1978

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ABSTRACT

Computer simulation of the thermodynamics and dynamics of an intercooled two-stage two-cylinder reciprocating air compressor system is reported and the obtained results discussed. The main emphasis is placed upon modeling the pressure-wave motion in the interconnecting pipework for which the governing equations were solved by means of two different methods: the characteristic mesh method and the finite difference method which utilizes Richtmyer's two-step version of the Lax-Wendroff scheme. For the treatment of boundaries several different modi fications of the numerical schemes were explored in order to improve the accuracy and to increase the computing speed. Both methods yielded predictions of the pressure-crank angle diagrams in cylinders and other positions in the compressor installations which were generally in acceptable agreement with the experimental records. Deficiencies were notable only in the predictions of the higher frequency pressure pulsations.

INTRODUCTION

Notable progress has been reported in literature during the past few years on further developments of the methods of simulation of the physical processes occurring in reciprocating compressors, e.g. [1], [2], [4], to [6], [9]. The reported methods seem to be capable of yielding satisfactory predictions of the overall performances of the complex compressor systems under various working conditions. However, needs for further improvements of both the physical models and computing method are still required. Deficiencies are evident in the case of modeling the valve behaviour and gas flow through the valves, gas leakage and real properties of gases, while the further refinements of the numerical schemes should be aimed at improving the accuracy and the computing economy.

Simulations of the compressor are generally performed by decomposing the cyclic process into the following basic phases: the thermodynamics of expansion and compression, the dynamics of valve motion and the flow through the valves, the gas flow and the associated wave motion in the suction and discharge piping systems, heat transfer to the surroundings and, finally processes in the auxiliary equipment. Each phase is modeled and programmed independently with adjustable boundary conditions so that composite models of compressors having different configurations and arrangements could easily be created by matching the appropriate basic processes.

Although some degree of uncertainty is inherent in the model of each phase, it is notable that the modeling of the pressure-wave phenomena exerts the major influence upon the quality of simulation and consumes most of the computing time The work described here is concentrated along these lines: it concerns the prediction of the overall performances of a two-stage intercooled reciprocating compressor, with a particular emphasis placed upon the investigation of various numerical methods for calculating the unsteady pressure-wave propagation.

BASIC EQUATIONS AND NUMERICAL SCHEMES

The standard equations governing the transient thermodynamic process in the compressor cylinders and the dynamics of the valve motion have been reported elsewhere e.g. [1] [9]. Here only the equations describing the gas flow in the interconnecting ducts will be quoted. The quasi-one dimensional unsteady nonhomentropic flow with heat transfer can be represented by the continuity, momentum and energy equation, conveniently written in the conservative form [6], [7].

$$\frac{\partial}{\partial t}(\rho F) + \frac{\partial}{\partial x}(\rho u F) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho u F) + \frac{\partial}{\partial x}(\rho u^2 F + pF) = -2\rho F \frac{f}{D} u\,|u| - p\frac{dF}{dx} \tag{2}$$

$$\frac{\partial}{\partial t}(\rho e_t F) + \frac{\partial}{\partial x}\left[uF\,(\rho e_t + p)\right] = \rho\, qF \tag{3}$$

Various numerical methods have been used for the solution of the hyperbolic equation set (1) to (3). Initially we followed the approach of Benson et al. [1],[2] which utilizes the characteristic mesh method originally proposed by Courant et al. [3]. For this purpose the governing equations were transformed into the characteristic form yielding explicit increments of the Riemann variables λ and β and the entropy along the two characteristics and the path line, respectively.

The application of this method to the prediction of the pressure change in a two stage compressor system has been reported earlier [9] together with modifications that were introduced to enable simultaneous calculation of all three characteristic variables at the same grid points. The results obtained showed an acceptable agreement with the experimental records, but indicated notable damping of the higher frequency pressure pulsations. Some improvements were achieved by decreasing the size of the time step but at the expense of a substantial increase of the computing time.

