1994

Development of a Transient Simulation Model of a Freezer Part I: Model Development

X. Yuan
Xi'an Jiaotong University

D. L. O'Neal
Texas A&M University

DEVELOPMENT OF A TRANSIENT SIMULATION MODEL OF A FREEZER PART I: MODEL DEVELOPMENT

Xiuling Yuan
Dept. of Power and Machinery
Xian Jiaotong University
Xi'an, China

Dennis L. O'Neal
Dept. of Mechanical Engineering
Texas A&M University
College Station, TX 77843-3123

Abstract

A dynamic and distributed parameter model was developed to analyze a freezer system. Basic conservation equations were used to develop a mathematical model of the freezer, which consisted of models of the condenser, the evaporator, the compressor model and the capillary. Based upon the freezer model, a computer program was developed to predict changes of pressure, temperature, flow rate, etc., with both time and space.

Nomenclature

A	area (m^2)
J	conversion factor $(1 w\cdot m/w\cdot s)$
M	density of mass flow $(kg/m^2\cdot s)$
ΔX	difference of quality
Q	heat transfer rate (w)
L	length (m)
$\dot{m}$	mass flow rate (kg/s)
Nu	Nusselt number
Pr	Prandtl number
P	pressure (pa)
X	refrigerant quality
Re	Reynolds number
h	specific enthalpy (J/kg)
c	specific heat $(J/kg°C)$
Cp	specific heat at constant pressure $(kJ/kg\cdot k)$
T	temperature $(K°C)$
U	overall heat transfer coefficient $(w/m^2\cdot k)$
$w\cdot u$	velocity (m/s)

Subscripts

a	circumstance
c	critical state
e	evaporator
so	inlet of suction value
i	inside, inlet, control volume
L	liquid phase
o	outside, outlet
r	refrigerant
s	saturated state
t	time
w	tube wall
υ	vapor phase

Greek Letters

ε	calculation accuracy
ρ	density (kg/m^3)
μ	dynamic viscosity $(N\cdot S/m^2)$
υ	specific volume (m^3/kg)
λ	thermal conductivity $(w/m°\cdot C)$

Introduction

Improvement in the efficiency of small refrigeration plants plays an important part in the savings of electric energy. It is necessary for the designers to know the operational characteristics of the individual parts in the system to predict the performance of the system. Optimization of performance can be achieved through a dynamic simulation of the system. This paper presents a mathematical model to predict transient behavior of a freezer. A second paper provides some limited experimental validation.

Mathematical Model of Freezer System

A schematic view of the refrigeration system is shown in Figure 1. The operational states and parameters of a refrigeration system vary with both time and space from its start-up to shut-down. The system was divided into four main parts: compressor, condenser, evaporator, and capillary tube. With the exception of the compressor, each component was subdivided into several control volumes along its axial direction (one-dimensional analysis). The continuous physical process of the refrigeration system was analyzed by assuming a series of quasi-static processes within short time intervals. Figure 2 shows the ith control volume represented by a length of δX. Governing equations of the control volume were established for each components and solved using an implicit finite difference method. The models of each component were coupled to form a complete model of the closed system. The model was based on the following assumptions: (1) The pressure drop of the refrigerant produced in the tube of the condenser and evaporator was ignored, (2) The influence of oil on the refrigeration circuit was neglected, (3) Refrigerant flow was assumed to be one-dimensional, (4) Two-phase of vapor and liquid were at a thermodynamic equilibrium state in each section, (5) Thermophysical properties were uniform at any cross-section, and (6) The thermal resistance of the metal tube was neglected.

Mathematical Model of the Condenser and Evaporator

The process of heat transfer occurring in the condenser and evaporator is complicated by the existence of phase change. The governing equations for the refrigerant inside the control volume were developed by the application of the basic conservation equations. The refrigerant flow inside the tube was considered axially one-dimensional. The viscous dissipation and pressure dissipation and axial heat conduction of the fluid were neglected. The continuity and energy equations were given below:

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}=0 \tag{1}$$

$$\frac{\partial(\rho h-p)}{\partial t}+\frac{\partial(\rho u h)}{\partial x}+\frac{Q_i}{V_i}=0 \tag{2}$$

We get the following governing equations for the tube wall with assumption (6):

$$\left(\rho A C_p\right)_w \frac{\partial T_w}{\partial t}=\frac{Q_i}{\Delta x}-\frac{Q_o}{\Delta x} \tag{3}$$

