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## Design and Optimization of Lightweight Porous Damping Treatments

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## 1pCA3 Design and Optimization of Lightweight Porous Dampers



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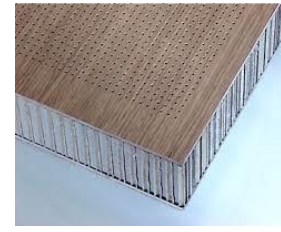
# PUBLICATIONS & PRESENTATIONS

1. Y. Xue and J. S. Bolton, “Structural vibration damping by the use of poro-elastic layers: a summary,” Invited Technical Report of Inter-Noise, Chiba, Japan, August 2023.
2. Y. Xue and J. S. Bolton, “Design and optimization of lightweight porous dampers,” Invited Technical Report of the 184th ASA Meeting, Chicago, IL, USA, May 2023.
3. Y. Xue, J. S. Bolton, T. Herdtle, S. Lee and R. W. Gerdes, “Structural damping by lightweight poro-elastic media,” *J. Sound Vib.* **459**, 114866 (2019), <https://doi.org/10.1016/j.jsv.2019.114866>.
4. Y. Xue, J. S. Bolton, T. Herdtle, S. Lee and R. W. Gerdes, “Structural damping by layers of fibrous media applied to a periodically-constrained vibrating panel,” *J. Phys. Conf. Ser.* **1264**, 012043 (2019), <https://iopscience.iop.org/article/10.1088/1742-6596/1264/1/012043> (journal paper) and <https://docs.lib.purdue.edu/herrick/204/> (presentation @ RASD 2019).
5. Y. Xue, J. S. Bolton and T. Herdtle, “Design of lightweight fibrous vibration damping treatments to achieve optimal performance in realistic applications,” *SAE Technical Paper 2019-01-1524*, <https://doi.org/10.4271/2019-01-1524> (journal paper) and <https://docs.lib.purdue.edu/herrick/199/> (presentation @ SAE-NVC 2019).
6. Y. Xue, J. S. Bolton and Y. Liu, “The acoustical coupling of poro-elastic media in a layered structure based on the transfer matrix method,” *Proceedings of Inter-Noise 2019*, paper 1857, Madrid, Spain, <https://docs.lib.purdue.edu/herrick/200/>.
7. Y. Xue, J. S. Bolton, T. Herdtle, S. Lee and R. W. Gerdes, “A comparison between glass fibers and polymeric fibers when serving as a structural damping medium for fuselage-like structures,” *Proceedings of Inter-Noise*, paper 1478, Chicago, IL, August 2018, <https://docs.lib.purdue.edu/herrick/179>.
8. Y. Xue and J. S. Bolton, “Fibrous material microstructure design for optimal structural damping,” *J. Acoust. Soc. Am.* **143**(3), 1715, *Proceedings of the 175th ASA Meeting*, Minneapolis, MN, May 2018, <https://docs.lib.purdue.edu/herrick/176>.
9. Y. Xue and J. S. Bolton, “Microstructure design of lightweight fibrous material acting as a layered damper for a vibrating stiff panel,” *J. Acoust. Soc. Am.* **143**(6), 3254-3265 (2018), <https://doi.org/10.1121/1.5038255>.
10. Y. Xue, J. S. Bolton, R. W. Gerdes, S. Lee and T. Herdtle, “Prediction of airflow resistivity of fibrous acoustical media having two fiber components and a distribution of fiber radii,” *Appl. Acoust.* **134**, 145-153 (2018), <https://doi.org/10.1121/1.5038255>.
11. Y. Xue and J. S. Bolton, “Fibrous material microstructure design for optimal damping performance,” *Proceedings of the 5th Symposium on the Acoustics of Poro-Elastic Materials (SAPEM)*, Le Mans, France, December 2017, <http://docs.lib.purdue.edu/herrick/168>.
12. T. Herdtle, Y. Xue and J. S. Bolton, “Numerical modeling of the acoustics of low density fibrous media having a distribution of fiber sizes,” *Proceeding of the 5th Symposium on the Acoustics of Poro-Elastic Materials (SAPEM)*, Le Mans, France, December 2017, <http://docs.lib.purdue.edu/herrick/167>.
13. Y. Xue, J. S. Bolton, R. W. Gerdes, S. Lee and T. Herdtle, “Prediction of airflow resistivity of fibrous acoustical media having double fiber components and a distribution of fiber radii,” *Proceedings of Inter-Noise*, pages 5649-5657, Hong Kong, August 2017, <http://docs.lib.purdue.edu/herrick/165>.

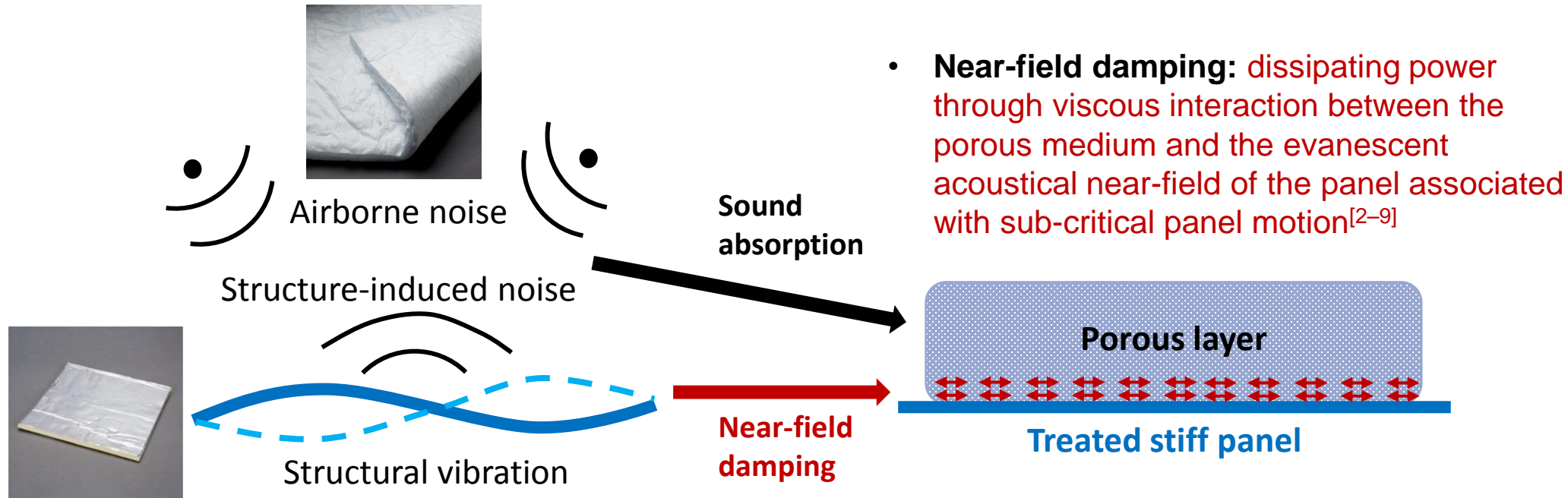
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# Challenge

- **Advanced Noise Control Materials<sup>[1]</sup>**
  - **What's important about a noise control material?**
    - **Safety**
    - **Cost**
    - **Weight**
    - **Volume**
    - **Recyclability**
    - ...
    - ...
    - **Acoustical Performance**



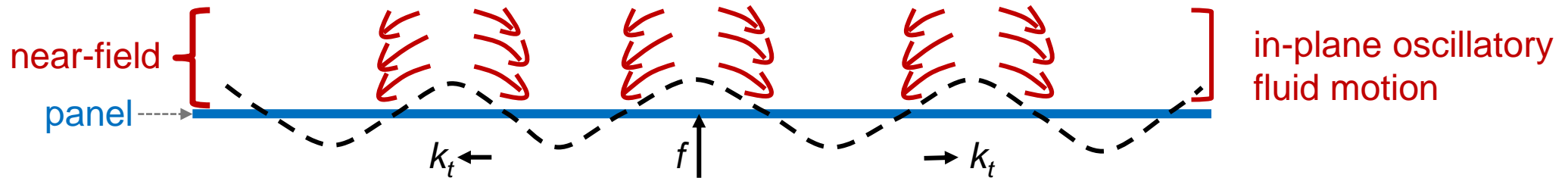
# Objective: Multifunctionality



- **Objectives: modeling, predicting and optimizing** the near-field damping performance of conventional sound absorbing materials (fiber, foam, etc.), so that a properly-designed porous layer can achieve both structural damping and sound absorption at the same time  
→ **save weight and cost**

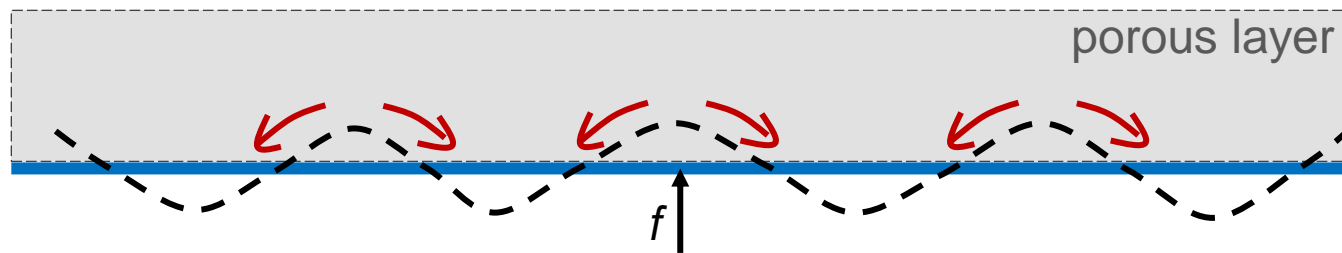
# What is “Near-Field Damping” (NFD)?

