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A Set of 74 Test Functions for Nonlinear Equation Solvers

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**A SET OF 74 TEST FUNCTIONS FOR
NONLINEAR EQUATION SOLVERS**

John R. Rice
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ABSTRACT

This report gives 74 functions appropriate to use to test programs which solve one nonlinear equation in one real variable. The functions are given in a Fortran subroutine as a two-dimensional array.

A SET OF 74 TEST FUNCTIONS FOR NONLINEAR EQUATION SOLVERS

INTRODUCTION AND SUMMARY: This report gives 74 functions in FORTRAN code which have been gathered as a set of test functions for a polyalgorithm to solve $F(x) = 0$. These functions are arranged in a two-dimensional array whose indices are passed through the block common

```
COMMON /FDATA/ J,JJ
```

and thus $F(x)$ can be used in its simplest form. The ranges of the indices are

J = 1	JJ = 1 to 14
J = 2	JJ = 1 to 12
J = 3	JJ = 1 to 10
J = 4	JJ = 1 to 24
J = 5	JJ = 1 to 14

These functions may be classified into overlapping groups according to various properties the functions possess. This classification is given below with the notation that 3-8 denotes the eighth element of the third group (i.e., $J = 3$, $JJ = 8$).

SIMPLE (25 members)

1- 7	1-11	2- 2	2- 5	2- 6	3- 3	3- 6
4- 2	4- 3	4- 7	4- 9	4-10	4-11	4-12
4-14	4-15	4-17	4-18	4-19	4-20	4-21
4-22	4-23	4-24				

CLUSTERED ROOTS (7 members)

1- 6	1-10	5- 1	5- 2	5- 3	5- 4	5- 5
------	------	------	------	------	------	------

MULTIPLE ROOTS (14 members)

1- 2	1- 3	1- 4	1- 5	1- 6	1- 9	1-10
4- 1	4- 4	4- 5	4- 6	4- 8	4-16	5- 8

FRACTIONAL ORDER ROOTS (7 members)

1- 3	1-10	2- 8	3- 4	3- 7	5-11	5-12
------	------	------	------	------	------	------

DISCONTINUITIES (4 members)

1- 5	2- 9	2-10	3- 2
------	------	------	------

ASYMPTOTES TO ZERO (6 members)

1-12	5- 6	5- 7	5- 8	5- 9	5-10
------	------	------	------	------	------

ROUND OFF SENSITIVE (5 members)

1- 8	1- 9	2- 4	2- 7	3- 5
------	------	------	------	------

NON-FUNCTIONS (3 members)

3- 8	3- 9	3-10
------	------	------

PATHOLOGICAL (11 members)

1- 3	1- 5	2- 1	2- 4	2- 9	3- 1	3- 2
3- 4	4-13	5-11	5-12			

BADLY SCALED (6 members)

1- 1	1-11	2- 1	2- 3	5-13	5-14
------	------	------	------	------	------

FROM 'REAL' PROBLEMS (2 members)

