Analysis of the Influence of Seat-Plating Or Cushioning on Valve Impact Stresses in High Speed Compressors

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ANALYSIS OF THE INFLUENCE OF SEAT-PLATING OR CUSHIONING ON VALVE IMPACT STRESSES IN HIGH SPEED COMPRESSORS

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INTRODUCTION

Failure of compressor valve reeds or plates due to impact between the valve and the seat has been recognized as one of the major causes of valve failure. Design procedures have been recommended that limit the impact-velocity (commonly known as setting velocity) of the valves [6]. An empirically established relationship has been defined in literature [3] for this setting velocity and is given as:

$$V_s = \omega H$$

where ω is the rotational speed of the compressor [rad/sec] and H is the maximum valve displacement. It has been recommended that this setting velocity should be below 0.1 to 0.2 [m/sec].

The stresses in compressor valves have been investigated in the past [5]. However, main emphasis has been on the stresses that occur due to the operating loads. The very nature of impact stresses makes it extremely difficult, if not impossible, to measure these stresses. An attempt was made in the recent past to predict the impact stresses occurring due to direct colinear impact between the valve and the valve seat [1].

A natural question arises as to what would be the effect of cushioning or seat-plating on these stresses. The present paper is an attempt to extend the work done in reference [1] to investigate the above question. Seat-plating and/or cushioning, in effect, means putting a sandwich material between the valve plate and the valve seat. The generalized case of impact process between three materials of different material properties will be analyzed. The formulas for impact stresses developing in each of the three materials will be established and it will be shown that the resulting maximum stresses do indeed depend upon the material properties of the cushioning or plating material.

THEORETICAL ANALYSIS

It is assumed that the impact is colinear. This is true in case of rigid ring plate valves and in case of reed valves, it will be true only as a first approximation, since in this case the impact velocities are approximately equal to the product of the time derivative of the first mode participation factor and the first mode [4]. Let h_1, h_2, and h_3 be the thicknesses of valve plate, cushion or seat-plating material, and valve seat, respectively. Figure 1 shows the coordinates and the velocity distribution at the beginning of impact. For convenience in writing, we will call valve plate as plate, and valve seat as seat hereafter. Let V_o be the impact velocity of the plate just before the impact begins. Let the plate, the cushion, and the seat be represented by suffixes 1, 2, and 3, respectively. Then, the governing equations of

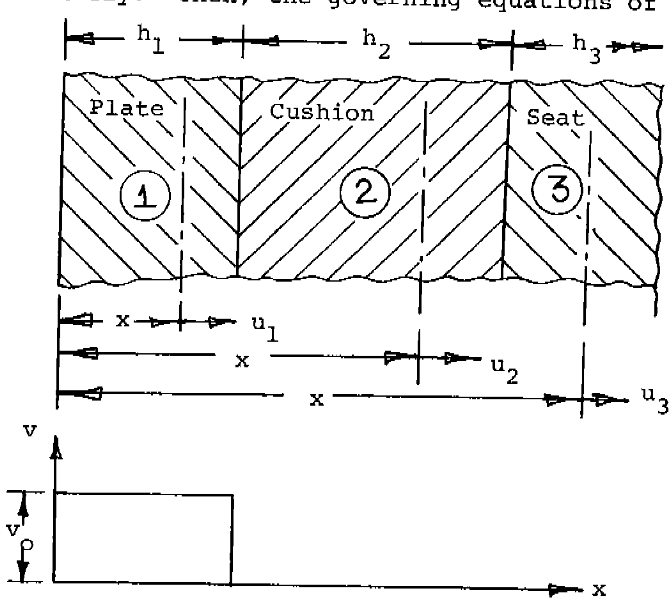

Figure 1. Coordinates and Velocity Distribution at the Beginning of Impact.

motion for the three materials can be written as follows [2].

