be observed in this

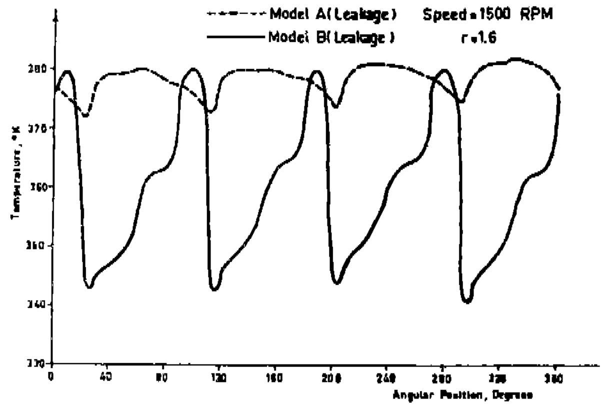

Figure 6. Variation of Outlet Port Temperature with θ_r.

as well. Figure 7 is an example of the predicted leakage flow rate during 180 degrees of revolution of a Roots blower. It is clearly seen that an appreciable amplitude increase in the variation is observed in case of Model B.

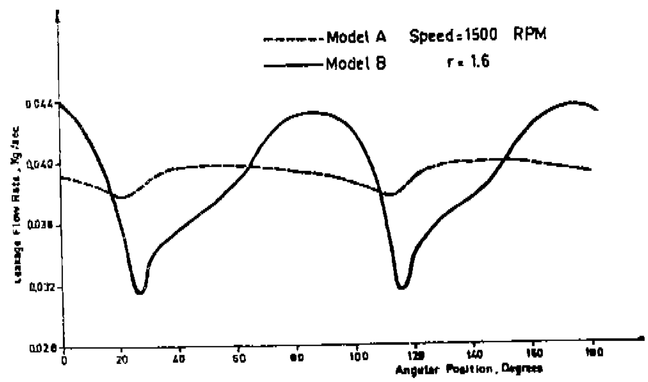

Figure 7. Variation of Leakage Flow Rate with θ_r.

Variation of predicted volumetric efficiency with pressure ratio and speed is given in figures 8 and 9. A qualitative comparison of these results with the ones existing in the open literature is satisfactory. It is felt that a satisfactory quantitative comparison is only possible if the leakage flow is predicted accurately. Due to the lack of well documented test case in open iterature no quantitative comparison is presented. However, experimental work will be started in a near future. Figure 10 shows the effect of speed on the mean leakage flow. The mean leakage flow increases

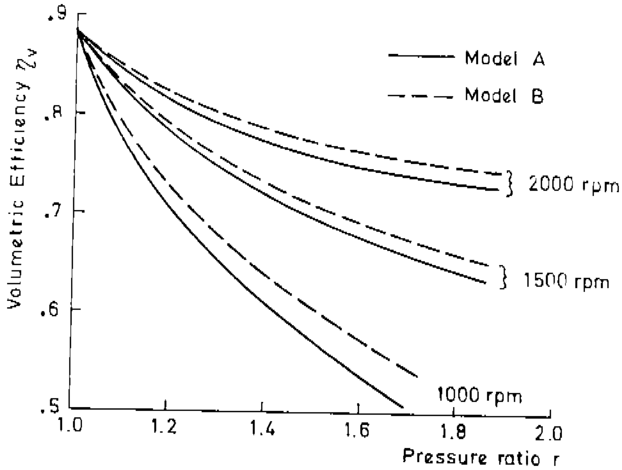

Figure 8. Variation of η_v with Pressure Ratio.

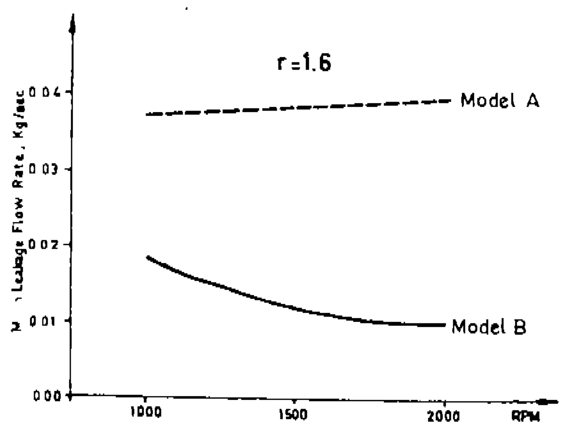

Figure 10. Variation of Leakage Flow with Rotor Speed.

