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Optimization of Compressor Design Including Economic Factors

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ABSTRACT

A new approach to choosing the optimum design of reciprocating compressors is discussed. Economic criteria are included together with optimisation procedures which account for thermodynamic criteria of performance. These procedures are applied to a mathematical model of a compressor to produce a “best” design. Such factors as initial cost, interest rates, depreciation, inflation, running costs, load factors and compressor life can either be applied to the results of a completed optimisation based on thermodynamic criteria only or incorporated directly into the procedures and permitted to affect the optimisation during the “search” phase. It is demonstrated that economic factors affect the “best” design and that thermodynamic efficiency may require to be sacrificed to achieve the optimum overall efficiency.

INTRODUCTION

To quote from reference (1). “The compressor design process is a complex procedure. The designer must select values for the design variables, i.e. bore and stroke sizes, valve port diameters and locations, etc, to produce a compressor of specified capacity with highest Coefficient Of Performance (COP) and lowest cost. The interacting effects of the variables are not simple and thus determining the “best” values of the variables is a complex task. To analyse the compressor design, the designer has historically resorted to building a prototype compressor. Using his experience and intuition, the designer successively modified the compressor and retested it. This “cut and try” sequence generally required several stages before an acceptable compressor was developed and was expensive and time consuming. To reduce the amount of compressor testing and development time, compressor designers are now using compressor modelling and simulations to reduce the amount of testing. The simulations allow the design engineer to evaluate design decisions quickly. The logical next step in compressor design has been the use of univariant searches to attempt to optimize the compressor design in terms of improved performance or some other objective. Univariant searches consist of keeping all design variables fixed except one. The simulation is then run for different values of the free variable. Once the optimum value of the free variable is found it is fixed and the univariant search is repeated with another variable as the free variable. This process is then repeated until all variables have been searched. Univariant searches do not continuously account for the interaction of design variables. To achieve the true optimum it is more efficient to consider all the variables simultaneously with the variable interactions taken into account continuously. Optimization or “non-linear programming” technology permits an analysis of this type. The objective function and constraint functions can be highly nonlinear functions of several design variables thus making nonlinear programming an efficient way to consider the many variables of the compressor simulation.”

Papers presented at Purdue Compressor Technology Conferences (1, 2, 3, 4, 5) have considered the optimisation of compressor design to meet thermodynamic criteria (within the practical constraints which must be imposed). If the thermodynamic performance and reliability of compressors from various manufacturers show little difference in thermodynamic performance then economic considerations become dominant. The present paper is an initial attempt to involve economic criteria.

ECONOMIC FACTORS

The inclusion of economic factors was approached from two standpoints, an indirect method and a direct method. In the indirect method an adequate mathematical model of the compressor is used with an appropriate search routine to maximise an objective function which, within the constraints imposed, gives the “best” thermodynamic design; economic factors are then applied to the results of the search. With the direct method both economic and thermodynamic criteria are included in an overall objective function. Hence both economic and thermodynamic variables are permitted to influence the design during the search. Based on the premise that economic factors are likely to change more rapidly due to market conditions than thermodynamic criteria and constraints, the indirect method has the advantage that a design optimised on thermodynamic criteria can be considered as completed and can have changing or different economic factors.
accounted for at a later date without expensive computer resources. In the direct method all the variables are permitted to interact during the search to achieve the best overall design; if the economic criteria change then the whole optimisation procedure has to be rerun. The direct method presupposes that all the relevant variables can be identified and quantified at the outset, together with the constraints on them. It was shown (5) that it is difficult to specify a suitable objective function even when the indirect method is used wherein only thermodynamic criteria have to be defined; the problem is even greater with the direct method. Since market forces increasingly press for better thermodynamic performance at less cost, it is considered that either or both of the methods will be applied increasingly.