A fully implicit finite difference formulation of the governing equations has been adopted here for the heat exchangers, while each of the individual components is updated in an explicit fashion. The discretized formulation of equations (1) through (3) were:

$$\dot m_{i+1/2}^{n}-\dot m_{i-1/2}^{n}+\frac{V_i}{\Delta t}\left(\rho_i^{n}-\rho_i^{n-1}\right)=0 \tag{4}$$

$$\dot m_{i-1/2}^{n} h_i^{n}-\dot m_{i-1/2}^{n} h_{i-1}^{n}+\frac{V_i}{\Delta t}\left(\rho_i^{n-1}h_i^{n}-\rho_i^{n-1}h_i^{n-1}\right)-\frac{V_i}{\Delta t}\left(p_i^{n}-p_i^{n-1}\right)+Q_i=0 \tag{5}$$

$$\left(M C_p\right)_w \frac{T_{wi}^{n}-T_{wi}^{n-1}}{\Delta t}=Q_i-Q_o \tag{6}$$

Heat Transfer Relationships in the Condenser

There were two flow steps inside the condenser. The first was single-phase convective heat transfer and the other was condensing heat transfer of saturated vapor. Standard heat transfer correlations were used to calculate the heat transfer coefficients. The heat transfer coefficient for single-phase heat transfer inside a tube is given by (3):

$$h_i=0.023\,Re^{0.8}\,Pr^{0.3}\,\lambda_r/d_i \tag{7}$$

The two phase condensation inside a tube was calculated from the following:

when $Re_L<5000$ and $1000<Re_V<20{,}000$ $\quad h_i=13.8 P_{rL}^{1/3}\cdot F^{1/6}\cdot Re_V^{0.2}\lambda_L/d_i$ (8)

$20{,}000<Re_V<100{,}000 \quad h_i=0.1 P_{rL}^{1/3}\cdot F^{1/6}\cdot Re_V^{2/3}\lambda_L/d_i$ (9)

when $Re_L>5000, \quad h_i=0.026 P_{rL}^{1/3}\,Re_m^{0.8}\lambda_L/d_i$ (10)

where $Re_L = d_i \rho_L u_L / \mu_L$

$$Re_V = d_i \rho_L u_L / \mu_L (\rho_L / \rho_V)^{0.5}$$

$$F = (h_V - h_L) / \left[C_{pL}(Tr - Tw)\right]$$

$$Re_m = d_i / \mu_L \left[\rho_L u_L + \rho_L u_L (\rho_L / \rho_V)^{0.5}\right]$$

The condenser was made of serpentine pipes that were added to the external surface of the test freezer. Natural convective and radiation heat transfer were modeled outside the tube.

Analysis of Heat Transfer in the Evaporator

The inside convection heat transfer coefficient for single-phase vapor heat transfer inside a tube was the same as eq. (7). Two-phase evaporation heat transfer inside a tube [3] was given by

$$h_i = K\left(\frac{\lambda_L}{d_i}\right)\left[0.102\left(\frac{Mdi}{\mu_L}\right)^2 \frac{J\Delta x(h_V - h_L)}{L}\right]^n \tag{11}$$

when $X_0 < 0.9$ then $K = 2.874 \times 10^{-4}$, n = 0.5

$X_0 > 1$ then $K = 3.29 \times 10^{-3}$, n = 0.4

The evaporator was distributed over five surfaces except the lid. Heat transfer from the outside consisted of natural convection heat transfer with the air of the freezer chamber and cooling of the insulation material. The calculated heat transfer with heat conduction [4] was

$$Q_1 = AU\Delta T \tag{12}$$

$$Q_2 = (mc)_m \Delta T_{n,n-1} \tag{13}$$

ΔT was the temperature difference between the refrigerant evaporator and the ambient air, while $\Delta T_{n,\ n-1}$ was the temperature difference of the in-cabinet air within each time interval Δt, and $(mc)_m$ was the equivalent heat capacity of the insulation material.

Initial Conditions and Boundary Conditions

The initial value of each parameter was described by experimental data. Boundary conditions on the inlet mass flow and enthalpy are provided by the compressor discharge conditions for the condenser and the capillary tube exit conditions for the evaporator. Pressure is provided by means of the exit flow boundary condition. For the condenser, the boundary condition is specified by the flow through the capillary tube. For the evaporator, the boundary condition is specified by the flow through the compressor.

The convective boundary conditions for the condenser were mass flow rate and discharge enthalpy of the compressor and mass flow rate of the capillary tube. The heat exchange boundary conditions for the condenser were natural convection and radiation between the tube wall and air. The convective boundary conditions for the evaporator were mass flow rate and outlet enthalpy of the capillary tube and mass flow rate of the compressor. The heat exchange boundary conditions for the evaporator were convective heat transfer with the air inside the freezer chamber and convection heat transfer with the insulation material.