for plate: $$\frac{\partial^2 u_1}{\partial t^2} = c_1^2 \frac{\partial^2 u_1}{\partial x^2} \quad (1)$$

for cushion: $$\frac{\partial^2 u_2}{\partial t^2} = c_2^2 \frac{\partial^2 u_2}{\partial x^2} \quad (2)$$

for seat: $$\frac{\partial^2 u_3}{\partial t^2} = c_3^2 \frac{\partial^2 u_3}{\partial x^2} \quad (3)$$

where u_1, u_2, and u_3 are the respective elastic displacements as shown in Fig. 1; and c_1, c_2, and c_3 are the wave velocities in the plate, the cushion, and the seat, respectively and are defined as follows:

$$c_1 = \sqrt{\frac{E_1}{\rho_1}} \quad (4)$$

$$c_2 = \sqrt{\frac{E_2}{\rho_2}} \quad (5)$$

$$c_3 = \sqrt{\frac{E_3}{\rho_3}} \quad (6)$$

where E_1, E_2, and E_3 are the Young's modulus of elasticity of materials 1, 2, and 3 and ρ_1, ρ_2, ρ_3 are corresponding mass densities. The solutions of equations (1), (2), and (3) are given as [2]:

$$u_1(x,t) = f_1(x - c_1t) + g_1(x + c_1t) \quad (7)$$

$$u_2(x,t) = f_2(x - c_2t) + g_2(x + c_2t) \quad (8)$$

$$u_3(x,t) = f_3(x - c_3t) + g_3(x + c_3t) \quad (9)$$

Functions f_1, f_2, and f_3 represent forward waves (i.e. waves moving in the positive x-direction) moving with velocity c_1, c_2, and c_3 respectively and g_1, g_2, and g_3 represent backward waves (i.e. waves moving in the negative x-direction) also moving with velocity c_1, c_2, and c_3 respectively. The velocities and stresses in the three materials will, then, be given by:

$$v_1(x,t) = \frac{\partial u_1}{\partial t} = c_1[-f_1'(x-c_1t) + g_1'(x+c_1t)] \quad (10)$$

$$v_2(x,t) = \frac{\partial u_2}{\partial t} = c_2[-f_2'(x-c_2t) + g_2'(x+c_2t)] \quad (11)$$

$$v_3(x,t) = \frac{\partial u_3}{\partial t} = c_3[-f_3'(x-c_3t) + g_3'(x+c_3t)] \quad (12)$$

$$\sigma_1(x,t) = E_1\frac{\partial u_1}{\partial x} = \rho_1 c_1^2[f_1'(x-c_1t)+g_1'(x+c_1t)] \quad (13)$$

$$\sigma_2(x,t) = E_2\frac{\partial u_2}{\partial x} = \rho_2 c_2^2[f_2'(x-c_2t)+g_2'(x+c_2t)] \quad (14)$$

$$\sigma_3(x,t) = E_3\frac{\partial u_3}{\partial x} = \rho_3 c_3^2[f_3'(x-c_3t)+g_3'(x+c_3t)] \quad (15)$$

The primes on the functions f and g indicate the derivatives with respect to the arguments of the respective functions. For convenience in writing, hereafter we will omit these arguments. To obtain the stress functions for each of the three materials, the above equations have to be solved using the initial conditions as follows. For the plate, the initial conditions are: At t=0, $v_1=V_o$ and $\sigma_1 = 0$. Thus:

$$c_1[-f_1' + g_1'] = v_o \quad (16)$$

$$f_1' + g_1' = 0 \quad (17)$$

From (16) and (17), the initial magnitudes of f_1' and g_1' are given as:

$$f_1' = -\frac{v_o}{2c_1} \quad (18)$$

$$g_1' = \frac{v_o}{2c_1} \quad (19)$$

Conditions at Free Boundary

Since a free boundary cannot support any normal stress, we get at x=0 and for all t,

$$f_1' + g_1' = 0$$

or

$$f_1' = -g_1' \quad (20)$$

The interpretation of Equation (20) is that a wave incident on the free boundary is reflected back in the form of a wave of equal magnitude but opposite in nature and direction. For example a compression wave moving toward the free boundary will be reflected back as a tension wave of same magnitude moving away from the boundary.