THE INDIRECT METHOD

Figure 1 shows the initial cost of air compressors from eight manufacturers, plotted against size (swept volume). The cost of a simple single stage compressor with discharge pressure 100 lbf/in² gauge but with no special features has been estimated by the broken line. (The several compressors above the broken line are the more expensive non-simple machines, e.g. oil free, two stage, etc.)

Figure 2 is a plot of optimum specific capacity (ft³/min hp) versus compressor swept volume (Vsw) for an air compressor operating at compressor pressure ratio 7.9, i.e. discharge pressure 100 lbf/in² gauge. This field diagram (Figure 1b in Reference (5)) was obtained by optimising thermodynamic parameters only. Diagrams such as Figure 2 include a series of optimised designs which have been obtained with compressor speed and swept volume held at fixed values. Thus lines of constant speed and equal compressor capacity become available (e.g. 400 rev/min and 24.44 ft³/min) and it is then possible to identify the different combinations of speed and swept volume which will provide a particular capacity. The advantage of the procedure used, in addition to being relatively economical of computer time, is that it allows a range of capacities to be examined from this single field diagram. In Figure 2 performance continues to improve as compressor speed is reduced suggesting that further improvement in performance could be gained at even lower speeds; this result suggests that consideration of thermodynamic criteria alone is inadequate. Machines with very low speed may have better thermodynamic performance (and longer life) but are commercially unattractive due to increased weight and cost. (The optimisation procedures used (5) permit an objective function to be evaluated close to the practical constraints which are not to be exceeded ("inequality constraints"). However, if procedures are used which can handle "equality constraints" then, at the price of increased computer resources, the best design to meet a specified duty could be found, with swept volume and compressor speed also free to float as variables during the optimisation.)

Some lines of constant speed and of constant gas throughput are shown in Figure 2. At a chosen design value of gas throughput, e.g. 24.44 ft³/min the minimum power requirement may be ascertained for each speed in the range 600 to 300 rev/min. Thus the running cost at a design point can be determined. Additionally for each optimum design point A, B, C and D the swept volume is available from Figure 2 and the initial cost of the compressor from the broken line on Figure 1. The reference compressor had a bore of 6 in and a stroke of 4½ in (Vsw, = 127 in³). Hence for point A, at 600 rev/min, the optimum specific capacity was 4.405 ft³/min hp, the total power 5.548 hp, the swept volume Vsw = 99 in³ giving an initial cost.
of £425. In Table 1 points A, B, C, and D are compared in an incremental fashion.

Optimum Thermodynamic Point (Fig. 2)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed, rev/min</td>
<td>600</td>
<td>500</td>
<td>400</td>
<td>300</td>
</tr>
<tr>
<td>Stroke Volume, in³</td>
<td>99</td>
<td>108</td>
<td>132</td>
<td>170</td>
</tr>
<tr>
<td>Capacity, ft³/min</td>
<td>24.44</td>
<td>24.44</td>
<td>24.44</td>
<td>24.44</td>
</tr>
<tr>
<td>Initial cost, £</td>
<td>425</td>
<td>450</td>
<td>512</td>
<td>615</td>
</tr>
<tr>
<td>Change in £ initial cost, (X)</td>
<td>0</td>
<td>25</td>
<td>62</td>
<td>103</td>
</tr>
<tr>
<td>Optimum specific capacity, ft³/min hp</td>
<td>4,405</td>
<td>4,540</td>
<td>4,654</td>
<td>4,762</td>
</tr>
<tr>
<td>Power input, hp</td>
<td>5.548</td>
<td>5.383</td>
<td>5.251</td>
<td>5.132</td>
</tr>
<tr>
<td>Change in power, hp</td>
<td>0</td>
<td>0.165</td>
<td>0.132</td>
<td>0.119</td>
</tr>
<tr>
<td>Change per year, £ (100% LF)</td>
<td>0</td>
<td>32.34</td>
<td>25.87</td>
<td>23.32</td>
</tr>
<tr>
<td>A/Z = r (100% LF)</td>
<td>0</td>
<td>1.294</td>
<td>0.417</td>
<td>0.226</td>
</tr>
<tr>
<td>Change datum</td>
<td>AB</td>
<td>BC</td>
<td>CD</td>
<td></td>
</tr>
</tbody>
</table>

Table 1 Effect of accounting indirectly for economic factors on the thermodynamic optimisation field diagram in Figure 2 (for capacity 24.44 ft³/min.)