1-13	2- 7
------	------

(A number of other test functions from 'real' problems are available, but they require too much Fortran code for inclusion here).

```

FUNCTION F(X)
COMMON/ FDATA/ J, JJ
DATA PI /3.1415926 /
GO TO(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13), J
C          TEST FUNCTIONS IN USE SUMMER 1967
100 CONTINUE
GO TO(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13), J
1  F = (X-.12(1+.5))*(X+.2(1+.2))*1E-9
   RETURN
2  IF(ABS(X).GT.1.E9) X = 1.E8
   F = ABS(X-.9)**4.5
   RETURN
3  F = ABS(X-361.2)**.7*ABS(X+157.2)**1.5*((X-.1E-05)/ABS(X-.1E-10))
   RETURN
4  XALOG = 0.
   IF(X.NE.0.) XALOG = X*ALOG(ABS(X))
   F = (X-11.257)**3*(50.*X-.1*XALOG**2 -7.)
   RETURN
5  CONTINUE
   IF(ABS(X).GE.0.9) GO TO 501
   IF(X.LE.-.4) F = SIN(6.*X)
   IF(X.GE.-.4) F = .1
   IF(X.GE.-.1E-03) F = (X+.2E-08)/X*(X-.2E-08)/X
   IF(X.GE.-1.E-09) F = (X+.00001)*(1.+COS(400.*X))/2.
   IF(X.GE.-.1) F = ABS(X-.2)**.4*X
   IF(X.GE.-.5) F = ABS(X-.4)**2
501 IF(ABS(X).GE.0.9) F = ABS(X)
   IF(X.GT.313. AND.X.LT.403.) F = (X-300.)*(X-314.)*(X-399.)**2
   RETURN
6  F = ABS(X-17.0)**2*ABS(X-17.1)**1.8*(X-20.)
   RETURN
7  F = (X-1.)*(X-2.)*(X-3.)*(X-4.)*(X-5.)*(X-6.)
   RETURN
8  F = (((X-21.)*X+175.)*X-735.)*X+1624.)*X-1764.)*X+720.
   RETURN
9  Y = X + 1.1111
   F = 1. - Y*(100.-Y*(54.-Y*(12.-Y)))
   RETURN
10 Y = X - 1312.
   IF(ABS(Y).GT..1E+7) Y = SIGN(.1E+7, Y)
   F = ALOG10(1.+Y**2)*(Y+16.)*SQRT(ABS(Y-8.))
   RETURN
11 Y = SQRT(ABS(X-400.))
   IF(ABS(X).LE.400.) GO TO 1110
   F = ALOG10(97.)*COS(10.0 )+(X-400.)*(1.+3.*(X-400.))
   RETURN
1110 F = ALOG10(1.+Y**2)*COS(X/40.)
   RETURN
12 F = 1./(ABS(X**X)+1.)
   RETURN
C          FORESTRY - INVESTMENT RETURN FUNCTION
13 Y = 1. + X

```

```

F = 20./Y**15 + 36./Y**25 + 40./Y**33 + 475./Y**40 - 1.12*(Y**40-1.)
) / (X*Y**40) - 4.5 - 6./Y**4 - 2./Y**8
RETURN
C          TEST FUNCTIONS OF SEPT. 1967
200 GOTO(20,21,22,23,24,25,26,27,28,29,290,291),J
20 IF(ABS(X) .LT. 1.F+8) GO TO 101
F = 1.222F+0 - ABS(X)
RETURN
101 F = 1.+ X**4
RETURN
21 F = SIN(X)
RETURN
22 F = COS(.001*X)
RETURN
23 F = (X+1.)*(X+5.)*(X-27.)*(X-104.)
IF( ABS(F) .LT. 1.F-5*ABS(X) ) F = 13.
RETURN
24 DO 507 I = 1,40
507 DU = EXP(5.)*(DU+1.)/(ABS(DU)+COS(2))
Y = ABS(X)
F = SIN(X)*(X**2+1.) - EXP(SQRT(Y)/10.0*(X-1.))*(X**2-5.)
RETURN
25 IF( ABS(X) .GT. 6500.) +0 TO 601
F = SIN(X)*(X**2 + 1.) - EXP(SQRT(ABS(X)))*(X-1.)*(X**2-5.)
RETURN
601 F = 1.F+20+SIGN(1.,X-5.0)
RETURN
C          MINIMIZE INTEGRAL EXP(XT)(T*T+1) - (T**3+2) ON (0,1)
26 I700 = 1
Z = X*(1.000001) - 1.F-5*SIGN(1.,X)
702 G=0
DO 701 I = 2,40
T = FLOAT(I-1)/40.
701 G = G + ( EXP(Z*T)*(T**2+1.) - (2.+T**3) ) **4
G = (2.*G + 1. + (EXP(Z)*2.-3.)**4)/90.
GO TO (703,705), I700
703 G1 = G
Z = X*(1.00001) + 1.F-5*SIGN(1.,X)
I700 = 2
GO TO 702
705 F = (G - G1)/(2.F-5*(1.+ABS(X)))*SIGN(1.,X)
RETURN
27 F = (ABS(X-002.))**.4
RETURN
28 F = (X-2.)/(X-4.)*(X-5.)/(ABS(X-6.))**.6*SIGN(1.,X-6.)*(X*X+1.)
RETURN
29 IF( ABS(X) .LT. 10. ) GO TO 1010
F = ABS(X+0.)**2