Conditions at Interface 1-2

To find out what happens to the waves reaching the interface between materials 1 and 2 from either side, let us solve the equations for the cushion (i.e. material 2). As long as the plate and the cushion remain in contact due to the impact, the velocities and stresses in both the plate and the cushion at the contact point (i.e. $x=h_1$) should remain the same. Thus, we get:

$$c_1(-f_1' + g_1') = c_2(-f_2' + g_2') \quad (21)$$

$$\rho_1 c_1^2(f_1' + g_1') = \rho_2 c_2^2(f_2' + g_2') \quad (22)$$

We should also recognize the fact that when a wave is incident upon an interface between two materials, it breaks into two parts. One part is reflected back into the same material from where the original wave came and the other part is transmitted into the second material. This means that there is no opposing wave in the second material at this moment. Therefore, if we are considering f_1' as the incident wave, g_2' must be zero. Then, from (21) and (22) we get:

$$f_2' = \frac{c_1}{c_2}\left(\frac{2\rho_1 c_1}{\rho_1 c_1 + \rho_2 c_2}\right) f_1' \quad (23)$$

$$g_1' = -\left(\frac{\rho_1 c_1 - \rho_2 c_2}{\rho_1 c_1 + \rho_2 c_2}\right) f_1' \quad (24)$$

where g_1 is the wave-component reflected back into the plate and f_2' is the one transmitted into the cushion. The same two equations (23 and 24) will be true for a wave travelling from the cushion towards the plate and incident on interface 1-2, except that suffixes 1 and 2 as well as functions f' and g' will now be interchanged. Thus, we would have:

$$g_1' = \frac{c_2}{c_1}\left(\frac{2\rho_2 c_2}{\rho_2 c_2 + \rho_1 c_1}\right) g_2' \quad (25)$$

$$f_2' = -\left(\frac{\rho_2 c_2 - \rho_1 c_1}{\rho_2 c_2 + \rho_1 c_1}\right) g_2' \quad (26)$$

Conditions at Interface 2-3

The boundary conditions for the seat are also the same as those for cushion, i.e. the velocity and the stresses in both the cushion and the seat at the interface between them will be the same as long as the impact contact is maintained. The only other point to note is that we assume that there will never be any reflected wave g_3' in the seat simply because h_3, the seat thickness, is too large as compared to h_1, or h_2, as seat is part of the total cylinder head and therefore the forward wave f_3' would have dissipated long before it would have a chance to be reflected back. However, in cases where the back reflection cannot be neglected, the outlined procedure can easily be modified to take it into account. Thus, from (12) and (15) we get:

$$c_2(-f_2' + g_2') = c_3(-f_3' + g_3') \quad (27)$$

$$\rho_2 c_2^2(f_2' + g_2') = \rho_3 c_3^2(-f_3' + g_3') \quad (28)$$

$$g_3' = 0 \quad (29)$$

Solving for f_3' and g_2' in terms of f_2', we get:

$$f_3' = \frac{c_2}{c_3}\left(\frac{2\rho_2 c_2}{\rho_2 c_2 + \rho_3 c_3}\right) \quad (30)$$

$$g_2' = -\left(\frac{\rho_2 c_2 - \rho_3 c_3}{\rho_2 c_2 + \rho_3 c_3}\right) f_2' \quad (31)$$

Resulting Stresses

The above analysis clearly demonstrates that the plate and the cushion, in general, will have a forward moving wave and a backward moving wave at any instant and the total stress will be the sum of the two which can be obtained by superposition since the governing equation is a linear one. For the seat, there is only one wave, the forward moving wave.

However, we must recognize the fact that there is a time lag between the incident wave on one interface and the wave reflected from the other interface. This time lag is equal to $2h/c$, where h is the distance between the two interfaces and c is the wave velocity in the material connecting the two interfaces. Therefore, to obtain the maximum stress in any of the three materials, the total stress function $(f'+g')$ should be plotted over a period of time. One such plot is shown for an example case in Figure 3.