From equation (A1) in Appendix A, the number of years required to balance each different initial cost with the corresponding running costs can be calculated. Thus, assuming a 100% load factor, a change from A to B breaks even in 0.84 years, a change from B to C breaks even in 2.65 years and a change from C to D breaks even in 4.90 years. For the calculation the monetary interest rate was taken to be 10%. (A limited working week of 40 hours and a year of 50 weeks is equivalent to an annual load factor of 22.8%) The number of years to break even over the range of load factors at the four compressor speeds can be determined and is plotted in Figure 3.

If additional capital is invested in an efficient compressor with a high specific capacity the investment would pay a dividend after a calculable period of years. Alternatively for 'pay-off' periods of say 5, 10 and 20 years the load factors to 'break-even' for each of the optimised designs can be calculated as illustrated in Table 2. A longer pay-off period makes the use of a more efficient machine more profitable for a particular load factor. Hence it is important to estimate what the load factor is likely to be during the assumed life of the compressor.

Table 2 Load Factor range for 3 pay-off periods (see Fig. 3)

<table>
<thead>
<tr>
<th>Period (years)</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimised Design (rev/min)</td>
<td>Load Factor Range (%)</td>
<td>X x L/F.</td>
<td></td>
</tr>
<tr>
<td>A (600)</td>
<td>0 - 16.8</td>
<td>0 - 8.4</td>
<td>0 - 4.2</td>
</tr>
<tr>
<td>B (400)</td>
<td>16.8 - 53.0</td>
<td>8.4 - 26.5</td>
<td>4.2 - 13.3</td>
</tr>
<tr>
<td>C (400)</td>
<td>53.0 - 98.0</td>
<td>26.5 - 49.0</td>
<td>13.3 - 24.5</td>
</tr>
<tr>
<td>D (300)</td>
<td>98 - 100</td>
<td>49 - 100</td>
<td>24.5 - 100</td>
</tr>
</tbody>
</table>

THE DIRECT METHOD

Approximating the initial cost of compressors of differing stroke volume by the broken straight line in Figure 1 simplifies the algebra of some aspects of economic optimisation by allowing swept volume to be regarded as an economic variable.

Hence \[ \frac{d}{d(hp)} (\text{initial cost}) = \frac{d}{d(V_{sw})} (X) = A \]

Also \[ \frac{d}{d(hp)} (\text{annual running cost}) = \frac{d}{d(hp)} (-A) = B \]

(A is negative because it is defined (Appendix A) as a cost saving)

From equation (A1) in Appendix A

\[ X = \frac{A \cdot LF \cdot X}{(1 + n)} \]

differentiating with respect to input power

\[ \frac{dX}{d(hp)} = \frac{dA \cdot LF \cdot X}{(1 + n)} = \frac{B \cdot LF \cdot X}{(1 + n)} \]

\[ \frac{d(V_{sw})}{d(hp)} = \frac{dX}{d(hp)} \cdot \frac{d(V_{sw})}{dX} = \frac{B \cdot LF \cdot X}{(1 + n)} \]
Let \( \frac{1}{Y} = \text{specific capacity (ft}^3/\text{min hp) and } X' = \frac{V_{sw}}{\text{capacity (ft}^3/\text{min)}} \)

\[
\frac{dY}{dX'} = \frac{d(hp)}{d(V_{sw})} = \frac{(1 + n)a}{B \cdot LF \cdot x} = K
\]

\[Y = KX' + \text{Konst} \quad \ldots \quad (1)\]