- In the subcritical frequency range – structural wavelengths smaller than acoustical wavelength



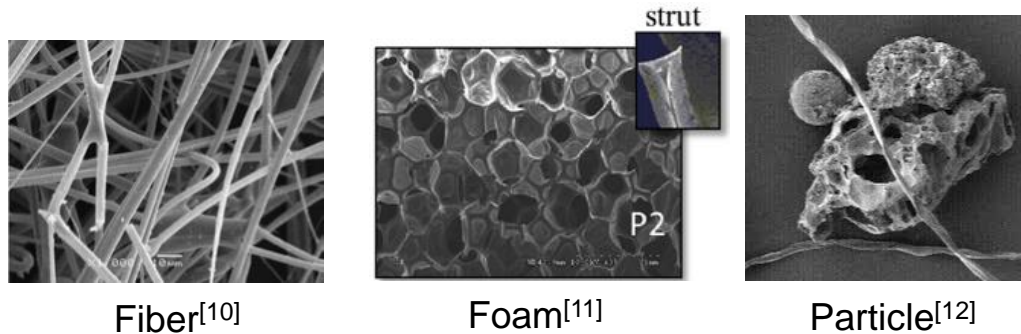
- Exponential decay with vertical position  $e^{-\sqrt{k_t^2 - k^2}}$ , at critical frequency  $k_t = k$

- Near-field depth increase as frequency approaches critical



- Place porous layer in near-field – viscous interaction with in-plane fluid motion dissipates energy and so damps panel motion

# Porous Media



Fiber / strut / particle / pore size  
Solid material density, etc.

Microscopic geometry

Parameters

Macroscopic (bulk) properties<sup>[13]</sup>

- Thickness
- Flow resistivity
- Porosity
- Tortuosity
- Viscous characteristic length
- Thermal characteristic length
- Bulk density
- Young's modulus
- Poisson's ratio
- Loss factor (**mechanical**)



Building Connection (Modeling for Design)

Acoustical properties

Performance

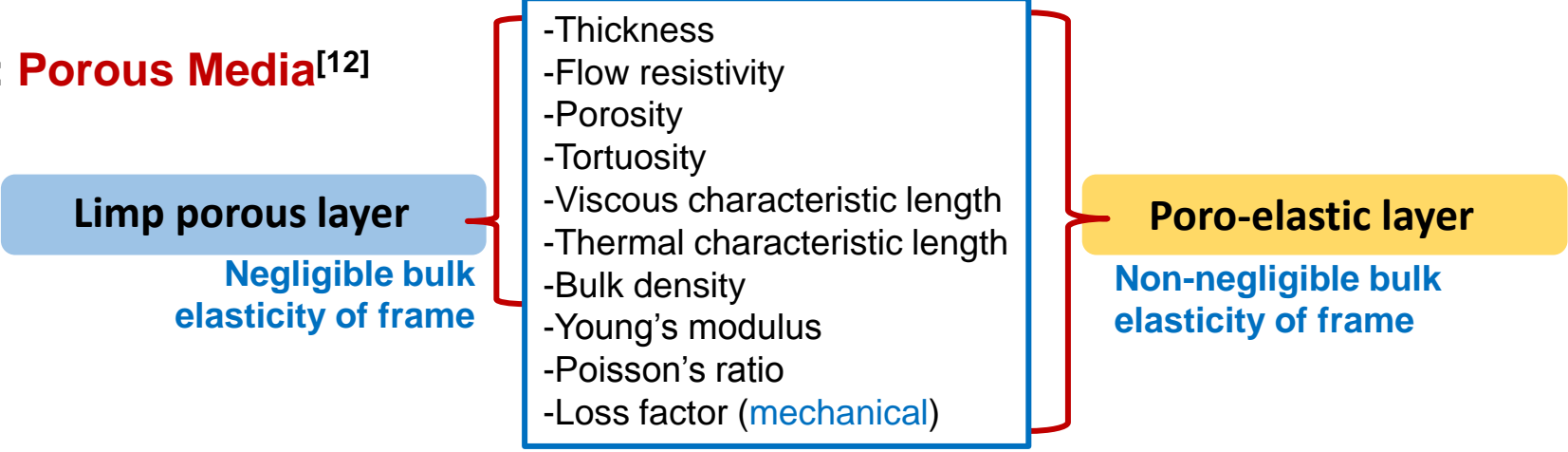
Damping properties

Acoustic pressure  
Acoustic particle velocity  
Acoustic impedance

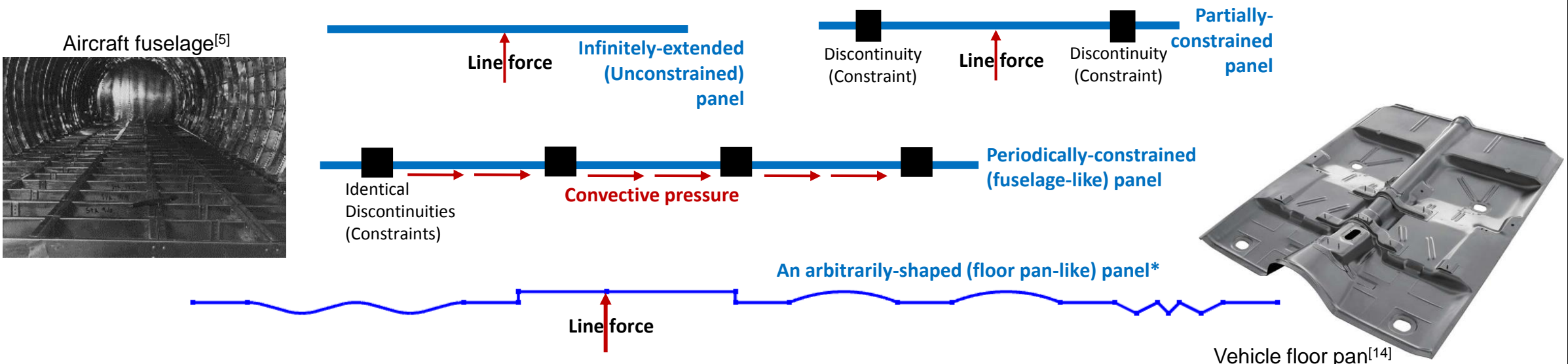
Reduction of the panel's vibration  
Power dissipation within the porous layer  
Layered system energy loss factor

# Near-Field Damping (NFD) by Porous Media

- Modeling Targets: **Porous Media**<sup>[12]</sup>



- Modeling Targets: Vibrating **Structures** under **Excitations**

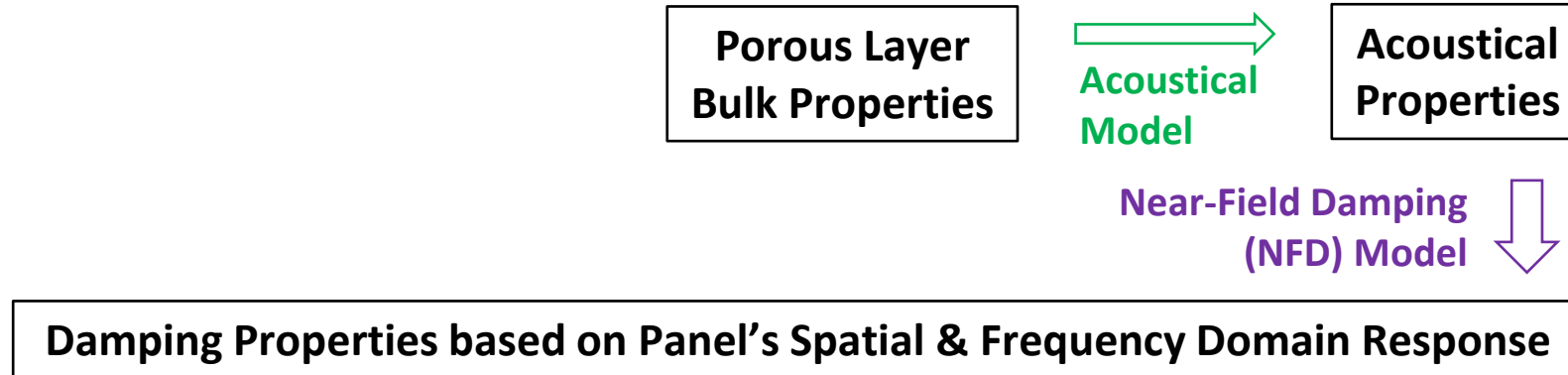


\* Numerical simulation based on commercial software may be involved as complexity of structure increases



# General Approach

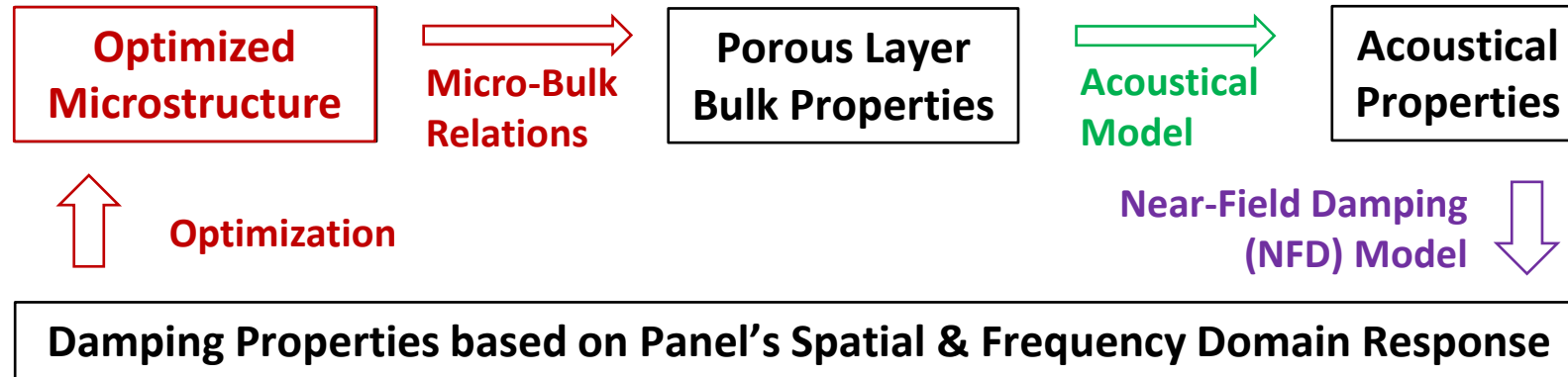
- Analytical modeling to build the connection