Example Case

Let us analyze an example case of a soft cushion and the plate and the seat of harder material. Let us assume the following data:

$$\rho_1 c_1 = \rho_3 c_3 = 2\rho_2 c_2$$

$$c_1 = c_3 = 2c_2$$

$$h_2 = 2h_1$$

Then, at the beginning of the impact we have:

$$f_1' = -\frac{V_o}{2c_1} \quad \text{(compressive wave)}$$

$$g_1' = \frac{V_o}{2c_1} \quad \text{(tensile wave)}$$

$$f_2' = 0$$

$$g_2' = 0$$

$$f_3' = 0$$

Readers who have an acoustics background should note that this case is more difficult than the three media case in acoustics since there we are treating steady state harmonic motion which simplifies matters considerably. Here, we have an initial value problem and the waves are not harmonic, which forces us to follow the waves as they develop. Let us use subscripts I, II, III, etc. to denote the number of reflections. These reflections are sketched in Figure 2. Then, after the first reflection, the new stress wave functions will be:

$$(f_1')_I = -g_1' = f_1' = -\frac{v_o}{2c_1}$$

$$(g_1')_I = -\left(\frac{\rho_1 c_1 - \rho_2 c_2}{\rho_1 c_1 + \rho_2 c_2}\right) f_1' = -\frac{f_1'}{3}$$

$$(f_2')_I = \frac{c_1}{c_2}\left(\frac{2\rho_1 c_1}{\rho_1 c_1 + \rho_2 c_2}\right) f_1' = \frac{8}{3} f_1'$$

$$(g_2')_I = -\left(\frac{\rho_2 c_2 - \rho_3 c_3}{\rho_2 c_2 + \rho_3 c_3}\right)(f_2')_I = \frac{8}{9} f_1'$$

$$(f_3')_I = \frac{c_2}{c_3}\left(\frac{2\rho_2 c_2}{\rho_2 c_2 + \rho_3 c_3}\right)(f_2')_I = \frac{8}{9} f_1'$$

After the second reflection, we will have the following functions:

$$(f_1')_{II} = -(g_1')_I = \frac{f_1'}{3}$$

$$(g_1')_{II} = \frac{c_2}{c_1}\left(\frac{2\rho_2 c_2}{\rho_1 c_1 + \rho_2 c_2}\right)(g_2')_I = \frac{8}{27} f_1'$$

$$(f_2')_{II} = -\left(\frac{\rho_2 c_2 - \rho_1 c_1}{\rho_2 c_2 + \rho_1 c_1}\right)(g_2')_I = \frac{8}{27} f_1'$$

$$(g_2')_{II} = -\left(\frac{\rho_2 c_2 - \rho_3 c_3}{\rho_2 c_2 + \rho_3 c_3}\right)(f_2')_{II} = \frac{8}{81} f_1'$$

$$(f_3')_{II} = \frac{c_2}{c_3}\left(\frac{2\rho_2 c_2}{\rho_2 c_2 + \rho_3 c_3}\right)(f_2')_{II} = \frac{8}{81} f_1'$$

The above process will continue as further reflections and transmissions of waves continue. However, since the magnitude of these stress functions keeps on reducing drastically with the number of reflections, only the first two reflections will be sufficient to give the maximum value of stresses in this case. To obtain total stress at any instance, the sum of the forward and backward moving waves has to be multiplied by the corresponding value of ρc^2. Figure 3 shows the plot for this case. The maximum stresses for this example case are:

(i) Plate Material:

$$(\sigma_1)_{max} = \rho_1 c_1^2 (f_1' + g_1')_{max}$$

$$= \rho_1 c_1^2 \left(\frac{2}{3} f_1'\right)$$

$$= \rho_1 c_1^2 \times \frac{2}{3} \times \left(-\frac{v_o}{2c_1}\right)$$

$$= -\frac{\rho_1 c_1 v_o}{3} \text{ [compressive]}$$

(ii) Cushion:

$$(\sigma_2)_{max} = \rho_2 c_2^2 (f_2' + g_2')_{max}$$

$$= \rho_2 c_2^2 \left(\frac{8}{3} + \frac{8}{9}\right) f_1'$$

$$= \frac{\rho_2 c_2}{\rho_1 c_1} \frac{c_2}{c_1} (\rho_1 c_1^2) \left(\frac{32}{9}\right) f_1'$$

$$= \frac{1}{2} \times \frac{1}{2} \rho_1 c_1^2 \times \frac{32}{9}\left(-\frac{v_o}{2c_1}\right)$$

$$= -\frac{4}{9} \rho_1 c_1 v_o$$

(iii) Seat Plate:

$$(\sigma_3)_{max} = \rho_3 c_3^2 (f_3')_{max}$$

$$= \rho_3 c_3^2 \left(\frac{8}{9} f_1'\right)$$

$$= -\frac{4}{9} \rho_1 c_1 v_o$$

The corresponding stresses without using the cushion and keeping the plate and the seat of the same material would be:

$$\sigma_{plate} = -\frac{\rho_1 c_1 v_o}{2}$$

$$\sigma_{seat} = -\frac{\rho_1 c_1 v_o}{2}$$

This shows that the soft cushion does indeed reduce the stress level in the valve plate and also, to a reduced degree, in the valve seat.

SUMMARY AND DISCUSSION

The effect of inserting a cushion between the valve plate and the seat or plating the seat on the impact stresses generated in the valve as well as the plate was discussed. The analysis is based on the assumption of colinear impact between all the three materials.

An example case of a soft cushion of twice the thickness of the valve plate was analyzed. The results do indeed indicate that the impact stresses are reduced both in the valve plate as well as in the seat. The amount of reduction in stress level will depend upon the softness (defined in terms of ρ and c) of the cushion material relative to the plate and seat materials, as well as possibly on the thickness of the cushion. The effect of varying the thickness is presently under investigation.

This is not to suggest that the above analysis will give us any breakthrough in the valve design. It has been intuitively felt in industry here and there that a softer seat would reduce the stress levels. In fact, it is our belief that seat-cushioning might even have been tried before by some. This paper is an attempt to explain what might have been felt intuitively. Besides, the problem of getting proper soft materials for cushions which may not deteriorate under the hostile atmosphere of high temperatures, lubricants, and refrigerants, etc. adds a new dimension to the problem. Still, we feel that the present analysis is a step in the right direction.

NOMENCLATURE

v_s = setting velocity [m/sec]

ω = rotational speed of compressor [rad/sec]

H = maximum valve displacement [m]

h_1 = thickness of valve plate [m]

h_2 = thickness of cushion material or seat plating [m]

h_3 = thickness of valve seat [m]

v_o = velocity of valve plate at the start of impact [m/sec]

v = velocity at any instant [m/sec]

x = coordinate [m]

u = elastic displacement [m]

t = time [sec]

c = wave speed [m/sec]

E = Young's modulus [N/m^2]

ρ = mass density [kg/m^3]

f, f' = waves travelling in positive x-direction

g, g' = waves travelling in negative x-direction

σ = stress [N/m^2]

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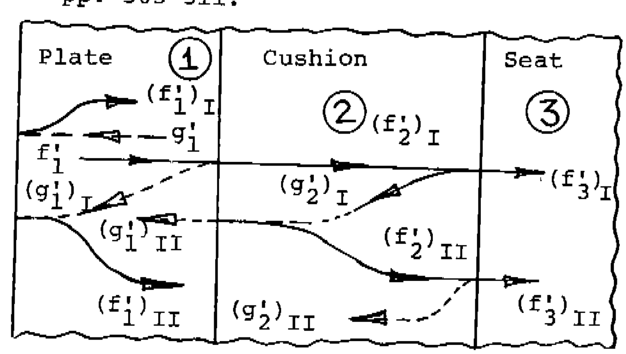