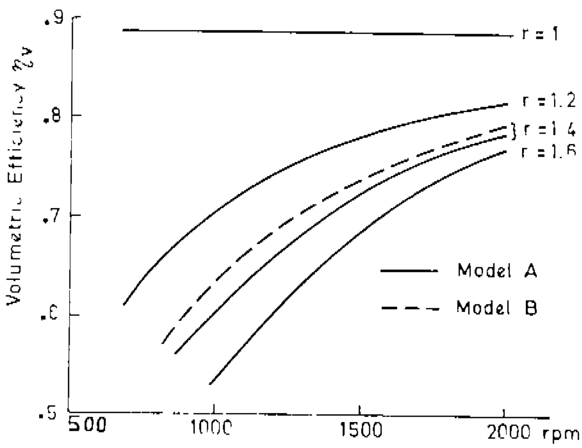

Figure 9. Variation of η_v with Rotor Speed.

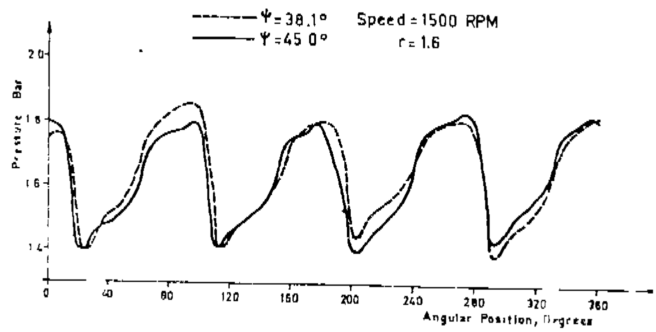

Figure 11. Effect of Pressure Angle on Outlet Port Pressure.

slightly in a linear manner with speed if no wave action is taken into account in the pipe system. However a reduction of mean leakage mass flow is seen for the case when wave action is considered. This is of course, due to the dynamic interaction between the blower and its pipe system. Figure 11 shows the effect of pressure angle on the pressure fluctuations. Slight variation of the wave shape is observed due to the change in the nature of the displacing volume.

The model, as it is, does not include heat transfer and heat generation due to friction. A quantitative comparison may need these effects to be introduced to the model. The assessment of leakage, considering each leakage path separately will lead to a better simulation. The existing computer program can be modified for this purpose easily.

NOTATION

A_b	Variable portion of areas A_y and A_z
A_c	Variable portion of areas A_y and A_z
A_o	Total leakage area at cold running conditions
A_r	Rotor cross sectional area
A_t	Total cross-sectional area between rotors and casing
A_{tip}	Area of half circular tip portion of rotor
A_{waist}	Area of half circular waist portion of rotor
A_{inv}	Area of involute portion of rotor
A_x	Trap pocket area decrease
A_y	Summation of trapped pocket and outlet port areas
A_z	Crossectional area of inlet tract
B	Blockage factor
c_p	Specific heat at constant pressure
D	Rotor diameter

d	Backflow corner distance
k	Ratio of specific heats
L	Rotor length
$\dot{m}$	Mass flow rate
p	Pressure
R	Gas constant, Rotor radius
r	Pressure ratio accross the blower
r'	Polar radius of involute geometry
r_b	Base circle radius
r_p	Pitch circle radius
r_w	Waist or tip circle radius
T	Temperature
t	Time
V	Volume
β	Angle (see figure 1)
η_V	Volumetric efficiency
ψ	Pressure angle
μ	Angle (see figure 3)
ν	Involute profile angle
θ	Involute geometry angle
θ_r	Displacement angle

Subscripts

o	Stagnation
it	inlet track
op	outlet port

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