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X. Liu
Carrier Corporation

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Xin Liu
Carrier Corporation, United Technologies
P.O.Box 4808, Carrier Parkway
Syracuse, NY 13221

ABSTRACT

The LMTD concept is based on the assumptions of a constant heat transfer coefficient and constant specific heat of fluids. However, in the phase change heat transfer, the assumption of constant specific heat does not apply and the heat transfer coefficient is not constant. This analysis shows that the average heat transfer coefficient based on the LMTD for those cases depends on the specific operation conditions, particularly, the temperature distribution or the vapor quality distribution. Therefore, one should be very cautious when comparing the average heat transfer coefficients based on the LMTD and when applying the average heat transfer coefficients determined on a specific condition to other conditions.

NOMENCLATURE

A	area
C	specific heat
h_{lg}	latent heat
l	length direction
L	total length of heat exchanger
$\dot{m}$	mass flow rate
P	pressure
Q	heat transfer rate
T	temperature
U	heat transfer coefficient
v	volume
x	vapor quality

Greek and symbols

Δ	difference
$(\pm)_{pc}$	positive for parallel flow and negative for counter-current flow
$(\pm)_x$	the sign defined in Table 1

Subscripts

c	cold
E	outlet side of hot fluid
g	vapor or gas
h	hot
l	liquid
o	inlet side of hot fluid
s	single phase
sat	saturation
tp	two-phase

INTRODUCTION

The concept of the log mean temperature difference (LMTD) has been widely used in heat transfer, especially in heat exchanger applications (Bowman *et al.*, 1940, and Kays and Landon, 1965). The LMTD is derived based on two major assumptions:

1. a constant heat transfer coefficient, U; and
2. a constant specific heat C.

Those are good approximations for a heat exchanger with single phase heat transfer on both sides. However, for a heat exchanger with two-phase flow on one side or both sides, the heat transfer coefficient,

$$U = \frac{1}{\frac{A_{tp}}{A_s h_s} + \frac{1}{h_{tp}}} , \tag{1}$$

is generally not constant. The magnitude of the heat transfer coefficient for two-phase flow can vary by a factor of several times as the vapor quality changes from 0 to 1. In addition, the specific heat is not directly related to the temperature change of the fluid because of the latent heat removal in the process. The average heat transfer coefficient of a heat exchanger is very often defined based on the LMTD temperature difference. For example, the LMTD has been used to determine the average heat transfer coefficient on the refrigerant side to compare the heat transfer characteristics of different refrigerants (Torikoshi *et al*, 1994, Sundaresan *et al*, 1994, Doerr & Pate, 1994). In some other practical cases (for example, the brazed heat exchanger), only the inlet and outlet temperatures are available, the intermediate temperature is difficult to measure, and the average heat transfer coefficient based on the LMTD is often used.

It becomes problematic when applying the LMTD to those situations with large quality changes between inlet and outlet to determine the average heat transfer coefficient. The application of the LMTD violates the assumptions from which it is derived. These assumptions guarantee that the heat transfer coefficient is independent of particular operation conditions. If we abandon these assumptions, then the questions we need to answer are the following:

1. does the average heat transfer coefficient based on the LMTD in those cases depend on the specific operation conditions?
2. what is the relationship between the average heat transfer coefficient and local heat transfer coefficient, and what kind of average of local heat transfer coefficients will give the equivalent average heat transfer coefficient based on the LMTD?

The following analysis try to answer the above two questions and shows that the average heat transfer coefficient based on the LMTD generally depend on the specific operation conditions.