Fig. 2: Wave reflections

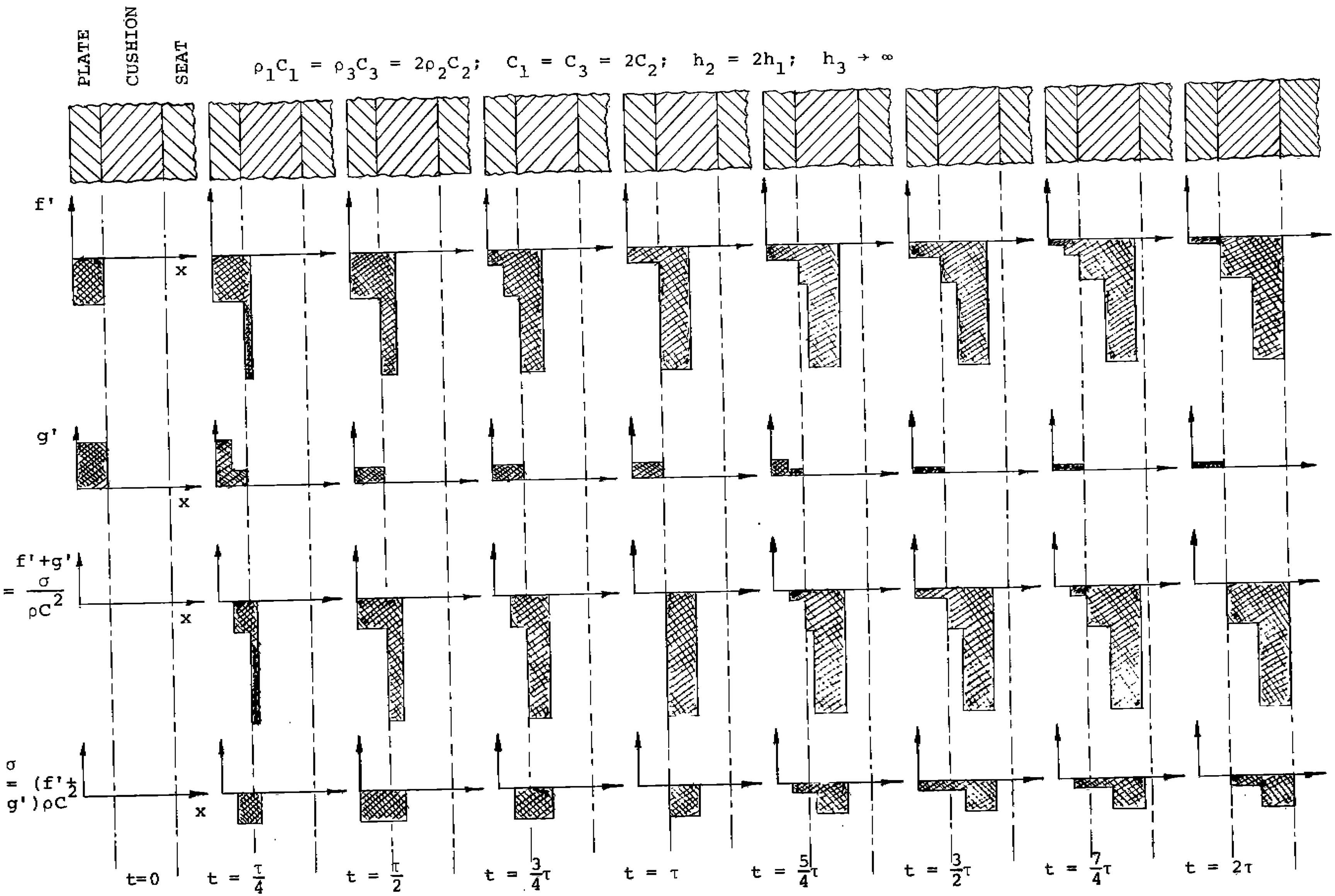

Fig. 3: Stress Wave Propogation for identical seat and plate materials and 'soft' cushion.