More recently McLaren et al. [4] to [6] reported the successful application of Richtmyers two-step version of the Lax-Wendroff finite difference scheme for the solution of the equations (1) to (3). By recognizing that the governing equations have the common conservative form

$$\frac{\partial f}{\partial t} + \frac{\partial g}{\partial x} = s$$

where $g = g(f)$ and $s = s(f)$, the application of the Lax-Wendroff scheme yields the values of the variable f (and consequently the variable g and source term s) at the next time instant and at the same space position:

$$f_i^{n+1} = f_i^n - \frac{\Delta t}{\Delta x} (g_{i+\frac{1}{2}}^{n+\frac{1}{2}} - g_{i-\frac{1}{2}}^{n+\frac{1}{2}}) + \frac{\Delta t}{2} (s_{i+\frac{1}{2}}^{n+\frac{1}{2}} + s_{i-\frac{1}{2}}^{n+\frac{1}{2}}) \quad (5)$$

For the evaluation of $g_{i\pm\frac{1}{2}}^{n+\frac{1}{2}}$ and $s_{i\pm\frac{1}{2}}^{n+\frac{1}{2}}$ the predictor step is utilized to yield:

$$f_{i\pm\frac{1}{2}}^{n+\frac{1}{2}} = \frac{1}{2} (f_{i\pm 1}^n + f_i^n) \mp \frac{\Delta t}{2\Delta x} (g_{i\pm 1}^n - g_i^n) + \frac{\Delta t}{4} (s_{i\pm 1}^n + s_i^n) \quad (6)$$

The main disadvantage of the above scheme is its unsuitability for the calculation of the variables at boundaries since the information required beyond the boundaries is generally unknown. McLaren et al [6] used the method of characteristics to calculate the boundary values.

In the present work various types of finite difference schemes for treatment of the edges were explored. The numerical properties of the tested schemes were firstly estimated analytically using the well known criteria developed for linear hyperbolic equations. More details were acquired by applying the schemes to the calculation of the pressure-wave motion generated by the sudden closing of the valve in a straight pipe in which a steady uniform nonhomentropic gas flow occurred prior to the valve closing. Various time- and space steps were tested and the amplitude damping coefficient evaluated and compared with the values obtained by means of the von Neumann analytic expression appropriate strictly only for the linear equations.

Two of the various schemes that were tested, proved to possess the desired properties. Both calculated the variable in the first step at the time half interval by means of the formula

$$f_I^{n+\frac{1}{2}} = f_I^n - \frac{\Delta t}{2\Delta x} (g_I^n - g_{I-1}^n) + \frac{\Delta t}{2} s_I^n \quad (7)$$

while the calculation of the second step utilized one of the following two formulas:

$$f_I^{n+1} = f_I^{n+\frac{1}{2}} - \frac{\Delta t}{\Delta x} (g_I^{n+\frac{1}{2}} - g_{I-\frac{1}{2}}^{n+\frac{1}{2}}) + \frac{\Delta t}{2} s_I^{n+\frac{1}{2}} \quad (8)$$

$$f_I^{n+1} = f_I^n - 2\frac{\Delta t}{\Delta x} (g_I^{n+\frac{1}{2}} - g_{I-1}^{n+\frac{1}{2}}) + \Delta t\, s_I^n \quad (9)$$

Both formulas (8) and (9) are basically extrapolating backward schemes of second-order accuracy and in this respect are consistent with the Lax Wendroff scheme which has been applied in the rest of the flow. The application of the schemes to the simple pressure-wave propagation in the pipe after closing one end yielded the amplitude damping coefficients for both schemes in close agreement with the analytical results and also gave an indication about the optimum sizes of the numerical steps for each scheme. No particular advantage with any of the two schemes was discernible in the performed test, but it was felt that formula (9) might yield somehwat superior behaviour in more complex flow situations and was used during the subsequent calculation of the overall compressor performance.

RESULTS AND DISCUSSION

The composite model in conjunction with several types of numerical schemes for the calculation of the pressure wave motion was applied to the simulation of a two-stage two-cylinder reciprocating air compressor with inter- and after-cooling. The specification of the compressor as well as the details concerning the modeling of the gas expansion and compression in the cylinders and the dynamics of the valves were reported earlier [9] together with some earlier predictions of performances by means of the characteristic mesh method.

Here only the recent results will be presented which were obtained by the two-step Richtmyer-Lax-Wendroff method (5) and (6) and the modified scheme for the treatment of boundary values according to formulas (7) and (9). Figure 1 shows the predictions of the pressure pulsations as a function of crank angle α in the 1st stage discharge chamber and the 2nd stage suction chamber obtained by both characteristic mesh method and the finite difference scheme. The experimental results recorded by "Kistler" piezoelectric transducers are also presented for comparison. Although both predicted results show

