

9-2021

## **FE Simulation of Split in Fundamental Air-Cavity Mode of Loaded Tires: Comparison with Empirical Results**

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Choi, Won Hong and Bolton, J Stuart, "FE Simulation of Split in Fundamental Air-Cavity Mode of Loaded Tires: Comparison with Empirical Results" (2021). *Publications of the Ray W. Herrick Laboratories*. Paper 241.  
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## FE Simulation of Split in Fundamental Air-Cavity Mode of Loaded Tires: Comparison with Empirical Results

Won Hong Choi, J. Stuart Bolton

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# Content

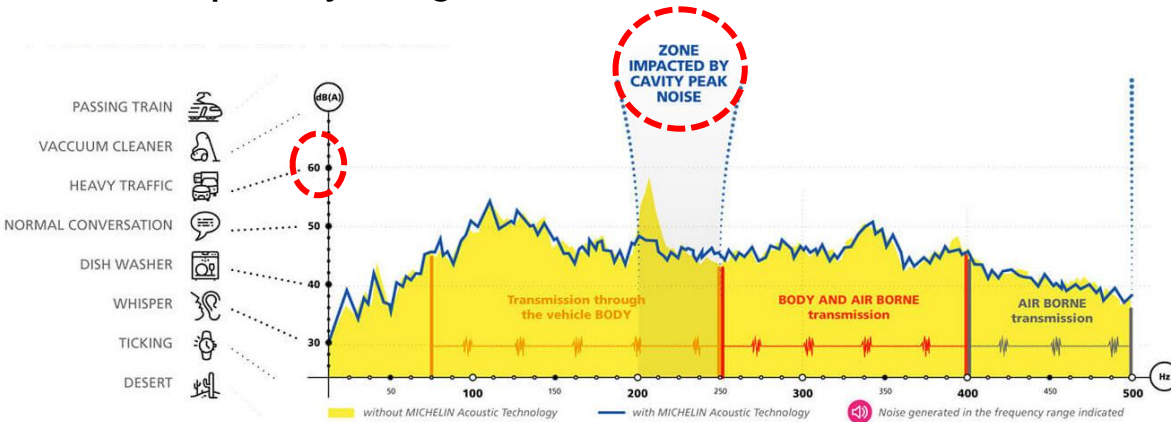
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- Introduction
- FE Model description
- Empirical measurement
- Simulation results: comparison with test results
- Conclusions

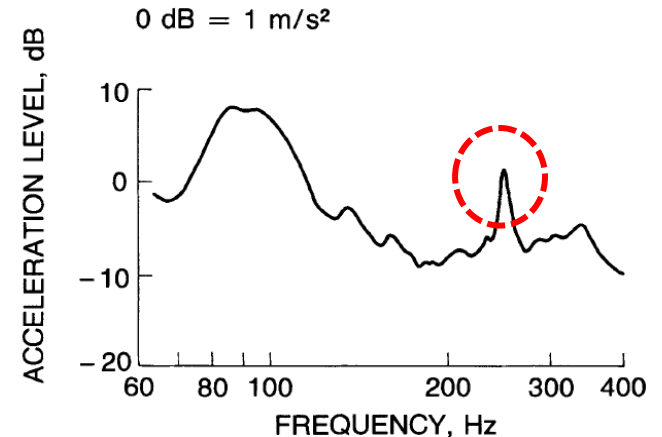


# Introduction

- Tire/road noise may be a dominant source of cabin and pass-by noise for Electric Vehicles since powertrain noise is largely eliminated
- Tire's cavity mode at around 200 Hz can contribute to vehicle interior noise due to dynamic forces transmitted through the suspension to interior in the low frequency range