Equation (1) gives lines of equal cost: Figure 4 shows the (wide) spread of cost of small compressors currently available (previously plotted in Figure 1). Values used for parameter K were \( n = 0.1, \ a = 2.0, \ B = 196 \) and \( x \cdot LF = 2 \) or 4 years. After determining each optimum thermodynamic design point, as in Figure 2, \( Y \) and \( X' \) in eq.1 could be calculated and points such as N and M placed on Figure 4. (Note that \( \frac{1}{Y} \) is equivalent to the objective function, the specific capacity, used in the thermodynamic optimisation, Figure 2). Since K is a fixed economic parameter the constant of integration Konst can be found and an economic comparison made between different designs by referring points such as N and M in Figure 4 back to the ordinate along a line of equal overall cost. Here the value of \( \frac{1}{Y} \) is equal to \( \frac{1}{Konst} \) and since it is desired to maximise this value, which will be termed the referred specific capacity, \( \frac{1}{Konst} \) is used as the objective function.

The effect on optimised compressor performance of the inclusion of economic factors in the objective function, using the direct method, is illustrated on Figures 4, 5, 6 and 7. Figure 5 is a plot of referred specific capacity to a base of compressor swept volume and was obtained by optimising the objective function, \( \frac{1}{Konst} \), which included both thermodynamic and economic criteria: it is of the same form as Figure 2. Figure 6 is a plot of specific capacity to a base of compressor swept volume and can be compared to Figure 2 directly. However, whilst the results plotted on Figure 2 were obtained by optimising an objective function based only on thermodynamic criteria, the results plotted in Figure 6 were obtained during the optimisations for Figure 5. Figure 7 relates the values of referred specific capacity to the corresponding values of specific capacity by lines of equal overall cost. It also includes lines of constant speed.

The results shown on Figure 2 and Figure 6 are very similar both qualitatively and quantitatively and examination of the values of volumetric efficiency for the range of results shown indicated that these too were very similar. It was shown (6) that when mechanical losses are not included in the objective function, low values of volumetric efficiency can result from the optimisation procedure. Since the same value of mechanical losses was used for both sets of results it was deduced that, in the present instance, this parameter was dominant in preventing the occurrence of low values of optimised volumetric efficiency. A higher value of mechanical efficiency i.e., a reduction in mechanical losses would have lowered the optimised volumetric efficiencies associated with the results shown on Figure 2 to a greater extent than those associated with Figure 6, because of the lack of economic criteria to moderate the fall. A higher value of mechanical efficiency would have caused the optimised values of specific capacity shown in Figure 2 to differ by a larger amount from those shown in Figure 6.

The specific capacity of two compressors N and M is compared with their corresponding referred specific capacity. Using thermodynamic criteria only it can be seen from Figure 6 that compressor N has a specific capacity of 4.65 ft\(^3\)/min hp and compressor M, 4.73 ft\(^3\)/min hp. On this basis, compressor M would be preferred. However when economic criteria are included then, as shown on Figure 5, compressor N has a referred specific capacity of 4.345 ft\(^3\)/min hp compared to 4.33 ft\(^3\)/min hp for compressor M. On this basis compressor N would be preferred.

The line of constant gas throughput of 24.55 ft\(^3\)/min shown in Figure 5 differs from that shown in Figure 6. The line in Figure 5 indicates that a peak of specific capacity is reached at 400 rev/min; this peak is not present in Figure 6. The value of the load factor (\( x \cdot LF \)) used was 4. The result agrees with that obtained using the indirect method, Figure 3 and Table 2, which showed that the best design speed for the load factor range of 2.65 to 4.90 for a gas throughput of 24.44 ft\(^3\)/min was 400 rev/min.

The effect is further illustrated in Figure 7 in which the corresponding specific capacity and referred specific capacity of the two compressors N and M are shown related by lines of equal overall cost.