PSEUDO-SPECIFIC HEAT

The specific heat is often defined for the situations without phase change as

$$C = \frac{1}{\dot{m}} \frac{dQ}{dT} \quad . \tag{2}$$

The heat associated with phase change, the latent heat, does not directly relate to the temperature change. However, the temperature of the fluid does change in a two-phase flow situation due to the pressure drop associated with the two-phase flow. Therefore, a pseudo-specific heat may be defined as

$$C = \frac{1}{\dot{m}} \frac{dQ}{dT} = \frac{dQ/dl}{\dot{m}\frac{dT}{dP}\frac{dP}{dl}} \ , \tag{3}$$

where dT/dP can be determined from Clapeyron equation:

$$\frac{dT_{sat}}{dP_{sat}} = \frac{T_{sat}(v_g - v_l)}{h_{lg}} \ , \tag{4}$$

and dP/dl can be determined from the two-phase pressure drop correlations (Carey, 1992), which are functions of vapor quality, x.

The term, dT/dP, determined from Clapeyron equation can be treated as approximately a constant since the change of saturation temperature, T_{sat}, is often not very large compared to the absolute saturation temperature. The two-phase pressure drop, dP/dl, roughly increases with vapor quality. If the heat transfer rate increases with vapor quality at the same rate as the pressure drop, the pseudo-specific heat is a constant. Otherwise, the pseudo-specific heat has to be considered as a function of vapor quality and other parameters.

To apply the LMTD to both single and two-phase heat transfer without modifications, the assumptions can be revised as

1. the heat transfer coefficient is constant;
2. the specific heat or pseudo-specific heat for both fluids are constant.

If either or both of the above assumptions does not hold, we need to investigate whether or not the average heat transfer coefficient based on the LMTD is independent of operation conditions and establish the relationship between the average heat transfer coefficient and local heat transfer coefficient.

VARIABLE HEAT TRANSFER COEFFICIENT

In this section we consider the situation in which the specific heat or the pseudo-specific heat defined in Equation 3 for both fluids is constant, and the heat transfer coefficient is not a constant, but a function of vapor quality. The heat transfer rate of two-phase flow can be related to the vapor quality change:

$$\begin{aligned} dQ &= U\Delta T dA \\ &= (\pm \dot{m} h_{lg})_x dx \ , \end{aligned} \tag{5}$$

where dA/dl is the surface area change with length and it is simply the perimeter for tubes with constant diameter, and the $(\pm)_x$ sign is determined by the flow arrangement and whether the fluid is cold or hot as shown in Figure 1 and Table 1.

Table 1 the sign of term $(\pm)_x$

Flow arrangemen	Fluid condition: Hot	Fluid condition: Cold
Parallel	-	+
Counter-current	-	-

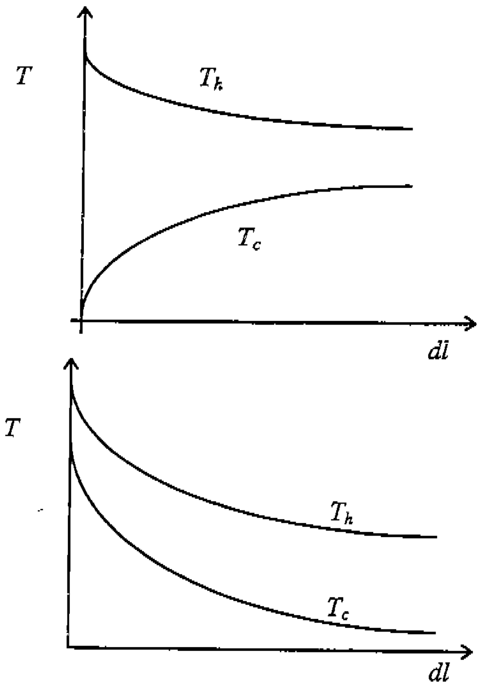

Figure 1. Parallel and counter-current flow arrangements

The temperature change in hot or cold fluid can be expressed as

$$dT = \frac{(\pm)dQ}{\dot{m}C} \quad , \tag{6}$$

where ± is determined by hot or cold fluid and flow arrangement. The temperature difference between the hot and cold fluids is

$$\begin{aligned} d\Delta T &= dT_h - dT_c \\ &= (\pm \dot{m}h_{lg})_x \left(-\frac{1}{\dot{m}_h C_h} - \frac{(\pm)_{pc}1}{\dot{m}_c C_c} \right) dx \ , \end{aligned} \tag{7}$$