Solution of Discrete Equations

The state changes of the refrigerant and tube wall in the ith control volume were defined. There were 15 control volumes in the condenser and ΔX was 0.78 m. There were 12 control volumes in the evaporator and ΔX was 2.8 m. The flow path of the refrigerant was considered a horizontally long tube. The series of solutions was obtained by using equations (4), (5), and (6) and auxiliary equations of heat transfer from inlet to outlet for the condenser and evaporator.

The different equations were solved by iteration. The enthalpy was selected as the convergence criterion. If the change in enthalpy between iterations was less than 1%, then calculation during time step Δt (~0.5 seconds) would be completed.

Model of Compressor

The model assumed a hermetically sealed compressor (Figure 3) which had the following assumptions: (1) The electric motor immediately reaches it normal operating speed, (2) The compression process was polytropic, (3) Electric motor losses were dissipated into the suction vapor and into the surrounding air, (4) the expansion process through the suction or discharge valve was adiabatic and isenthalpic, and (5) Property of the working fluid of each control volume was uniform.

The mass and energy balance equations for the suction chamber were:

$$\dot{m}_{so}^{n} - \dot{m}_{eo}^{n} + \left(\rho_2^n - \rho_2^{n-1}\right)\cdot\frac{V_{sump}}{\Delta t} = 0 \tag{14}$$

$$\dot{m}_{eo}^{n}\left(h_2^n - h_{eo}^n\right) + \rho_2^{n-1}\left(h_2^n - h_2^{n-1}\right)\frac{V_{sump}}{\Delta t} - \frac{V_{sump}}{\Delta t}\left(P_2^n - P_2^{n-1}\right) + q_{wi} = 0 \tag{15}$$

The same equations for the discharge chamber were:

$$\dot{m}_{chg}^{n} - \dot{m}_{c}^{n} + \left(\rho_6^n - \rho_6^{n-1}\right)\frac{V_{chg}}{\Delta t} = 0 \tag{16}$$

$$\dot{m}_{c}^{n}\left(h_6^n - h_5^n\right) - \rho_6^{n-1}\left(h_6^n - h_6^{n-1}\right)\frac{V_{chg}}{\Delta t} + \frac{V_{chg}}{\Delta t}\left(P_6^{n-1} - P_6^n\right) + q_{w2} = 0 \tag{17}$$

The overall energy equation for the compressor housing was:

$$(c\rho v)_w \frac{dT_w}{dt} = q_m + q_{w1} + q_{w2} - q_a \tag{18}$$

The model of the compressor consisted of the above equations, cylinder control equations and auxiliary heat transfer equations. The solutions were obtained by solving these equations.

Mathematical Model of the Capillary Tube

The capillary tube was assumed to be adiabatic and one-dimensional. The outlet refrigerent velocity of the condenser was assumed negligible.

There were two stages of flow in the capillary tube: subcooling liquid flow and uniform phase mixed flow of saturated liquid and vapor, respectively. The continuity, momentum and energy equations must be solved for both subcooled liquid and saturated flow in the capillary tube. For subcooling, the mass conservation equation was $dw = 0$ and the energy equation was $dh = 0$. The momentum equation was:

$$dP + f\cdot\frac{\rho w^2}{2}\frac{dL}{D} = 0 \tag{19}$$

For homogenous saturated vapor and liquid mixture, the continuity equation was $d(\rho w) = 0$. The momentum and energy equations were:

$$vdP + wdw + f \cdot \frac{w^2}{2}\frac{dL}{D} = 0 \tag{20}$$

$$d\left(h + \frac{w^2}{2}\right) = 0 \tag{21}$$

In the liquid segment, the specific volume and flow velocity of the liquid were constant. The liquid flow was considered isenthalpic. During the two phase stage, the mixture of liquid and vapor flowing had to be choked flow. Once choked conditions were met, the total mass flow was set.

Coupling Solution of Mathematical Model of Cooling System

The relations among these individual models were coupled (Figure 4). According to analysis, pressure, enthalpy, and mass flow rate were major coupling parameters. Coupling each of the component models produced a mathematical model of the whole system. The compressor inlet was defined as the inlet of the cycle and was the starting point of the model calculation. The evaporator outlet was defined as the outlet of the cycle that was the terminal point of the simulation. The terminal parameters were coupled with the starting point parameters and a closed cycle was formed again.

Conclusion

This paper describes a dynamic and distributed parameter model that was developed to analyze a freezer system. A freezer transient operation analysis computer program was developed which can predict the variations of operation and performance parameters with time and space. It laid a foundation for optimization design of freezers.

References

1. Josiassen, N.J., "Simulation of Condition Sequence During Start-up of an Evaporation Refrigeration System", *Proceedings of the Purdue Compressor Technology Conference*, 1978, pp. 309-316.