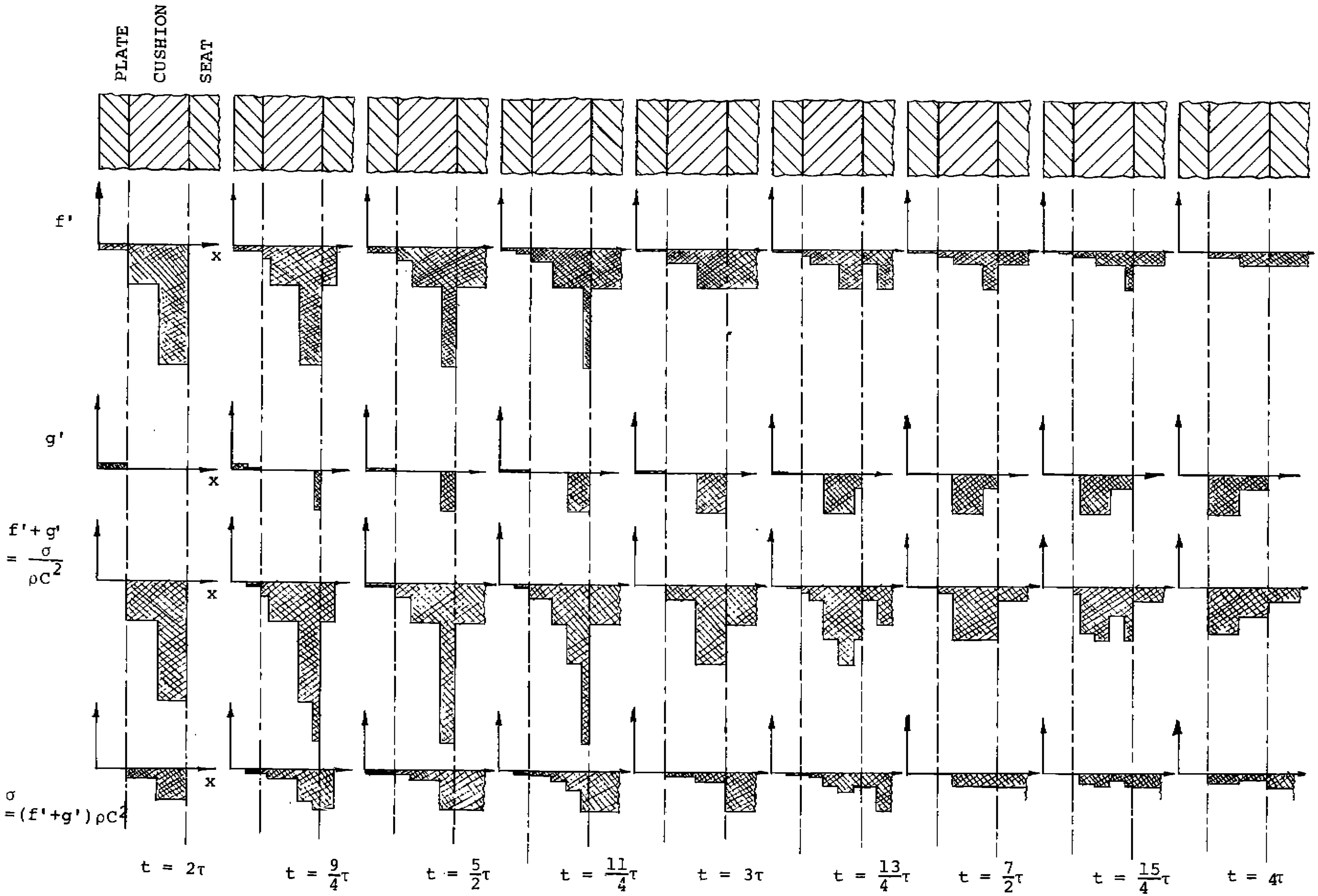

Fig. 3 -- Continued.