- **Acoustical Model (bulk-acoustical relations):** including Johnson-Champoux-Allard (JCA) model<sup>[15]</sup>, Biot theory<sup>[15–20]</sup> and B.C.s implementation <sup>[18,21]</sup>
- **NFD: acoustical-damping relations** including Euler-Bernoulli beam theory, wavenumber-space Fourier transform<sup>[22]</sup> and power analysis<sup>[23]</sup>
- **Acoustical Model + NFD** provides an bulk-damping model to predict the damping performance for porous media regardless of their microstructures

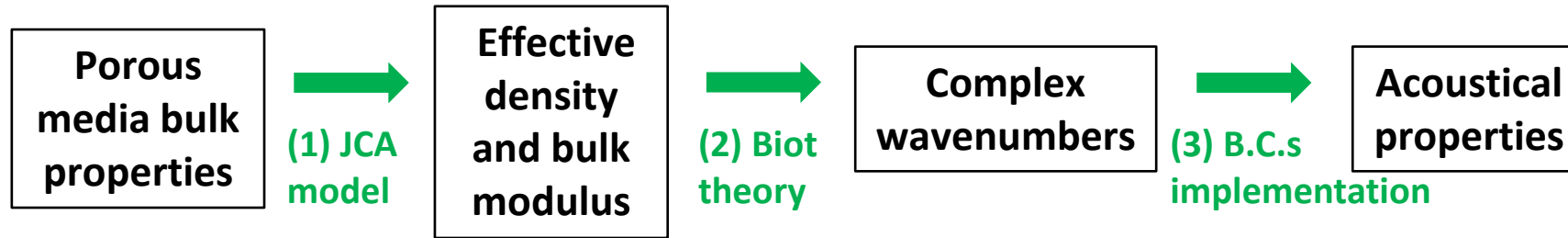
# General Approach

- Analytical modeling to build the connection



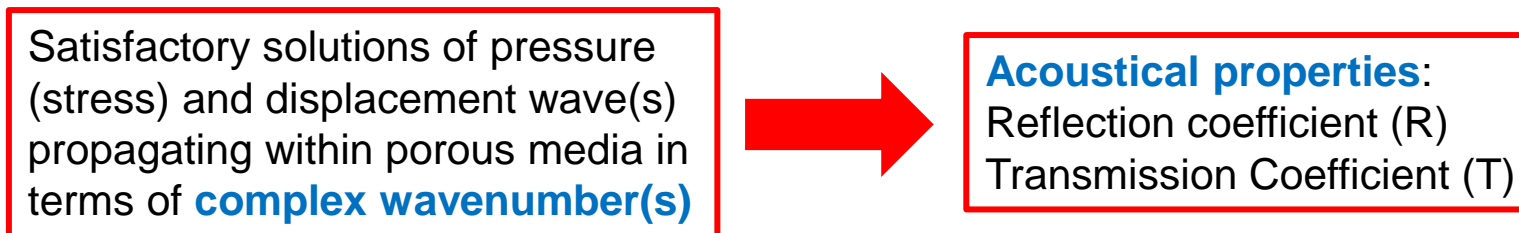
- Micro-bulk relations:** for porous media made of fibers<sup>[24]</sup>
- **Acoustical Model** + **NFD** + **Micro-bulk relations** provides an micro-damping model to maximize fibrous media's damping performance by optimizing their microstructures
- Fibrous layered damper design concept is summarized based on the parametric study and optimization process by using **Acoustical Model** + **NFD** + **Micro-bulk relations**

# Acoustical Modeling – B.C.s Implementation



- **Acoustical Model (bulk-acoustical relations):** including Johnson-Champoux-Allard (JCA) model<sup>[15]</sup>, Biot theory<sup>[15–20]</sup> and **B.C.s implementation**<sup>[18,21]</sup>

**focus here**

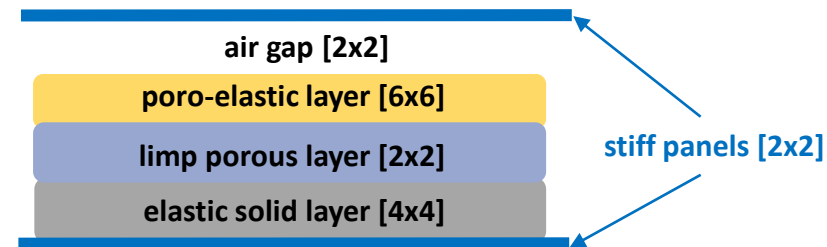


# Modeling of Multilayered Acoustical Systems

- Literature Review

- Classic models from Mason 1927<sup>[27]</sup> (origin of transfer matrix, used for acoustic filters)
- Transfer matrices: **[2x2]** (fluid-like layers, e.g., limp porous)<sup>[21]</sup>, **[4x4]** (elastic solids)<sup>[28–31]</sup>, **[6x6]** (poro-elastic)<sup>\*[32]</sup>

- **Challenge:** how to couple layers with different dimensions?



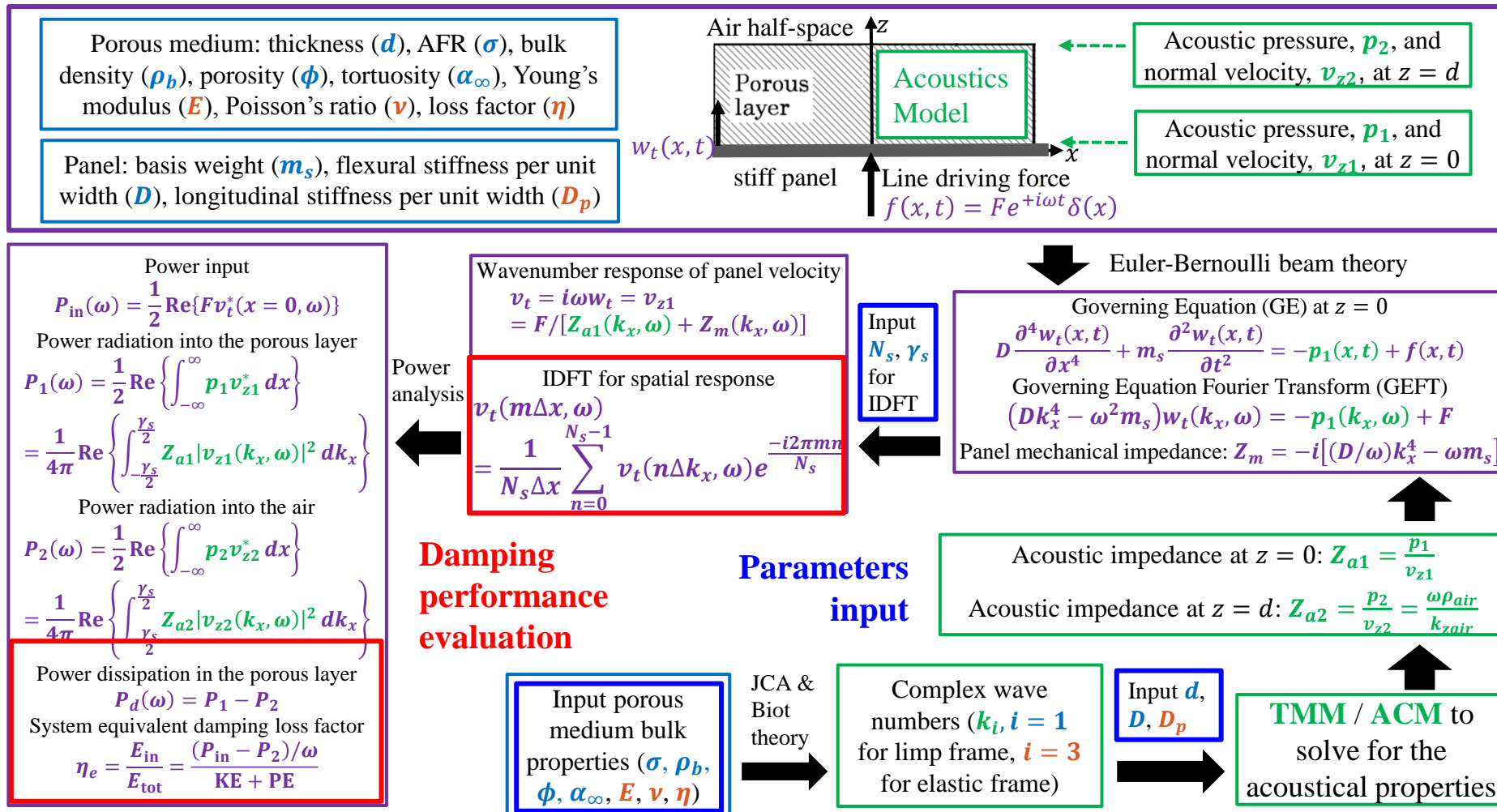
Lauriks et al. <sup>[33]</sup> (1992)	Brouard et al. <sup>[34]</sup> (1995)*	Bolton et al. <sup>[18]</sup> (1996)**	Dazel et al. <sup>[35]</sup> (2013)	Proposed TMM in this study	Song et al. <sup>[46]</sup> (2023)
Explicit expression	Implicit expression	Explicit expression	Implicit expression	Implicit expression	Implicit expression
By matrix order reduction	By B.C.s global assembly	By B.C.s global assembly	By recursive matrix operator	By matrix order reduction	By layer merge operation

\* Further summarized in Allard and Atalla's book<sup>[16]</sup> (2009)

\*\* Also referred to as the classic Arbitrary Coefficient Method (ACM)

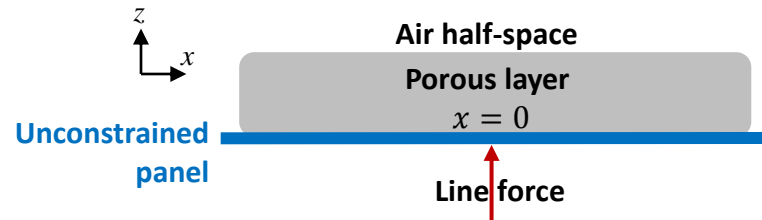
# Combination of Acoustical Models with the NFD Model