where $(\pm)_{mc}$ is positive for a parallel flow and negative for a counter-current flow. Integration of the above equation for $x = x_o$ at $\Delta T = \Delta T_o$ gives

$$x = \frac{\Delta T - \Delta T_o}{(\pm \dot{m}h_{lg})_x \left(-\frac{1}{\dot{m}_h C_h} - \frac{(\pm)_{pc}1}{\dot{m}_c C_c} \right)} + x_o \ . \tag{8}$$

The heat transfer coefficient in two-phase flow heat transfer is a function of vapor quality. Using the above transformation from vapor quality to temperature difference, we can convert the heat transfer coefficient as a function of vapor quality to that as a function of the temperature difference:

$$U'(x) \rightarrow U(\Delta T) \tag{9}$$

and then

$$d\Delta T = \Delta TU(\Delta T) \left(-\frac{1}{\dot{m}_h C_h} - \frac{(\pm)_{pc}1}{\dot{m}_c C_c} \right) dA \tag{10}$$

Integrating and rewriting yields

$$\int_{\Delta T_o}^{\Delta T_E} \frac{d\Delta T}{\Delta TU(\Delta T)} = \frac{\Delta T_h - \Delta T_c}{Q} A \ . \tag{11}$$

If the heat transfer coefficient is a constant, we will obtain the LMTD. However, if the heat transfer coefficient is not a constant, the integration will not give the LMTD. If we force an average heat transfer coefficient defined based on the LMTD

$$Q = \overline{U} A \frac{\Delta T_h - \Delta T_c}{\ln\left(\frac{\Delta T_h}{\Delta T_c} \right)} \quad , \tag{12}$$

then the average heat transfer coefficient has to be

$$\overline{U} = \frac{\ln\left(\frac{\Delta T_h}{\Delta T_c} \right)}{\int_{\Delta T_o}^{\Delta T_E} \frac{d\Delta T}{\Delta TU(\Delta T)}} \tag{13}$$

and it is a function of the temperature difference distribution. For a variable heat transfer coefficient, this result suggests that the average heat transfer coefficient defined based on the LMTD would be different for different operation conditions.

Equation 13 also implies that, to optimize a heat exchanger design, it is important to minimize the integral in the equation, or, in other words, to optimize the distribution of the temperature difference in a heat exchanger.

Assume that the heat transfer coefficient can be simplified as a linear function of vapor quality

$$\begin{aligned} U(x) &= a' + b' x \\ &= a + b\Delta T \end{aligned}$$

where

$$a = a' + b' x_o + \frac{-b' \Delta T_o}{(\pm \dot{m} h_{lg})_x \left(-\frac{1}{\dot{m}_h C_h} - \frac{(\pm)_{pc} 1}{\dot{m}_c C_c} \right)}$$

$$b = \frac{b'}{(\pm \dot{m} h_{lg})_x \left(-\frac{1}{\dot{m}_h C_h} - \frac{(\pm)_{pc} 1}{\dot{m}_c C_c} \right)}$$

$$\int_{\Delta T_o}^{\Delta T_E} \frac{d\Delta T}{\Delta T U(\Delta T)} = \frac{1}{a} \ln \left(\frac{\Delta T_E}{\Delta T_o} \frac{U(\Delta T_o)}{U(\Delta T_E)} \right)$$

Finally the heat transfer rate is

$$Q = \frac{\Delta T_E - \Delta T_o}{\ln \left(\frac{\Delta T_E}{\Delta T_o} \frac{U(\Delta T_o)}{U(\Delta T_E)} \right)} aA \quad .$$

If $b=0$, then $U(\Delta T_o) = U(\Delta T_E) = a$ and we would have the LMTD from the above equation.