```

```

RETURN
1010 F = SIGN(1.+X*X,SIN(X))
RETURN
200 F = SIGN(1.+X*X,SIN(X))
RETURN
201 IF( ABS(X) .GT. 08.0 ) X = 88.0*SIGN(1.0,X)
F = EXP(-X)*(1.+5*SIN(X))
RETURN

```

C
C

```

FUNCTIONS OF FFP, 196P
300 GOTO(30,31,32,33,34,35,36,37,38,39),J
20 F=1.
IF( X.GT.1000. .AND. X.LT. 1002. ) F= (X-1001.)*2.-1.
RETURN
21 +F( X.LT. -3. ) F=0.
IF( X.GE. -3. .AND. X.LT.-2. ) F=X+3.
IF( X.GE. -2. ) F= -1.
IF( X.GT. -1. .AND. X.LT.-1. ) F=1.
IF( X.GE. -1. .AND. X.LT.0. ) F=2.*X*X -1.

```

```

IF( X.GE. 0. .AND. X.LT.2. ) F=X-1.
IF( X.GE. 2. .AND. X.LT.4. ) F=X-3.
IF( X.GE. 4. ) F=COS(X-4.)
RETURN

```

```

22 F = SIN(PI/(AMAX)(1.E-12,ABS(X))*SIGN(1.,X)))*X
RETURN

```

DIST(X,1,1/10,1/100,---)

C

```

22 Y=-X
IF(Y.GE.0.) GOTO 016
Y=X-1.
IF(Y.GE.0.) GOTO 016
A=.1
010 Z=Y
Y= X-A
IF(Y.GE.0.) GOTO 014
A = .5)*A
GO TO 010
014 IF( Y.LE. -Z)GOTO 016
Y=-Z
016 F=Y
RETURN
24 Z = AMAX1(1.E-20,ABS(X))
F=X-1./SQRT(1.+1./(Z*7))
RETURN
25 F= (X+1.)/(X*X+2. )
RETURN

```

DIST(X,1,10,100,---)

C

```

26 Y=1.-X
IF( Y.GE.0. ) GO TO 014

```



```

A=10.
715 Z=Y
    Y=A-X
    IF( Y.GE.0. ) GO TO 712
    A3AM10.
    GOTO 715
712 IF( Y.LE.-Z ) GO TO 714
    Y= -Z
714 F=Y
    RETURN
27 F = SIN(PI/SQRT(A+5(X))) * Y
    RETURN
22 F=0.
    RETURN
20 X=X+1.
    F=0.
    RETURN

```

```

C
C -PROBLEMS TAKEN FROM TRAU(0-5), RALSTON(6-15), HILDEBRANDT(16-23)
400 CONTINUE
401 GOTO( 4 ,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59
*      ,60,61,62,63,64 ),J

```

```

40 F = (X*(X-1.))**2
    RETURN
41 F = X*X - 1.
    RETURN
42 F = X**20 - 1.
    RETURN
43 F = EXP(X)*(Y-1.))**2
    RETURN
44 F = X*(X-1.))**2
    RETURN

```

```

45 F = EXP(X)*(X-1.))**2
    RETURN
46 F = SIN(X) - X/2.
    RETURN
47 F = (SIN(X) - X/2.))**2
    RETURN
48 F = X*EXP(-X)
    RETURN
49 F = (((4.*X - 9.)*X + 9.)*X - 18.)
    RETURN
50 F = ((X-5.)*X + 17.)*X + 21.
    RETURN