2. Yasuda, H., and Touber, S., "Simulation Model of a Vapor Compressor Refrigeration System," *ASHRAE Transactions*, vol. 89, part 2, 1983, pp. 409-425.

3. *ASHRAE Handbook of Fundamentals*, SI Edition, American Society of Heating, Refrigerating and Air Conditioning Engineers.

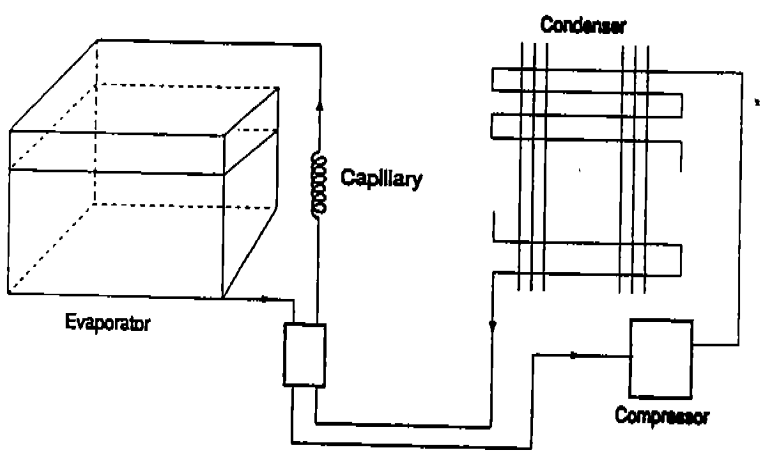

Figure 1 - Schematic of the freezer.

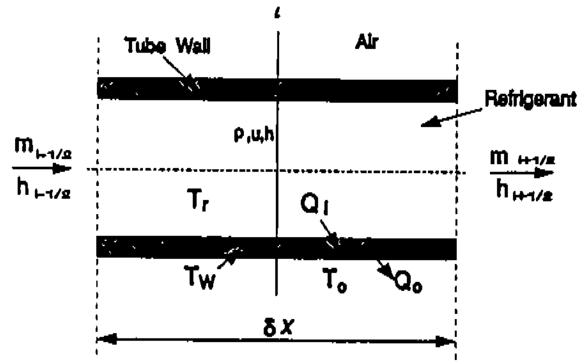

Figure 2. Control volume at the *i*th nodal point.

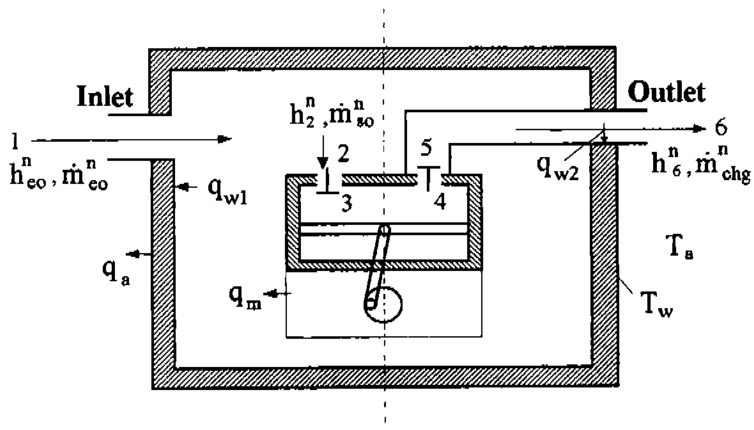

Figure 3 - Schematic view of the hermetically sealed compressor.

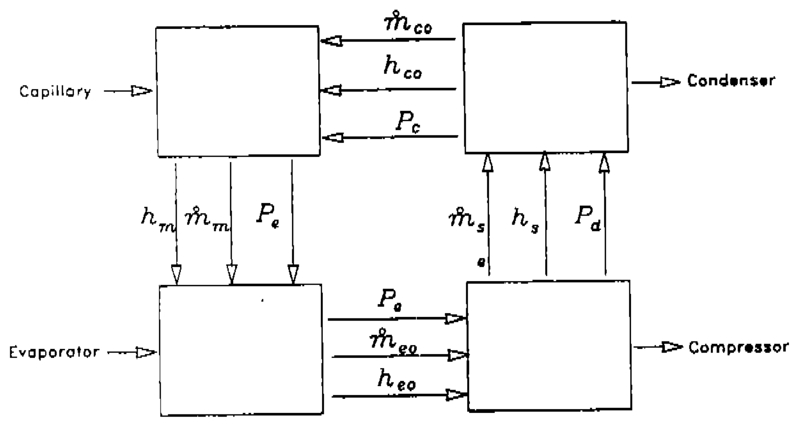

Figure 4 - Schematic view of the system coupling model.