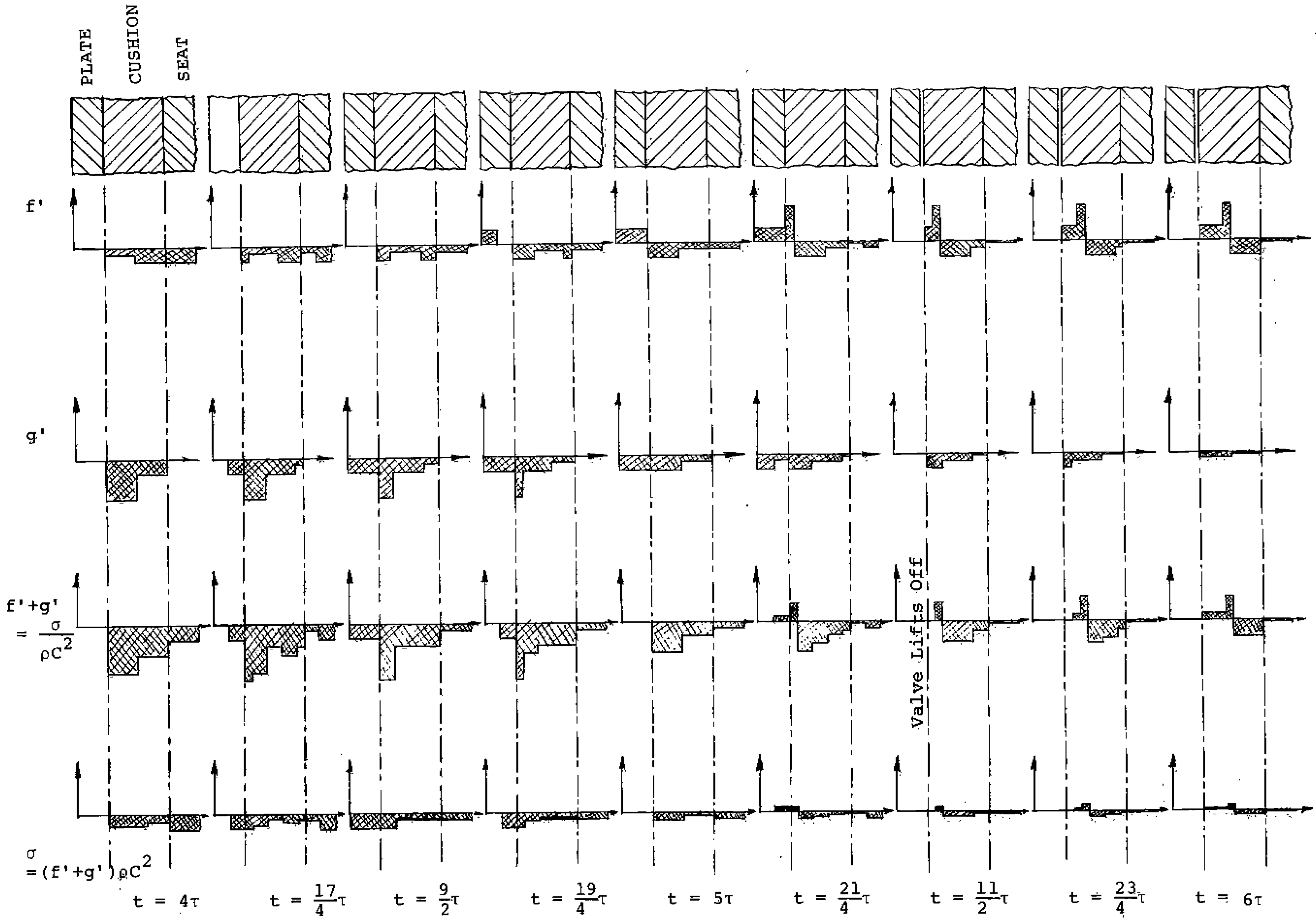

Fig. 3 -- Continued