Warmstarting the Constrained Optimal Filter Design Problem for Active Noise Control Systems in Conic Formulation

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Applications

For many practical active noise control applications:

• **Multichannel systems** : for large-size quiet zone.

• **Multiple constraints** : robust stability, enhancement, filter output power.
Background

One common approach for designing constrained multichannel controller: solve a constrained optimization problem

- Advantage: better noise control performance

- Challenge: significant computational effort (large channel number, filter order, number of the constraints)
Background

This work is a continuation of our previous work of convex & cone formulation:

• Zhuang and Liu, JASA 2021:

• Zhuang and Liu, InterNoise 2020:

• Zhuang and Liu, NoiseCon 2019:
Background

Control System Setup

Traditional Formulation

Convex Formulation

Cone Formulation

- Proposed previously
- Computational time: hours → seconds

Benefits of shorter computational time:
- Reducing time and cost during product design circle
- Make continuously design possible for time-varying environment.
Motivation

For proposed formulation, **warmstarting** strategies are difficult.

- **Cold start:**
  choosing initial guess **without** information of approximate location of optimal solution.
  e.g., use origin (0,0,...,0), or identity (1,1,...,1).

- **Warm start:**
  choosing initial guess **using** information of approximate location of optimal solution.
  e.g., the optimal solution of a similar but different environmental setup
Motivation

Why warmstarting strategies are important?

- **Commercial product design:**
  - Current product model may be a variation of previous models
  - Product differs from prototype by batch manufactural error

- **Time-varying applications:**
  - the optimal filter coefficients of previous environment condition can be used as the initial guess when the condition changes.
Review – Control diagram

- **Objective:** minimize the power of $\vec{e}$
- **Robust stability:** the feedback loop $W_x \hat{G}_{s_0}$
- **Output power:** Power of $W_x$ or $\vec{y}$
- **Disturbance enhancement:** $\vec{e}$ should not be amplified at certain frequency bands

If $\hat{G}_{s_0} = G_{s_0}$
Review – Convex and cone formulation

**Convex formulation**

**Cost function:** Quadratic function

**Constraints:**
- Enhancing Quadratic function
- Constraining total power of \(e\)

**Filter response:** Quadratic function
- The magnitude of frequency response

**Stability:** Max of eigenvalue
- Use Nyquist criterion

**Robustness:** Max of singular value
- \(M-\Delta\) structure and small gain theory

**Cone formulation**

**Cost function:** Linear

**Constraints:**
- Linear equalities or inequalities
- Second-order cones:
  \[ \{ (y, \tilde{x}) \in \mathbb{R} \times \mathbb{R}^{n_i-1} : y \geq \|\tilde{x}\|_2 \} \]
- Positive semidefinite cones:
  \[ \{ \text{vec}(X) \in \mathbb{R}^{n_i^2} : X \in \mathbb{R}^{n_i \times n_i} \text{ is positive semidefinite} \} \]
For cone programming algorithm, each iteration should:

- Inside the constraint boundaries
- Away from boundary as much as possible (follow the central path)

Use optimal point as new initial guess

Does not work!
Method – Warmstarting method

Use convex combination of cold start point and previous optimal point:
- Guarantees a usable initial guess (close enough to cold start)
- Very little extra computational effort for warm start point

Proposed by Anders Skajaa et al. in 2013
Method – Convert PSD cones to SOCs

Convex formulation

Cost function: Quadratic function

Constraints:

Enhancement: Quadratic function

Filter response: Quadratic function

The magnitude of frequency response

Stability: Max of eigenvalue

Use Nyquist criterion

Robustness: Max of singular value

$M - \Delta$ structure and small gain theory

• Need second-order cone (SOC) only

• The stability and robustness constraints can only be reformulated equivalently to positive semidefinite (PSD) cones

• Some relaxation must be done to convert them to second-order cones (SOCs)
Method – Convert PSD cones to SOCs

Stability: Max of eigenvalue
Use Nyquist criterion

Robustness: Max of singular value
$M - \Delta$ structure and small gain theory

Method 1: use max-norm properties:

$$\|M\|_{max} \leq \|M\|_2 \leq \sqrt{mn}\|M\|_2$$

PSD converts to SOCs:

$$\|W_x(f_k)\|_{max} \leq \frac{C(f_k)}{\sqrt{N_rN_s}\|\widehat{G}_{s0}(f_k)\|_2}$$

Method 2: use Frobenius norm properties:

$$\|M\|_2 \leq \|M\|_F$$

PSD converts to SOCs:

$$tr(\widehat{G}_{s0}(f_k)W_x^H(f_k)W_x(f_k)\widehat{G}_{s0}(f_k)) \leq C^2(f_k)$$

Open Loop Response

$$\|W_x(f_k)\widehat{G}_{s0}(f_k)\|_2 \leq C(f_k)$$
Result – Experimental setup

A multi-channel active noise control system on a wind channel
Result – Comparison of two methods

Method 1: use max-norm
Method 2: use Frobenius norm

- Converting constraints will sacrifice performance
- Method 2 has better performance (less conservative)
Result – Warmstarting performance

\[ S_{xx}^{new} \leq S_{xx}(E_n + \alpha P_n) \]

Auto spectral density function of newly generated noise signal

\[ E_n = \begin{pmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{pmatrix} \]

Each element of \( P_n \) is generated by a standard Gaussian process

Measured auto spectral density function - known optimal filter coefficients

Perturbation ratio - represents the changes of environmental setup
Result – Warmstarting performance

- Perturbation ratio = 0.1%
- Perturbation ratio = 1.0%
- Perturbation ratio = 5.0%

Proposed Method 2

![Graph showing iterations ratio (warm/cold) vs warm ratio for Proposed Method 2]

Original Formulation

![Graph showing iterations ratio (warm/cold) vs warm ratio for Original Formulation]

Warm ratio: closer to 1, initial point closer to previous optimal solution
When warm ratio is higher than 0.999, it goes outside the constraints.
Conclusion

• Two methods of converting the positive semidefinite cones into second order cones are proposed.

• After using the proposed formulation method 2, the iteration number can be reduced up to 45% when using the warmstarting strategy.

• For a relatively wide range of problem perturbation ratio (from 0.1% to 5%), the warmstarting method is robust when choosing the same warm ratio parameter.
Thank you!