**CONCLUSIONS**

When adequate mathematical models of a compressor are available economic factors can be included either indirectly or directly in arriving at a "best" design. In the indirect approach economic considerations are applied to modify an optimised objective function based on thermodynamic criteria such as specific capacity. In the direct approach the economic variables are included in the multivariont problem and an optimimum overall design determined within the constraints imposed. Hence, as this initial study shows, the optimum design overall may specify mechanical or operational dimensions different from those which meet the more limited objective of optimum thermodynamic performance. It is believed that the philosophy of this early paper will be increasingly implemented as the software for the necessary mathematical models and optimisation procedures continue to develop and become more readily available.

**APPENDIX A**

**Discounted Cash Flow**

One of the methods to estimate the profitability of an investment scheme makes comparison between the net cash investment and the return on the investment by discounting to their present value all cash flows during the operational period.
FIG. 4 SPECIFIC CAPACITY FOR VARIOUS COMPRESSORS (POINTS) AND LINES OF EQUAL OVERALL COST.

FIG. 5. OPTIMUM DESIGN BASED ON THERMODYNAMIC AND ECONOMIC CRITERIA.

FIG. 6 SPECIFIC CAPACITY OF OPTIMISED DESIGN IN FIGURE 5

FIG. 7. OPTIMUM SPECIFIC CAPACITY OVERALL.
Hence
\[
\text{Present value} = \frac{\text{Future value}}{(1 + n)^x}
\]

If the sum of the present values is greater than the nett cash investment then there is a return greater than the interest rate. The required rate of return will be stipulated by company policy.

The approach may be used to compare the merit of a high initial cost, high efficiency compressor with a cheaper less efficient machine. The difference in initial cost of the more efficient machine would correspond to the nett cash investment and the cost savings from the higher efficiency would be the equivalent of the yearly cash flows. Taxation must be taken into consideration: rates of taxation on profits may be as much as 50%. In this type of transaction a high proportion of the nett cash investment may be used as a capital allowance to offset taxes in the year of purchase. If this proportion is less than 100% then additional annual "writing down" allowances may be claimed.

Table A1 makes a comparison between two compressors P and Q. The anticipated life of each is 10 years. Compressor P cost £130 more to purchase initially than did compressor Q but due to better thermodynamic performance P produces annual savings in running costs which amount to £10 in the first year. An inflation rate of 10% causes the annual cost saving to increase by 10% per annum. An annual interest rate of 10% is assumed to remain constant, and the rate of taxation is 40%. Of the cost of the compressor 100% may be claimed as a capital allowance in the first year to offset tax. The residual values of each compressor after the ten years is negligible. The comparison shows that the sum of the present values (£105,123) is less than the nett cash investment (£130). Hence it is more profitable to purchase the cheaper compressor, Q, and to forgo the savings in running costs that P would have given. The 'break-even' point would occur after 14.3 years of compressor life: had the 100% capital allowance in the first year not been available the 'break-even' point would not have occurred until 22.5 years.

Table A1 is rewritten in a more general form in Table AII, using the nomenclature

\[ A = \text{cost saving in first year (E), with LF = 1} \]
\[ X = \text{nett cash investment (E)} \]
\[ r = \frac{A}{X} \]
\[ B = \text{rate of taxation} \]
\[ m = \text{annual inflation rate} \]
\[ n = \text{interest rate or required rate of return} \]
\[ x = \text{number of years of operation} \]
\[ LF = \text{load factor} \]

The total present value (if it is assumed that \( m = n \)) is
\[
\frac{BX + A}{(1 + n)} + \frac{A(1 + n - B)(x - 1)}{(1 + n)^2} - \frac{BA}{(1 + n)^2}
\]

which, to achieve the required rate of return over \( x \) years, must equal the nett cash investment, \( X \). From this equality, it follows that
\[
1 + n = rx \quad \text{(AII)}
\]