VARIABLE HEAT TRANSFER COEFFICIENT AND SPECIFIC HEAT ON BOTH SIDES

When both sides are two-phase flow and the pseudo-specific heat is not constant for both hot and cold fluids, we can rewrite the temperature difference as

$$d\Delta T = dT_h - (\pm)_{pc} dT_c = \left(\frac{dT_h}{dP_h} \right) \left(\frac{dP_h}{dl} \right) dl - (\pm)_{pc} \left(\frac{dT_c}{dP_c} \right) \left(\frac{dP_c}{dl} \right) dl \ , \quad (14)$$

where dT/dP can be determined by Clapeyron equation (Equation 4) and dP/dl can be determined from the pressure drop correlations. dP/dl is usually a function of vapor quality, as mentioned above. The term on the left, $d\Delta T$, can be written as

$$d\Delta T = \frac{d\Delta T}{dx} \frac{U \Delta T}{\left(\pm \dot{m} h_{lg} \right)_x} \frac{dA}{dl} dl \quad (15)$$

then substituting Equation 15 into Equation 14, we have the differential equation

$$\frac{d\Delta T}{dx} \frac{U \Delta T}{\left(\pm \dot{m} h_{lg} \right)_x} \frac{dA}{dl} = \left(\frac{dT_h}{dP_h} \right) \left(\frac{dP_h}{dl} \right) - (\pm)_{pc} \left(\frac{dT_c}{dP_c} \right) \left(\frac{dP_c}{dl} \right) \ . \quad (16)$$

Integrating for $x = x_o$ at $\Delta T = \Delta T_o$ and $x = x_E$ at $\Delta T = \Delta T_E$, we then have

$$\frac{\Delta T_E^2 - \Delta T_o^2}{2} = \frac{Q}{(x_E - x_o)} \int_{x_o}^{x_E} \frac{1}{U \frac{dA}{dl}} \left[\left(\frac{dT_h}{dP_h} \right) \left(\frac{dP_h}{dl} \right) - (\pm)_{pc} \left(\frac{dT_c}{dP_c} \right) \left(\frac{dP_c}{dl} \right) \right] dx \quad (17)$$

Taking the same definition of average heat transfer coefficient based on the LMTD as in Equation 12, the average heat transfer coefficient takes the following form

$$\overline{U} = \frac{\left(\Delta T_E + \Delta T_o \right) (x_E - x_o) \ln \left(\frac{\Delta T_E}{\Delta T_o} \right)}{\int_{x_o}^{x_E} \frac{1}{U \frac{dA}{dl}} \left[\left(\frac{dT_h}{dP_h} \right) \left(\frac{dP_h}{dl} \right) - (\pm)_{pc} \left(\frac{dT_c}{dP_c} \right) \left(\frac{dP_c}{dl} \right) \right] dx} \quad (18)$$

Obviously, similar to the case with variable heat transfer coefficient only, the average heat transfer coefficient based on the LMTD depends on the specific flow arrangement. It is a function of many parameters and is different from that with constant specific heat in several aspects:

1. The effect of pressure drop is coupling with U and dA/dl;

2. For a constant local heat transfer coefficient, the average heat transfer coefficient defined based on the LMTD is not equal to the local heat transfer coefficient, and would be different from the local heat transfer coefficient by a factor

$$\frac{\left(\Delta T_E + \Delta T_o \right) (x_E - x_o) \ln \left(\frac{\Delta T_E}{\Delta T_o} \right)}{\int_{x_o}^{x_E} \frac{1}{\frac{dA}{dl}} \left[\left(\frac{dT_h}{dP_h} \right) \left(\frac{dP_h}{dl} \right) - (\pm)_{pc} \left(\frac{dT_c}{dP_c} \right) \left(\frac{dP_c}{dl} \right) \right] dx}$$