- TMM / ACM + NFD** – based on a harmonic line force-driven, unconstrained panel



# Combination of Acoustical Models with the NFD Model

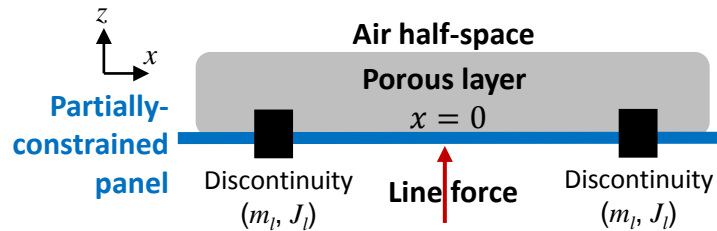
- TMM / ACM + NFD – modeling of different target structures



Governing Equation: unconstrained panel

$$D \frac{\partial^4 w_t(x, t)}{\partial x^4} + m_s \frac{\partial^2 w_t(x, t)}{\partial t^2} = -p_1(x, t) + \boxed{F e^{+i\omega t} \delta(x)}$$

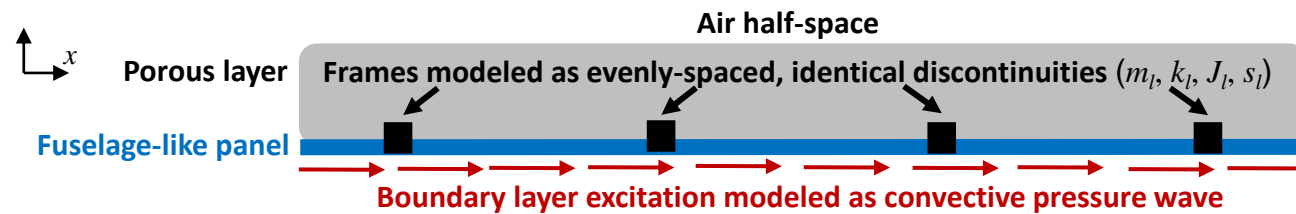
Line force



Governing Equation: adding two identical constraints

$$D \frac{\partial^4 w_t(x, t)}{\partial x^4} + m_s \frac{\partial^2 w_t(x, t)}{\partial t^2} = -p_1(x, t) + \boxed{F e^{+i\omega t} \delta(x)} + \sum_{j=1}^2 F_{l,j} \delta(x - x_{l,j}) + \sum_{j=1}^2 M_{l,j} \delta'(x - x_{l,j})$$

Reaction forces due to discontinuities



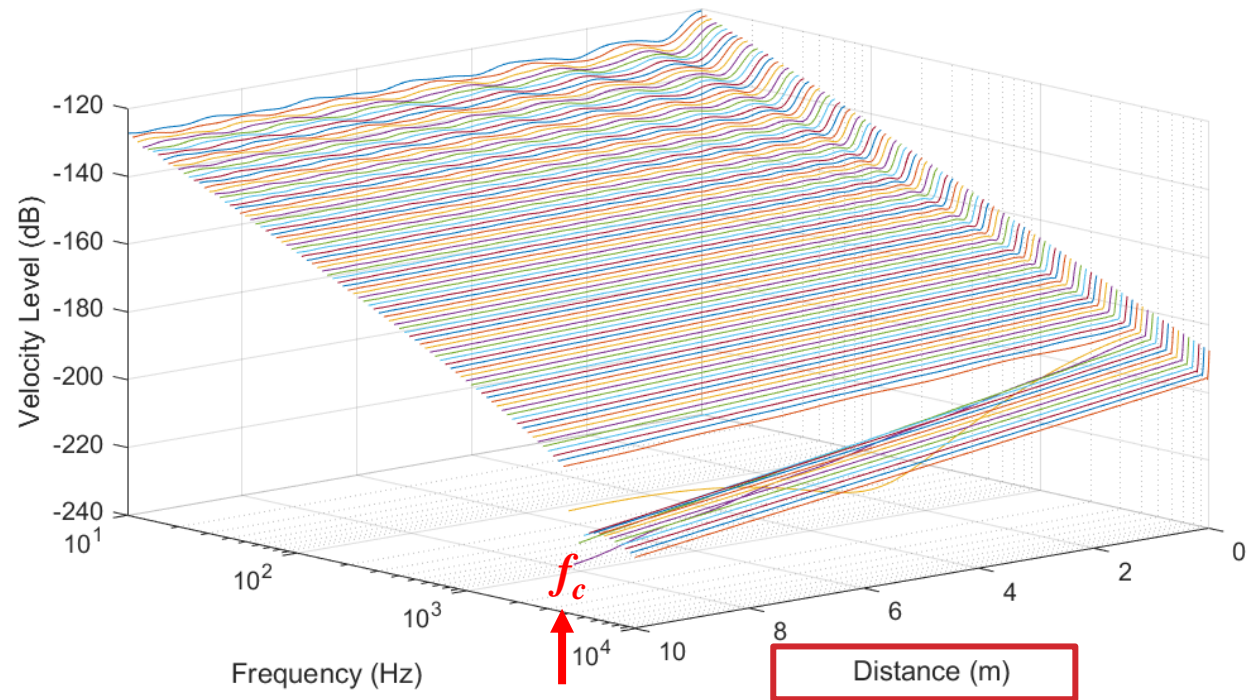
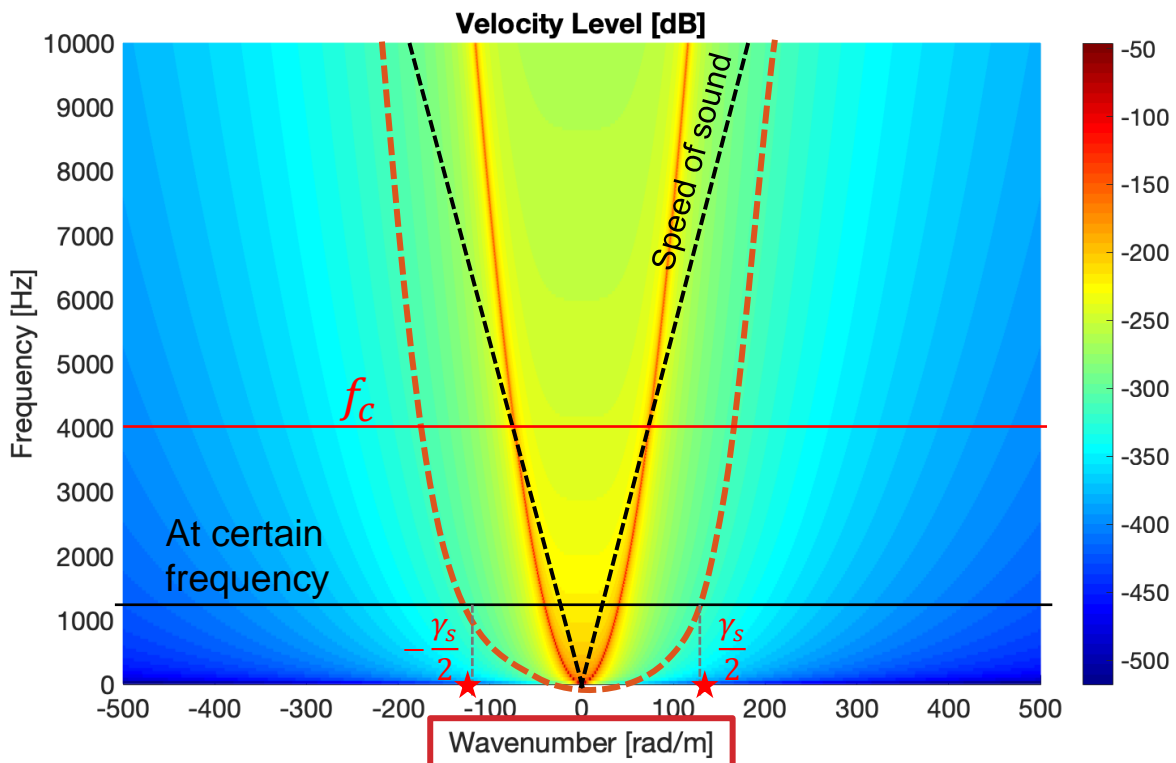
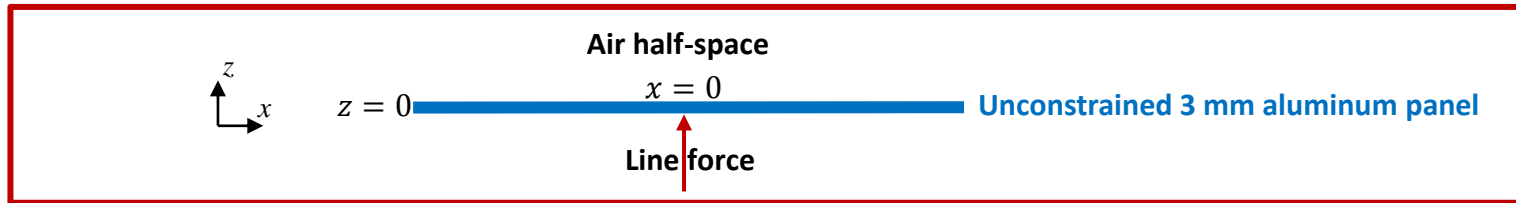
Governing Equation: adding periodic identical constraints

$$D \frac{\partial^4 w_t(x, t)}{\partial x^4} + m_s \frac{\partial^2 w_t(x, t)}{\partial t^2} = -p_1(x, t) + \boxed{F e^{+i\omega t} e^{-ik_v x}}$$