<table>
<thead>
<tr>
<th>Year</th>
<th>Cost Saving £</th>
<th>Tax on Cost Saving £</th>
<th>Capital Allowances £</th>
<th>Tax Saving £</th>
<th>Cash Flow £</th>
<th>Present Value Factor 10%</th>
<th>Present Value £</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10,0</td>
<td>-</td>
<td>130</td>
<td>52</td>
<td>62.0</td>
<td>0.909</td>
<td>56,364</td>
</tr>
<tr>
<td>2</td>
<td>11,0</td>
<td>4.0</td>
<td>-</td>
<td>-</td>
<td>7.9</td>
<td>0.826</td>
<td>5,785</td>
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<tr>
<td>3</td>
<td>12,10</td>
<td>4.40</td>
<td>-</td>
<td>-</td>
<td>7.70</td>
<td>0.731</td>
<td>5,785</td>
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<tr>
<td>4</td>
<td>13,31</td>
<td>4.84</td>
<td>-</td>
<td>-</td>
<td>8.47</td>
<td>0.683</td>
<td>5,785</td>
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<tr>
<td>5</td>
<td>14,64</td>
<td>5.32</td>
<td>-</td>
<td>-</td>
<td>9.32</td>
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<td>5,785</td>
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<tr>
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<td>5.86</td>
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<td>-</td>
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<td>0.564</td>
<td>5,785</td>
</tr>
<tr>
<td>7</td>
<td>17,72</td>
<td>6.44</td>
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<td>-</td>
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<td>8</td>
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<td>-</td>
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<td>-3,306</td>
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Total = 105,123

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<th>Year</th>
<th>Cost Saving £</th>
<th>Tax on Cost Saving £</th>
<th>Capital Allowances £</th>
<th>Tax Saving £</th>
<th>Cash Flow £</th>
<th>Present Value Factor 10%</th>
<th>Present Value £</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>X</td>
<td>BX</td>
<td>BX + A</td>
<td>BX + A</td>
<td>BX + A  ( (1 + n) )</td>
<td>BX + A  ( (1 + n) )</td>
</tr>
<tr>
<td>2</td>
<td>A(1 + n)</td>
<td>BA</td>
<td>-</td>
<td>A(1 + n - B)</td>
<td>A(1 + n - B)</td>
<td>A(1 + n - B) ( 1 + n )</td>
<td>A(1 + n - B) ( 1 + n )</td>
</tr>
<tr>
<td>3</td>
<td>A(1 + n)^2</td>
<td>BA(1 + n)</td>
<td>-</td>
<td>A(1 + n)(1 + n - B)</td>
<td>A(1 + n)(1 + n - B)</td>
<td>A(1 + n)(1 + n - B) ( 1 + n )</td>
<td>A(1 + n)(1 + n - B) ( 1 + n )</td>
</tr>
<tr>
<td>4</td>
<td>A(1 + n)^3</td>
<td>BA(1 + n)^2</td>
<td>-</td>
<td>A(1 + n)^2(1 + n - B)</td>
<td>A(1 + n)^2(1 + n - B)</td>
<td>A(1 + n)^2(1 + n - B) ( 1 + n )^2</td>
<td>A(1 + n)^2(1 + n - B) ( 1 + n )^2</td>
</tr>
<tr>
<td>...</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>x1</td>
<td>-</td>
<td>BA(1 + n)^{x-2}</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

TABLE A1: Annual Cash Flow Table for a Particular Case

TABLE A2: Annual Cash Flow Table for General Case
Figure 8 shows that the interest rate, \( n \), is of only secondary importance. The assumptions made in equation (A1) are (a) that taxation on earnings is levied in the year after they are obtained (b) the total amount of the initial cost of the compressor may be used as a capital allowance to offset taxes on profits in the year of purchase (c) the inflation rate and taxation rate remain constant (d) the inflation rate is equal to the required rate of return (or interest rate) (e) the residual value of the compressor is practically zero.

If none of the nett cash investment can be offset against tax then equation (A1) becomes:

\[
\frac{(1 + n)^2}{(1 + n - B)} = Rx  \quad \text{(A2)}
\]

REFERENCES