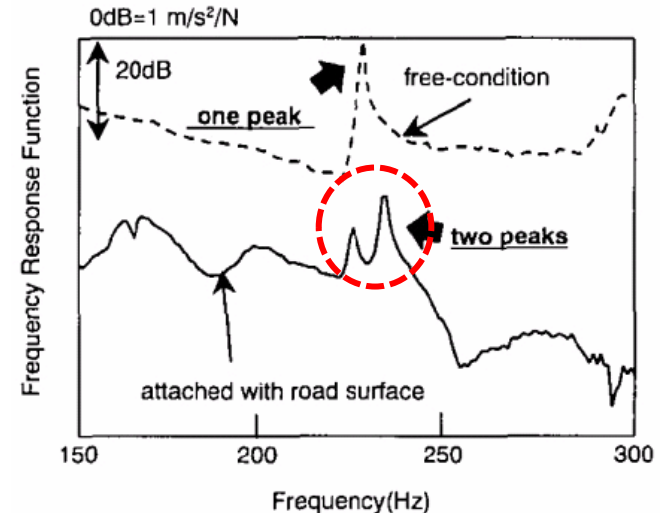
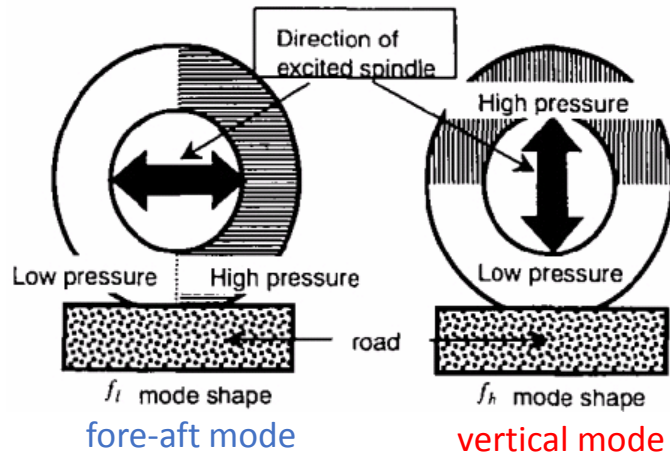
Introduction of cavity noise, courtesy of Michelin tire [1]



Air-cavity mode in acceleration, Sakata and et al [2]

# Introduction

- **Loading** breaks tire's geometrical symmetry, which **splits acoustic cavity mode**
- **Fore-aft mode** shifts to lower frequency, contributing to **horizontal force** at the hub
- **Vertical mode** shifts to higher frequency, contributing to **vertical force** at the hub



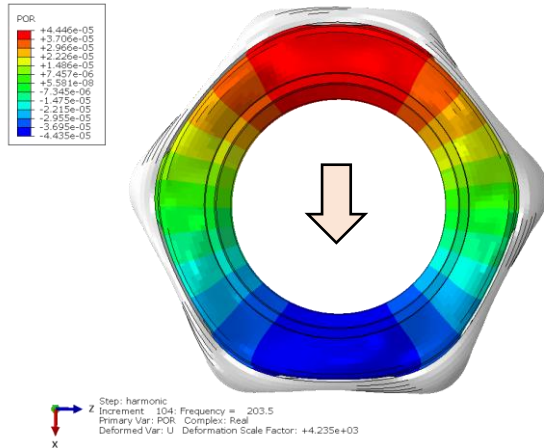
Two acoustic modes, Yamauchi and Akiyoshi [3]

Split in a cavity mode, Yamauchi and Akiyoshi [3]

# Introduction

- **Alignment** between structural modes and cavity modes can result in **amplified force** at the hub (Rui Cao and J. Stuart Bolton [4])
- Identify **mechanism of force generation** and ultimately **avoid any coupling** between these two features