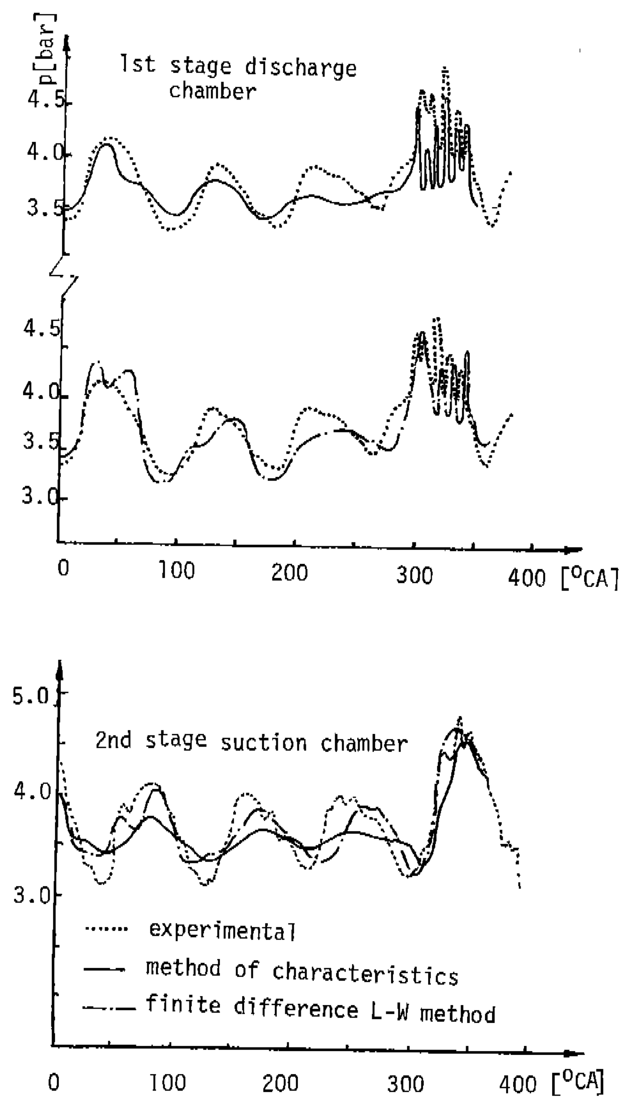

Fig.1: Pressure variation in 1st stage discharge-and 2nd stage suction chamber

reasonably good qualitative agreement with the experimental records, it is noticable that the finite difference scheme produced better general agreement. A certain degree of damping and signal smoothing is evident in both predicted curves. However, the finite difference scheme produces amplitudes and frequency which are reasonably close to the experimental values even during the period of the discharge valve opening, which is characterized by high frequency oscillations. The predicted results clearly indicate that a further improvement of the numerical schemes are both desirable and possible, but certain deficiency in modeling the discharge valve behaviour and uncertainties inherent in the empirical inputs concerning the valve properties, should not be discarded as possible sources of discrepancies between the modeled and measured curves.

Figure 2 shows the predictions and experimental records of the pressure-crank angle diagrams in both cylinders at three different compressor speeds. Only the finite difference predictions are shown for clarity. The general agreement with the experiments could be regarded as satisfactory. It is interesting to note that the predicted pressure oscillation during the discharge periods show somewhat smaller amplitudes than the experimental ones in the first cylinder, while in the second cylinder the discrepancy has the opposite trend. Since in both cases the same numerical procedure was applied, it is suspected that the valve characteristics (spring stiffness, damping factor) selected on the basis of the available manufacturers data may not be fully adequate. Further studies are required to eliminate or at least to reduce the indicated discrepancies but it is felt that the obtained results could be regarded as encouraging. It should be mentioned that the predicted global engineering parameters such as the air flow rate and the volumetric efficiency are in very close agreement with the experimental data. The calculations of four subsequent cycles required altogether about 40 CP seconds of the CDC CYBER 70 computer, which was considerably smaller than the earlier calculations which used the characteristic mesh method. The major reduction of the computer time is attributed to the substantial increase of both the time and distance steps as well as to the application of the variable-step Runge Kutta method for the calculation of the valve displacements.

CONCLUSIONS

The computer simulation of a two stage reciprocating system produced the prediction of the pressure-crank angle diagram in cylinders and other positions in the compressor which were in acceptable agreement with the experimental records. A substantial improvement of the accuracy of the predictions and the reduction in computing time was achieved by solving the equations governing the unsteady gas flow in the compressor piping system by means of finite difference schemes instead of using the method of characteristics.