Reaction forces due to discontinuities

# NFD Modeling Key Point

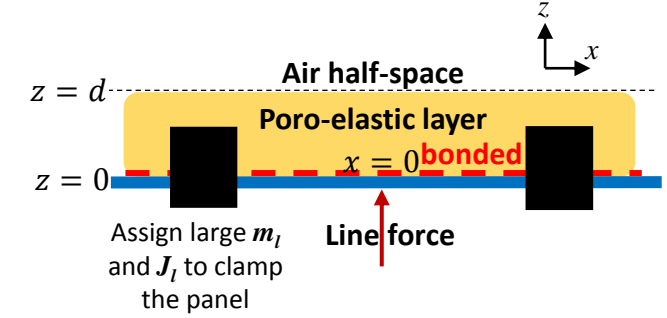
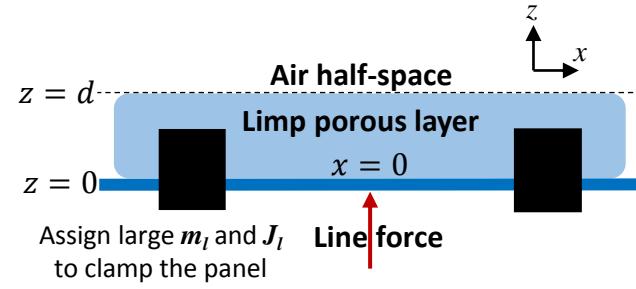
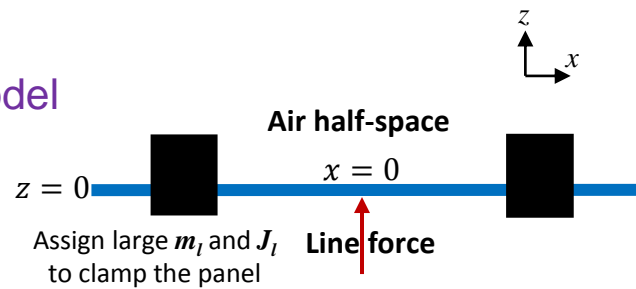
- An example to show wavenumber  $\leftrightarrow$  spatial domain Fourier transform



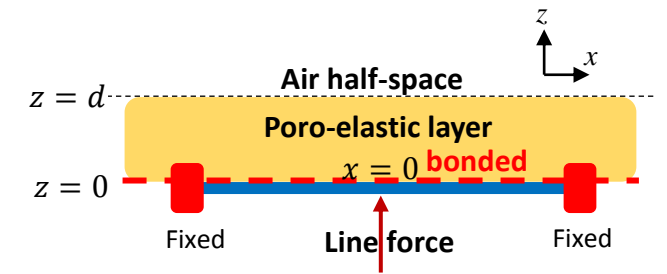
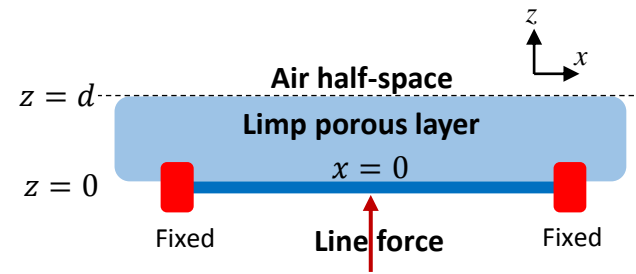
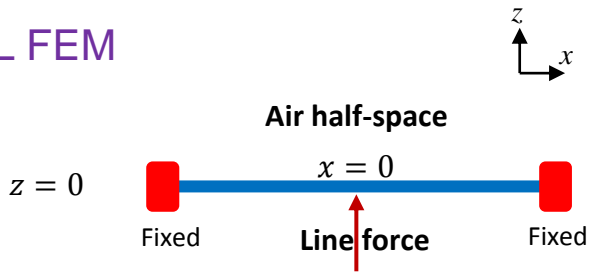
# NFD Model Validation – COMSOL

- Velocity response spectrum at  $x = 0$  of a partially-clamped, 1 mm aluminum panel

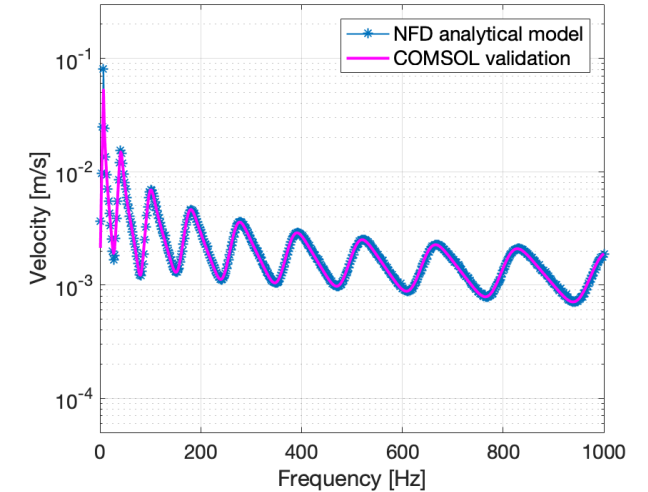
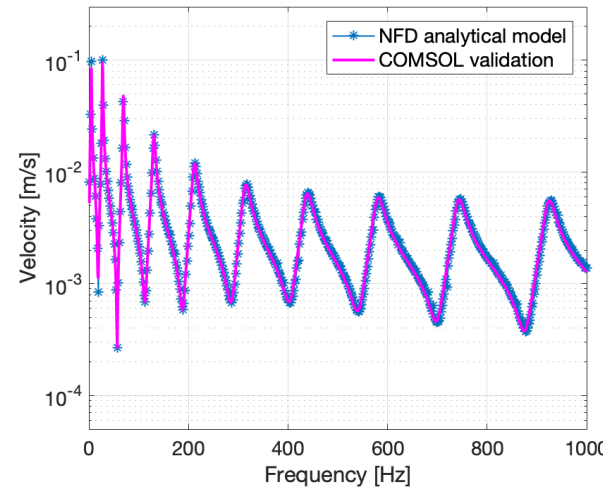
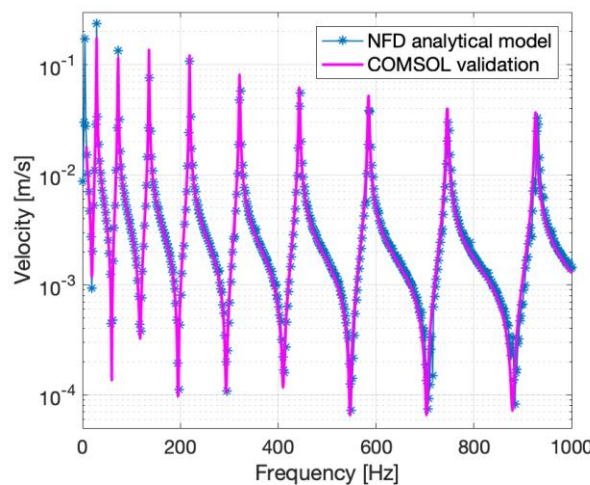
## NFD model



## COMSOL FEM



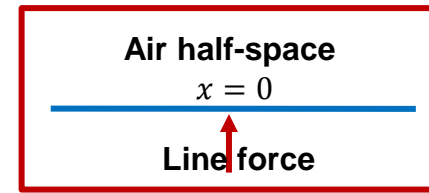
## Comparison



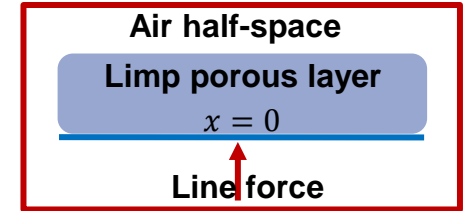


# Damping Effectiveness: Limp Porous Layer

- Velocity level differences at  $x = 10$  m
  - Difference between two cases for an aluminum panel
  - Significant attenuation in sub-critical frequency region
  - Higher critical frequency and stronger attenuation result from decreasing panel thickness



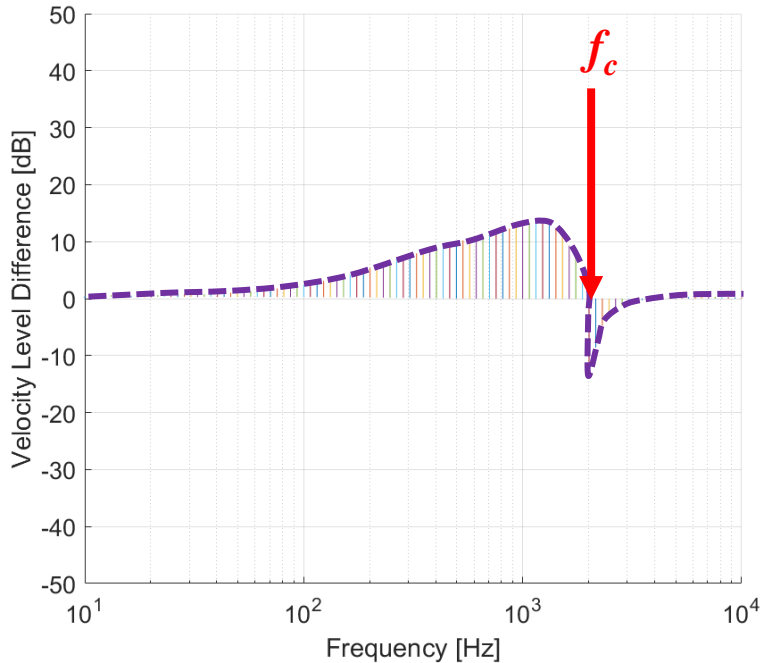
vs.