```

```

61 F = (((X-3.) * X -4.) * X -150.) * X -100.
RETURN
62 F=1.
DO 451 LL = 1,20
651 F = F*(X-FLOAT(LL))
RETURN
63 F = COS(X) - X*EXP(Y)
RETURN
64 F = TAN(X) - COS(X) - .5
RETURN
65 F = (COS(X) - X*EXP(Y))**3
RETURN
66 IF( Y .EQ. 0. ) X = 1.E-250
F = X + ALOG(ABS(X))
RETURN
67 F = (X*X -1.) * X -1.
RETURN
68 F = X - I(P(-X))
RETURN
69 F = TAN(X) - 1.01*X
RETURN
60 F = TAN(X) - 2.*X
RETURN
61 F = TAN(X) - 30.*X
RETURN
62 F = X**2 -2.*X -5
RETURN
63 F = X*(X-3.) -4.*(SIN(X))**2
64 CONTINUE
RETURN
C
C      FUNCTIONS OF FEBRUARY 1967
600 CONTINUE
GO TO(71,71,72,73,74,75,76,77,78,79,80,81,82,83) J
70 Y = X-112.
670 F = SIN(Y)*(1. +Y*Y) - EXP(SQRT(ABS(Y))/10.)*(Y-1.)*(Y*Y-5.)
RETURN
71 Y = X + 0422.
IF( ABS(Y) .GT. 2.5E+6 ) Y = SIGN(2.5E+6,Y)
GO TO 670
72 Y = X+3.
IF( ABS(Y) .GT. 1. ) GO TO 672
F = 2. *COS(6.*PI * Y)
RETURN
672 F = 1 + Y*Y
RETURN
73 Y = 10.*(X+6.2)
IF( ABS(Y) .GT. 1. ) GO TO 673
F = 2.*COS(0.*PI *Y)
RETURN
673 F = (1.+Y*Y)*(1.+(Y-1.)*(3.+(Y-1.)*(-2.-Y)))
RETURN

```

```

74 F = (X-1.1)*(X-1.11)*(Y-2.)*(X-2.002)*(X-2.00002)*(X-4.)*(Y-4.05)
   * (X-7.06)
   RETURN
75 Y = ABS(X)
   IF( Y .GT. 500. )      GO TO 575
   F = EXP(-Y)
   RETURN
575 F = .1E-175
   RETURN
76 Y = ABS(X)
   IF( Y .GT. 10000. )    GO TO 576
   F = (X-1.)*(X+7.4)*EXP(-Y)
   RETURN
576 F = .1E-175
   RETURN
77 F = (X-2.)**2*(X+10.)*(X-252.)/(X**6*(1.+9*SIN(X))+.001)*1.E+3
   RETURN
78 F = (X-400.)*(X+202.)*(Y+2)*(X-10000.)*.1E+5/
   * ((X**P + 1.)*(1.+P*COS(X)))
   RETURN
79 F = (X-2.)*(X+6.)*1.E+4/(1.+X**12.)
   RETURN
80 IF( X .GE. 2. )      F = YX-2.)*(1.+X*Y/EXP(SQRT(Y)))
   IF( X .LT. 2. .AND. X .GT. 1. )  F = SQRT(ABS(X-2.)*ABS(X-1.))
   IF( X .LT. 1. .AND. X .GT. 0. )  F = X**2.2*(1.-X)**1.4
   IF( X .LE. 0. .AND. X .GT. -1. ) F = (-X)**1.4*(X+1.)**.6
   IF( 0 .LE. -1. )      F = (-1.-X)**1.8*SIN(1./SQRT(-1.-X))
   RETURN
81 IF( X .GT. -1. )    F = ABS(X)**1.4*(X+1.)**.6
   IF( X .LE. -1. )    F = (1.+X)**2*(1. + .98*SIN(1./SQRT(-1.-X)))
   RETURN
82 Y = 400.*(X-.01)
   IF( ABS(Y) .LT. 1. )  F = (Y+.2)*(Y-.3)
   IF( ABS(Y) .GE. 1. )  F = .34*(1.+SIN(Y))+.1*Y*Y
   RETURN
83 Y = 1.E+9*(X-1.E-9)
   F = (Y-4.)*(Y+2.)*(Y+41.)
   RETURN
END

```