NOTATION

a	-	speed of sound
D	-	pipe diameter
e_t	-	total internal energy
f	-	friction factor, general symbol for transport property
F	-	pipe cross-sectional area
g	-	general symbol for function of transport properties
p	-	pressure
q	-	heat transfer per unit mass
s	-	symbol denoting the source term in conservation equations
t	-	time
x	-	coordinate along the gas flow
β, λ	-	Riemann variables
ρ	-	gas density
α	-	crank angle

ACKNOWLEDGEMENT

The authors wish to acknowledge the support of Association of Scientific Research of Bosnia and Hercegovina.

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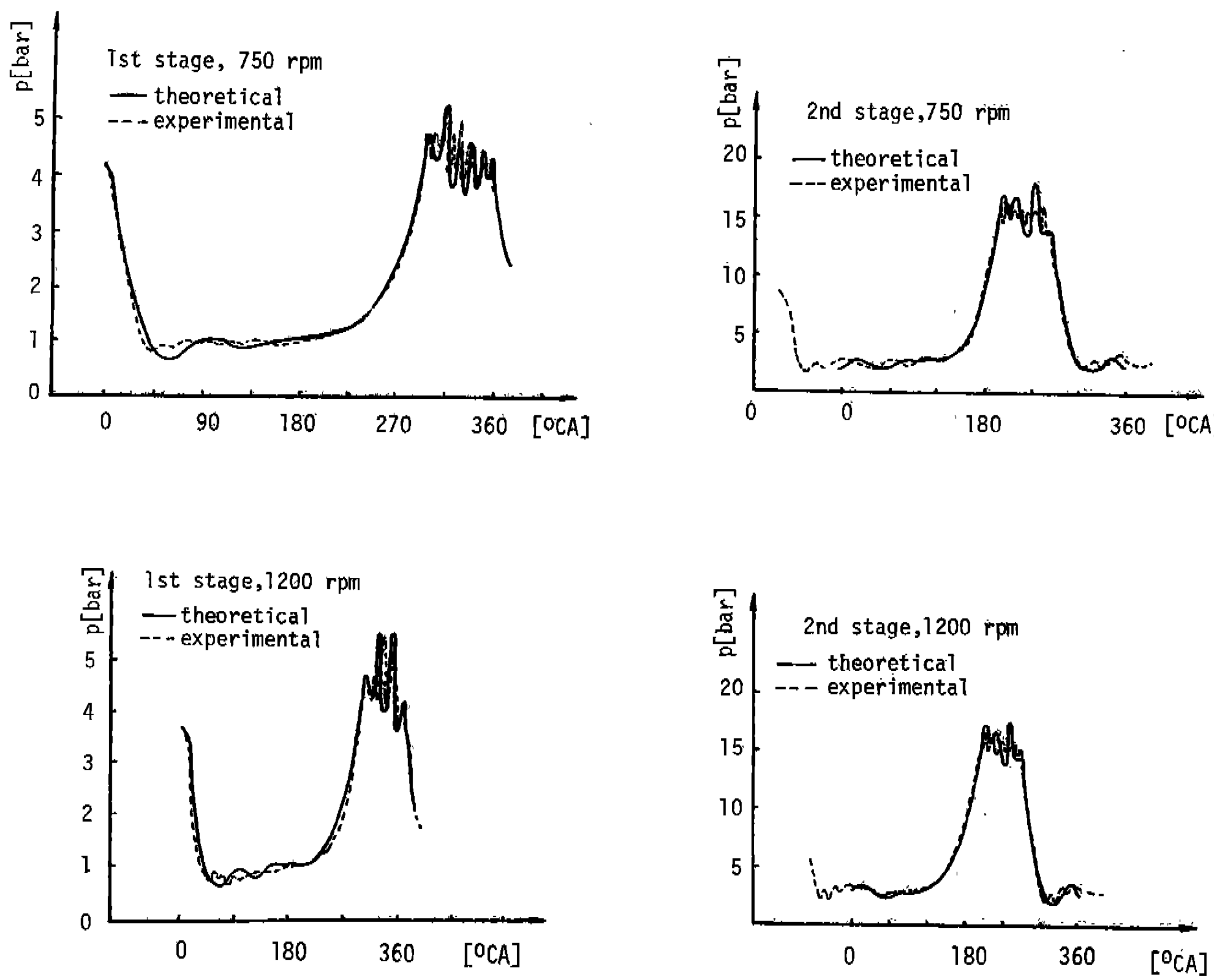

Fig.2: Predicted and recorded pressure variations in cylinders at two different speeds