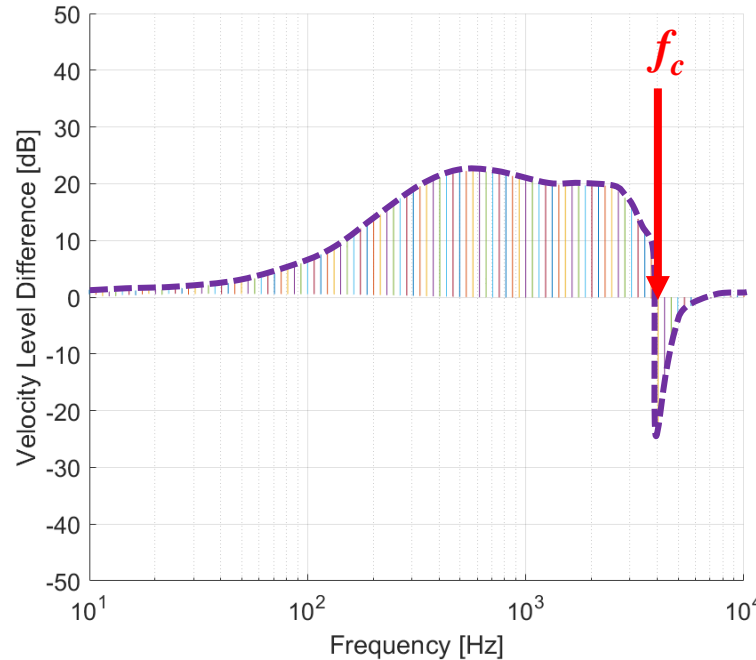
Limp porous layer bulk properties:

$\sigma$	$\phi$	$\alpha_\infty$	$\rho_b$	$d$
20000 Rayls/m	0.9871	1.2	10 kg/m <sup>3</sup>	3 cm

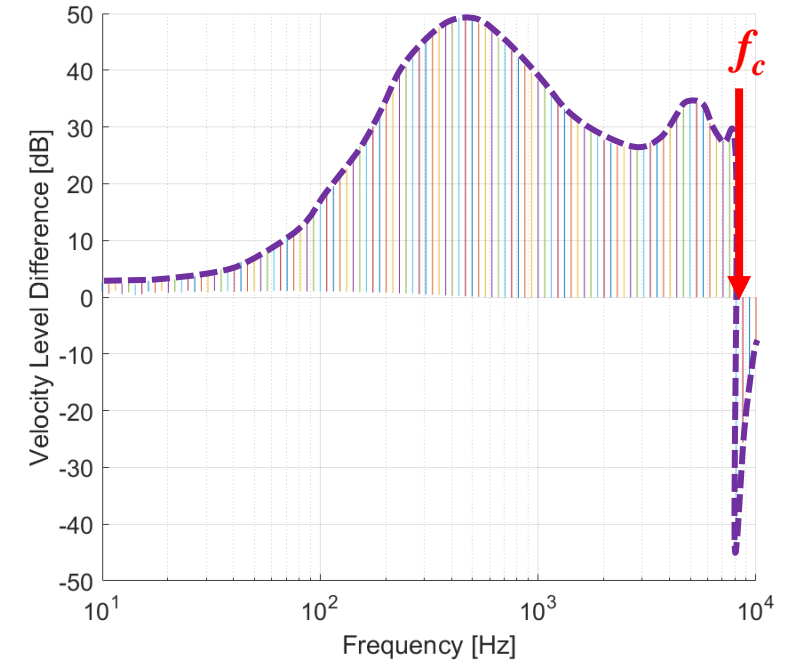
6 mm thick panel



3 mm thick panel

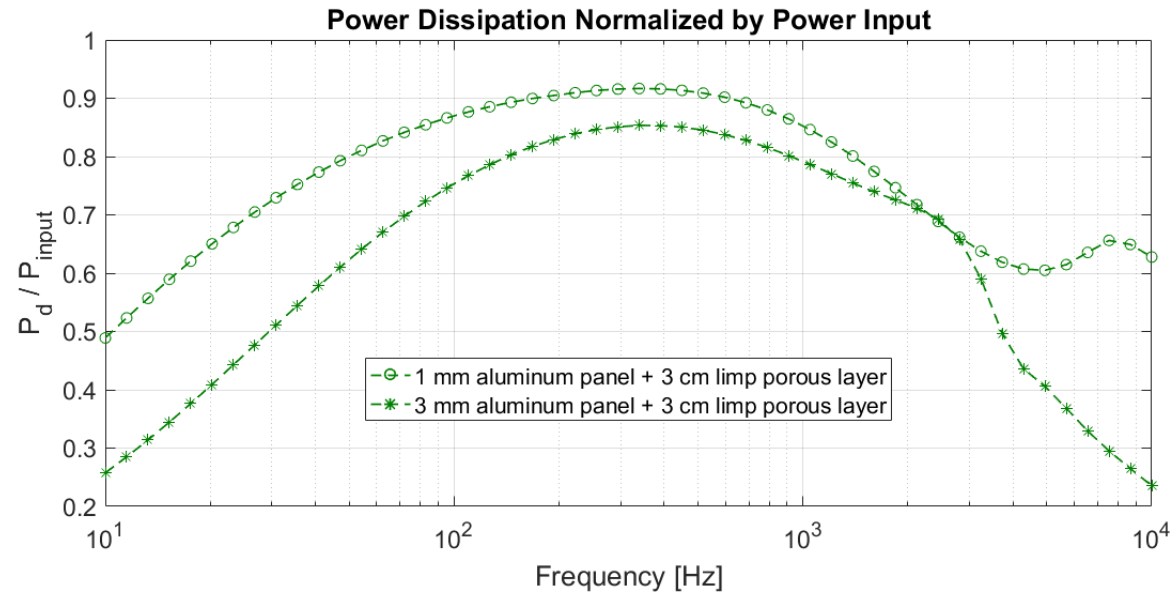
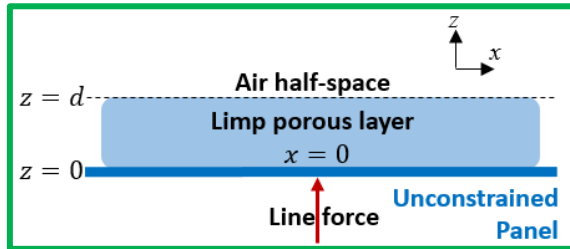


1.5 mm thick panel



# Damping Effectiveness: Limp vs. Elastic

- Power dissipation analysis

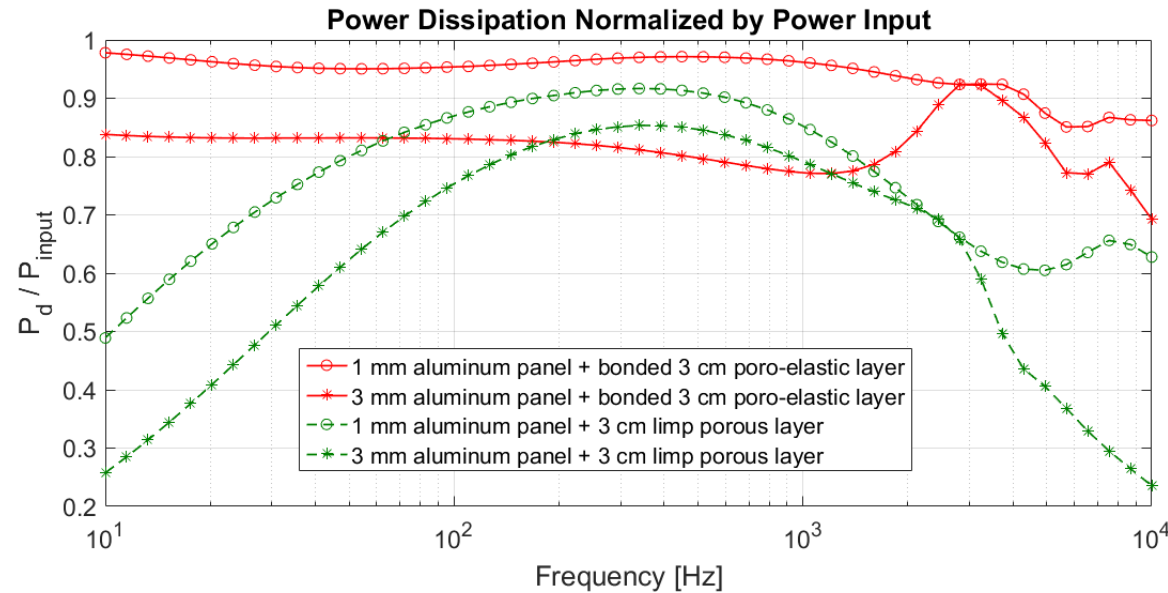
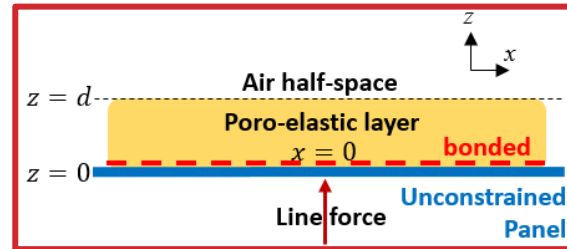
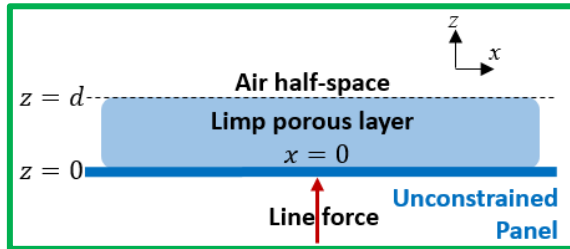


**Limp porous layer  
bulk properties:**

$\sigma$	$\phi$	$\alpha_\infty$	$\rho_b$	$d$
20000 Rayls/m	0.9871	1.2	10 kg/m <sup>3</sup>	3 cm

# Damping Effectiveness: Limp vs. Elastic

- Power dissipation analysis



Limp porous layer bulk properties:

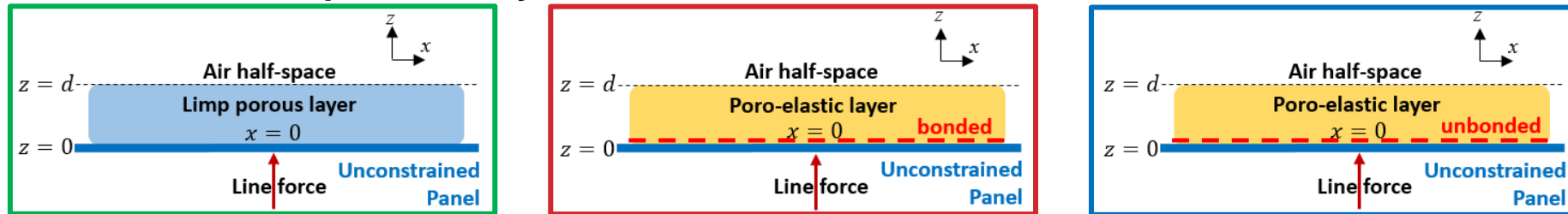
$\sigma$	$\phi$	$\alpha_\infty$	$\rho_b$	$d$
20000 Rayls/m	0.9871	1.2	10 kg/m <sup>3</sup>	3 cm

Poro-elastic layer bulk properties:

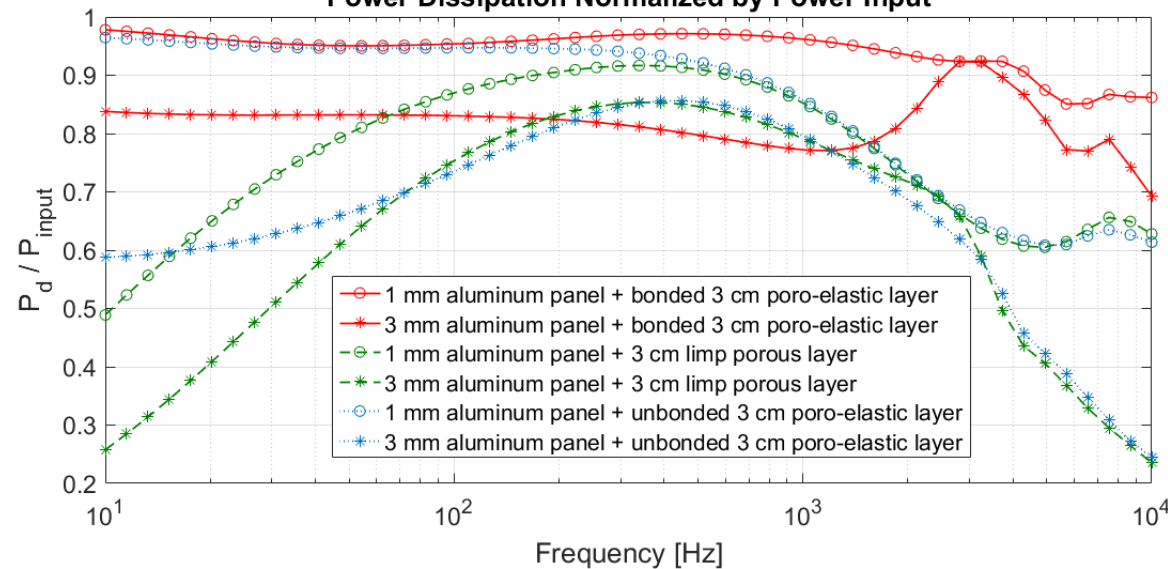
$\sigma$	$\phi$	$\alpha_\infty$	$\rho_b$	$d$	$E_1$	$\eta_m$	$\nu$
20000 Rayls/m	0.9871	1.2	10 kg/m <sup>3</sup>	3 cm	10 <sup>6</sup> Pa	0.3	0.3

# Damping Effectiveness: Limp vs. Elastic

- Power dissipation analysis



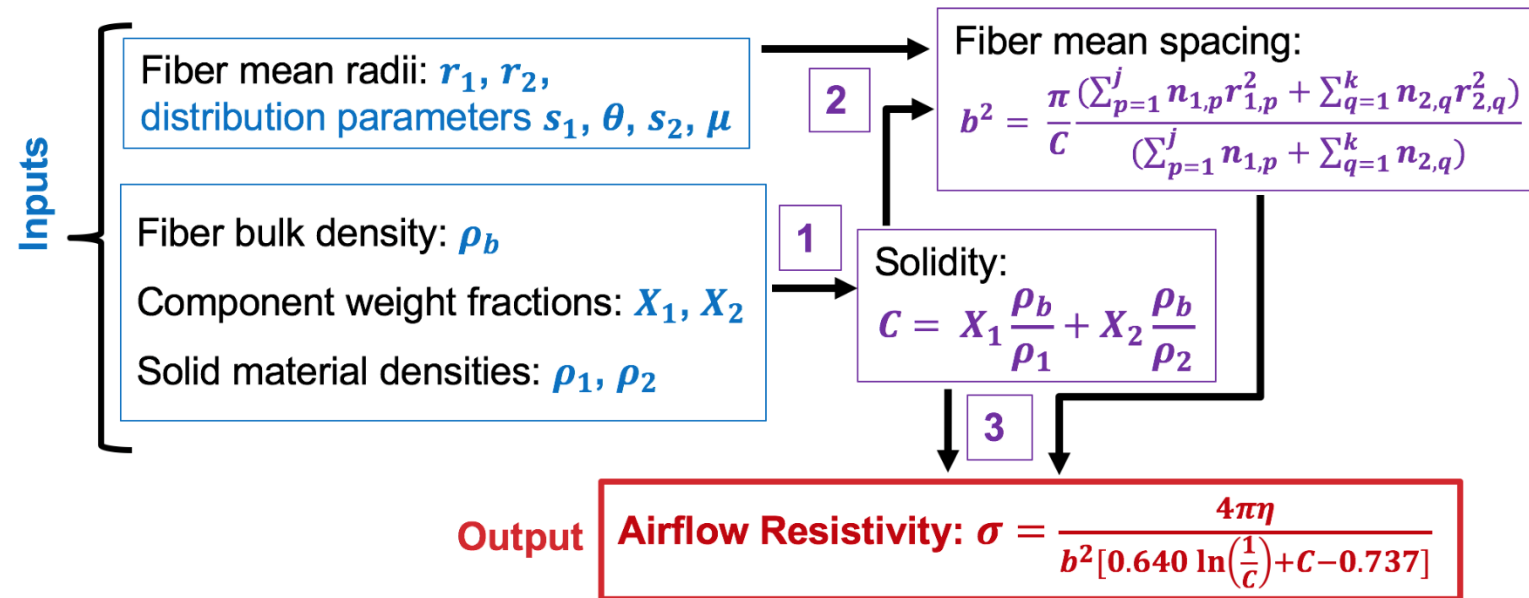
Power Dissipation Normalized by Power Input



➤ **Design concept: adding bulk stiffness to the porous layer and bonding it to the panel will create additional structural dissipation**

# Micro-Bulk Relations for Fibrous Media

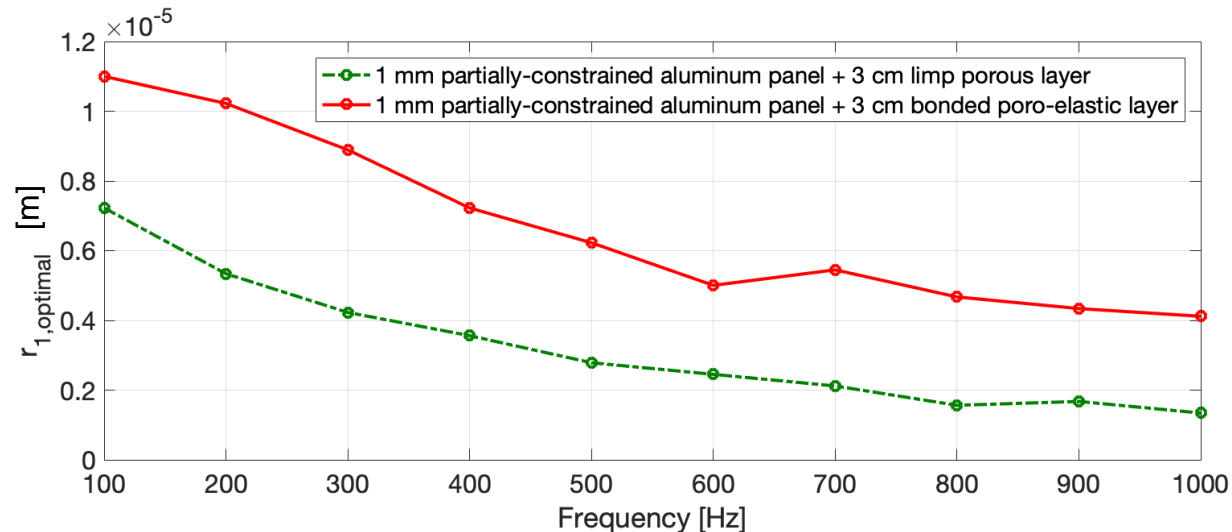
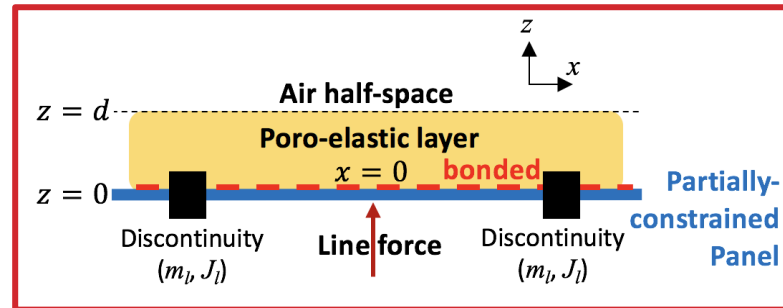
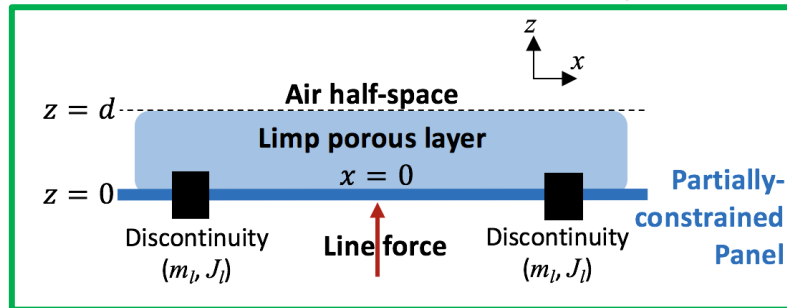
- **Airflow Resistivity (AFR) model** is modified based on Tarnow's model<sup>[24]</sup>
- It can be used for fibrous media with two fiber components and varying fiber radii (e.g., Thinsulate™)