VARIABLE HEAT TRANSFER COEFFICIENT AND VARIABLE SPECIFIC HEAT ON ONE SIDE

Assume a single phase flow on cold side and a two-phase flow on hot side,

$$
\begin{aligned}
d\Delta T &= dT_h - (\pm)_{pc} dT_c \\
&= \left(\frac{dT_h}{dP_h}\right)\left(\frac{dP_h}{dl}\right)dl - (\pm)_{pc}\left(\frac{U\Delta T}{\dot{m}h_{\mathrm{lg}}}\right)\left(\frac{dA}{dl}\right)dl \ . \qquad (19)
\end{aligned}
$$

Inserting Equation 15 into the above equation yields

$$
\frac{d\Delta T}{dx} = \frac{\left(\pm \dot{m}h_{\mathrm{lg}}\right)_x \left(\frac{dT_h}{dP_h}\right)\left(\frac{dP_h}{dl}\right)}{U(x)\Delta T \frac{dA}{dl}} - (\pm)_{pc}\frac{\left(\pm \dot{m}h_{\mathrm{lg}}\right)_x}{\dot{m}_c C_c} . \qquad (20)
$$

Equation 20 is a non-linear differential equation. The solution of this equation depends on the special forms of $U(x)$ and dP/dl. In the following we do not attempt to give a general solution, but instead, consider some special cases. We non-dimensionalize Equation 20 by letting

$$
\Delta T' = \frac{\Delta T}{\Delta T_a} \quad U' = \frac{U}{U_a} \quad P' = \frac{P}{P_a} \quad l' = \frac{l}{L} \ , \qquad (21)
$$

where the subscript a represents the characteristic parameter. Here we assume that the characteristic vapor quality is 1. Equation 20 can then be written as

$$
\begin{aligned}
&\frac{d\Delta T'}{dx} = E\frac{\left(\frac{dP_h'}{dl'}\right)}{U'(x)\Delta T'} - B \\
&\Delta T' = \frac{\Delta T_o}{\Delta T_a} \quad \text{at } x = x_o \ ,
\end{aligned} \qquad (22)
$$

where

$$
\begin{aligned}
E &= \frac{\left(\pm \dot{m}h_{\mathrm{lg}}\right)_x \left(\frac{dT_h}{dP_h}\right)\frac{P_a}{L}}{\frac{dA}{dl}\Delta T_a^2 U_a} \\
B &= (\pm)_{pc}\frac{\left(\pm \dot{m}h_{\mathrm{lg}}\right)_x}{\Delta T_a \dot{m}_c C_c}
\end{aligned}
$$

If

$$
E \ll B
$$

which means the temperature change on the side of two-phase flow is much smaller than that on the side of single-phase flow, we can expand $\Delta T'$ in terms of E, namely,

$$
\Delta T' = \Delta T'_o + E\Delta T'_1 + \ldots \qquad (23)
$$

$$
\frac{d(\Delta T'_o + A\Delta T'_1 + \ldots)}{dx} = E\frac{\left(\frac{dP_h'}{dl'}\right)}{U'(x)\left(\Delta T'_o + A\Delta T'_1 + \ldots\right)} - B \qquad (24)
$$

By balancing the terms of equal order in E, We have a zero order system of the form

$$
\begin{aligned}
&\frac{d\Delta T_o'}{dx} = -B \\
&\text{B.C.} \quad \Delta T_o' = \frac{\Delta T_o}{\Delta T_a} \quad \text{at } x = x_o \\
&\Delta T_o' = -Bx + \frac{\Delta T_o}{\Delta T_a}
\end{aligned} \qquad (25)
$$

and a first order system of the form

$$
\begin{aligned}
&\frac{d\Delta T_1'}{dx} = \frac{\left(\frac{dP_h'}{dl'}\right)}{U'(x)\Delta T'_o} \\
&\text{B.C.} \quad \Delta T_1' = 0 \quad \text{at } x = x_o \\
&\Delta T_1' = \int_{x_o}^{x} \frac{\left(\frac{dP_h'}{dl'}\right)}{U'(x)(-Bx)}dx
\end{aligned} \qquad (26)
$$