- **Step 1:**  $C$  calculation based on  $\rho_b, X_1, X_2, \rho_1, \rho_2$
- **Step 2:**  $b^2$  calculation based on  $r_1, r_2$ , distribution parameters and  $C$
- **Step 3:**  $\sigma$  calculation base on  $C$  and  $b^2$

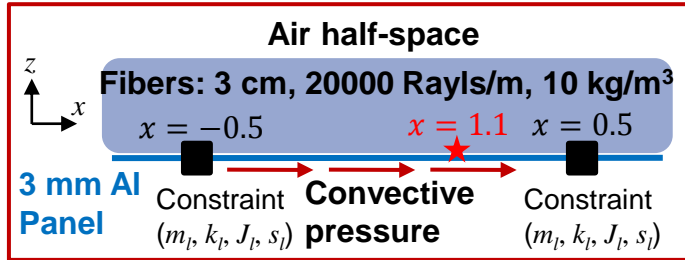
# Microstructure Design for Limp / Elastic Fibrous Damper

- Optimal fiber radii for a partially-constrained structure

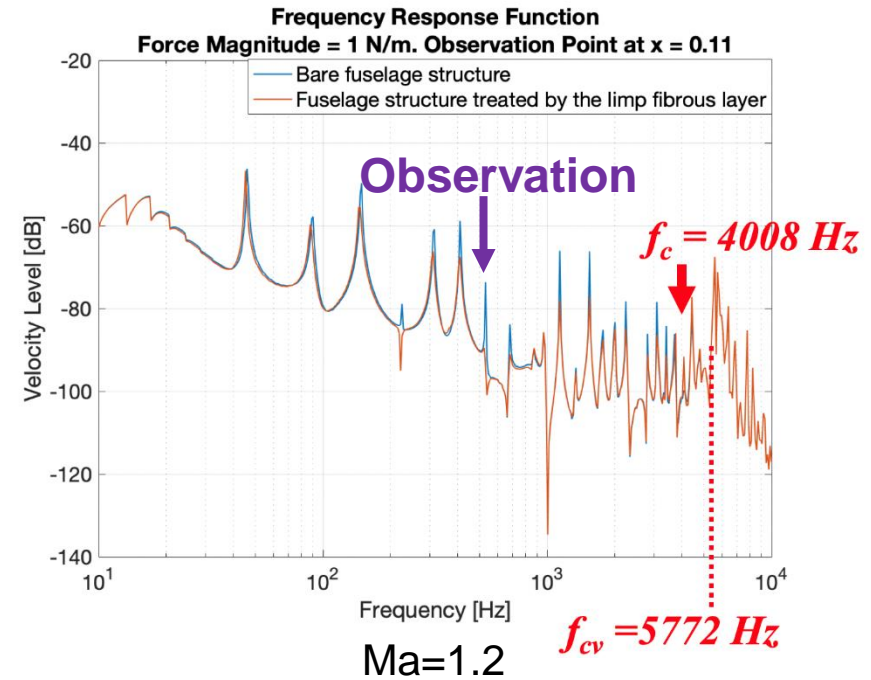
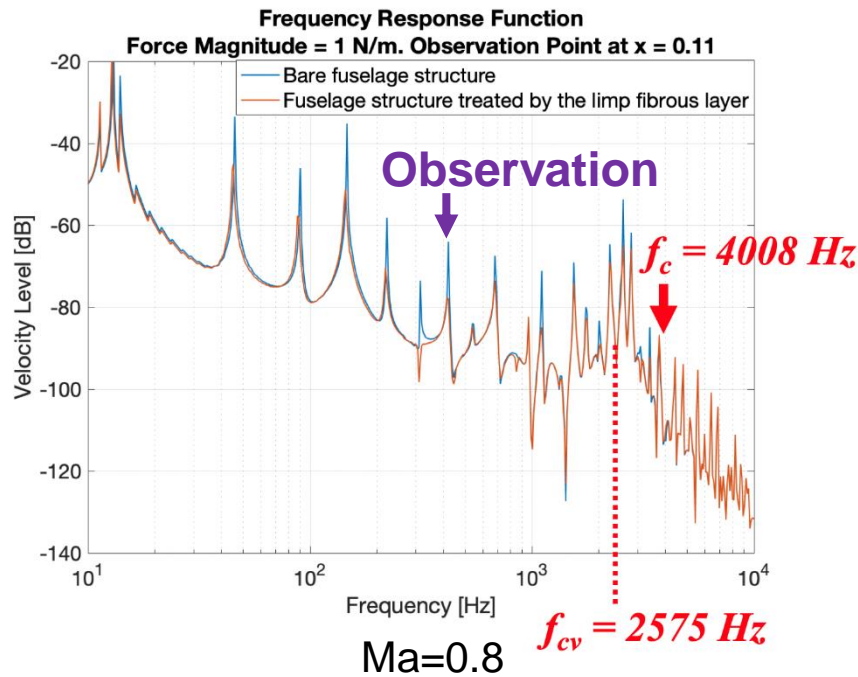


- Design concept: larger fiber size is better at damping lower frequency vibration, elastic fibers need larger fiber size to achieve the optimal damping

# Fuselage Structure Velocity Response Spectrum



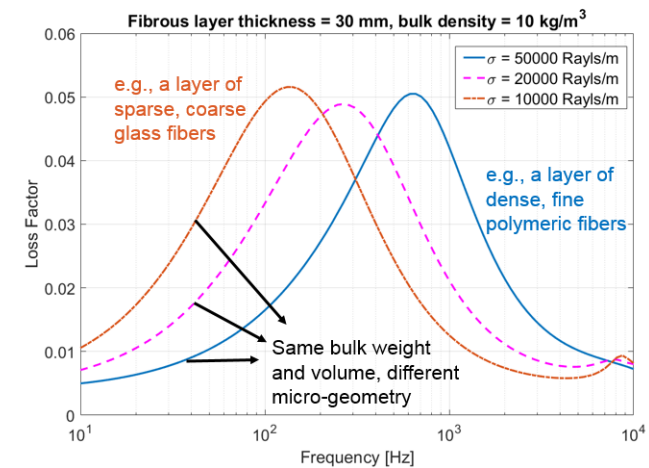
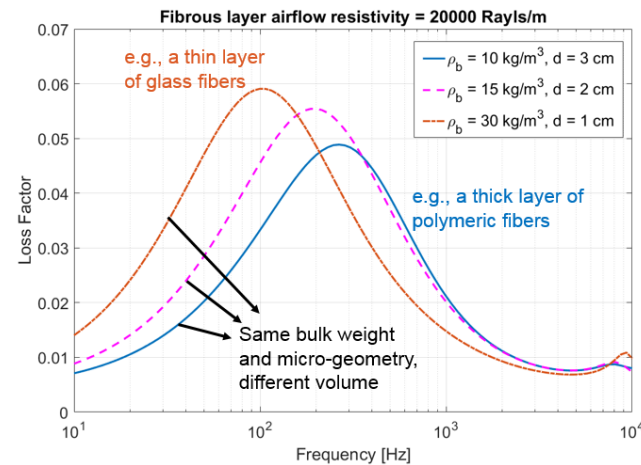
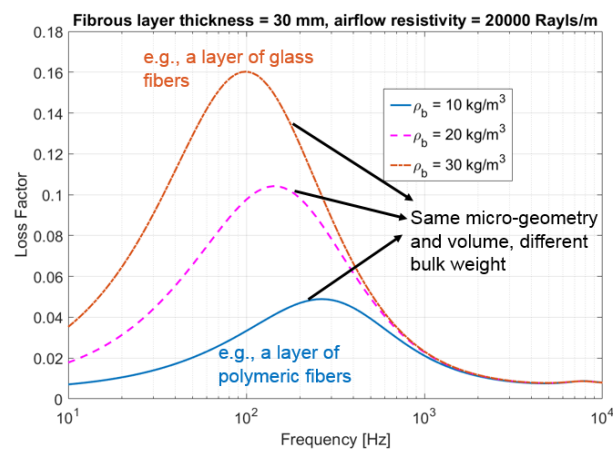
- Observation at **x = 0.11 m**
- Vibration peaks **below  $f_c$**  were reduced by **5–15 dB** by the fibrous layer



# Porous/Fibrous Damper Design Guidelines

## ❖ Based on Bulk Properties Parametric Study

- Fibrous dampers are more effective on thinner structures
- With limited space and the same microstructure, **making the fibers from heavier solid material (e.g., glass) will improve the low frequency damping (left)**
- With limited weight and the same microstructure, **a thin layer of heavy (e.g., glass) fibers gives better low frequency damping, while a thick layer of light (e.g., polymeric) fibers gives better high frequency damping (middle)**
- With limited space, limited weight and by changing microstructures, **a layer of sparse, coarse heavy fibers is better at reducing low frequency vibration, while a layer of dense, fine light fibers is better at reducing high frequency vibration (right)**





# SUMMARY

- Significant levels of damping can be achieved by properly designed porous treatment  
→ multi-functional (absorbing & damping) porous layer saves weight, space and cost
- Porous dampers are effective at reducing subsonic panel vibrations while absorbing the radiating sound from the panel in the supersonic region
- Analytical models that include the **AFR**, **TMM** and **NFD** provide a convenient toolbox for prediction and optimization of porous layer's near-field damping, and for designing the optimal macro/microstructure of the porous layer
- Parametric studies can be conducted by using this toolbox for optimization of porous layer's near-field damping (**in terms of system damping loss factor**), and for designing the optimal macro/microstructure of the porous layer
- Combined with finite element model, the design process can also be conducted on more realistic structures such as a floor pan-like structure

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