A solution to systems 25 and 26 may be written as

$$
\begin{aligned}
\Delta T' &= \Delta T_o' + E\Delta T_1' + \ldots \\
&= -Bx + E\int_{x_o}^{x} \frac{\left(\frac{dP_h'}{dl'}\right)}{U'(x)(-Bx)}dx + \frac{\Delta T_o}{\Delta T_a} + \ldots
\end{aligned} \qquad (27)
$$

The average heat transfer coefficient based on the LMTD can be calculated as

$$
\begin{aligned}
\overline{U} &= \frac{Q}{A\frac{\Delta T_E - \Delta T_o}{\ln\left(\frac{\Delta T_E}{\Delta T_o}\right)}} \\
&= \frac{(\pm \dot{m}h_{\mathrm{lg}})_x (x_E - x_o)\ln\left(\frac{\Delta T_E}{\Delta T_o}\right)}{AT_a\left[-B(x_E - x_o) + E\int_{x_o}^{x_E}\frac{\left(\frac{dP_h'}{dl'}\right)}{U'(x)(-Bx)}dx + \ldots\right]}
\end{aligned} \qquad (28)
$$

The average heat transfer coefficient in this case depends on the heat transfer coefficient distribution along the vapor quality distribution.

A similar procedures can be used for $E>>B$, which represents the case in which the temperature change on the two-phase side is much larger than that on the single phase side. When E and B are of the same order of magnitude, the problem is more complicated. For some special case, we may have an analytical solution as shown in the following example.

Assume $dA/dl=const$ and $(dP_h/dl)/U(x)$ can be linearized as $ax+e$, then we have

$$\frac{d\Delta T}{dx}=\frac{ax+e}{\Delta T}-b \quad ,$$

where

$$a=\frac{\left(\pm\dot{m}h_{lg}\right)_x\left(\frac{dT_h}{dP_h}\right)}{\frac{dA}{dl}} \qquad b=(\pm)_{pc}\frac{\left(\pm\dot{m}h_{lg}\right)_x}{\dot{m}_c C_c}$$

$$e=\frac{\left(\pm\dot{m}h_{lg}\right)_x\left(\frac{dT_h}{dP_h}\right)}{\frac{dA}{dl}}$$

Then the differential equation can be integrated as (Davis, 1962)

$$\ln\left(\frac{x+\frac{e}{a}}{x_o+\frac{e}{a}}\right)=\int_{\frac{\Delta T_o}{x_o+\frac{e}{a}}}^{\frac{\Delta T}{x+\frac{e}{a}}}\frac{v}{a+bv-v^2}dv \quad .$$

We might not get an explicit expression for ΔT for this case and numerical procedure might be required to calculate the average heat transfer coefficient.

CONCLUSION

The analysis presented in this paper shows that

1. the average heat transfer coefficient based on the LMTD for those situations with variable heat transfer coefficient or/and variable spcific or pseudo-specific heat depends on the specific operation conditions. The average heat transfer coefficient would be different in different conditions for the same local heat transfer coefficient. Therefore, one should be cautious when comparing the average heat transfer coefficients from different sources and when applying the average heat transfer coefficients to other conditions.

2. the relationship between the local heat transfer coefficient and the average heat transfer coefficient has been derived for the following cases:

a) variable heat transfer coefficient and constant specific heat or pseudo-specific heat (Equation 13);

b) variable heat transfer coefficient and variable pseudo-specific heat for both fluids (Equation 18);

c) variable heat transfer coefficient and variable pseudo-specific heat for one fluid and constant specific or pseudo-specific heat for the other fluid in some special cases.

This analysis assumes that the variable heat transfer coefficient is a function of vapor quality only. If the heat transfer coefficient depends on other variables, the above analysis has to be modified. For example, in convective evaporating heat transfer, the heat transfer coefficient is a function of heat flux. The heat flux can be expressed by the derivative of vapor quality:

$$q''=\frac{dQ}{dA}=\frac{(\pm\dot{m}h_{lg})}{\frac{dA}{dl}}\frac{dx}{dl} \quad .$$

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