Air-cavity mode coupled with structural resonances



Prediction by the validated FE Model



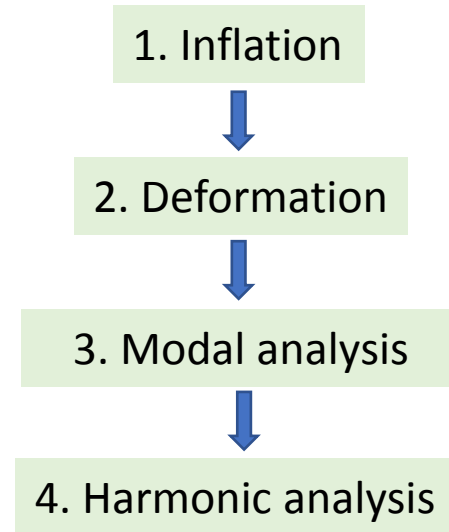
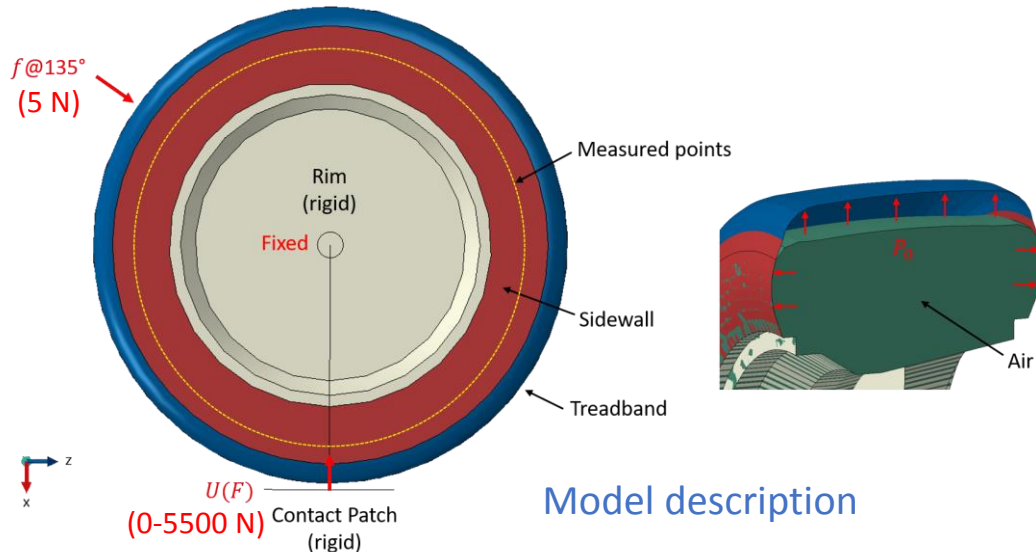
Forces at the hub



The influence of coupling on force level at the hub

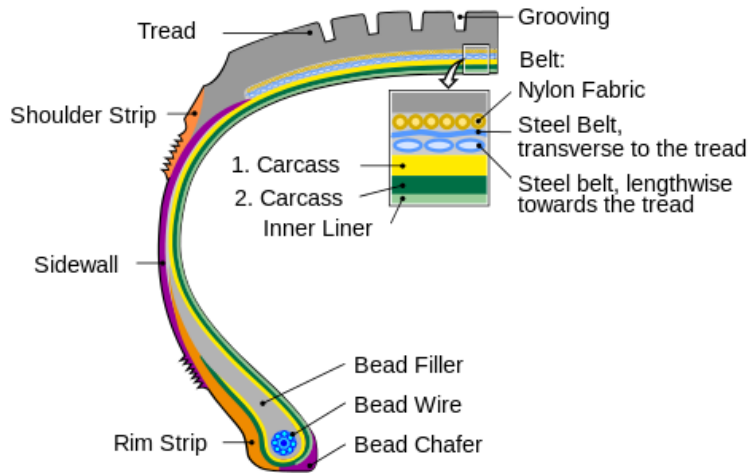
# FE Model Description

- An **axisymmetric model** was created based on the shape of a 2D cross-sectional area, consisting of treadband, sidewall, rim, air cavity and contact patch.
- A FE simulation has been implemented to attempt to **reproduce** both **structural modes** and **the split in the cavity mode** frequency for **deformed tire**

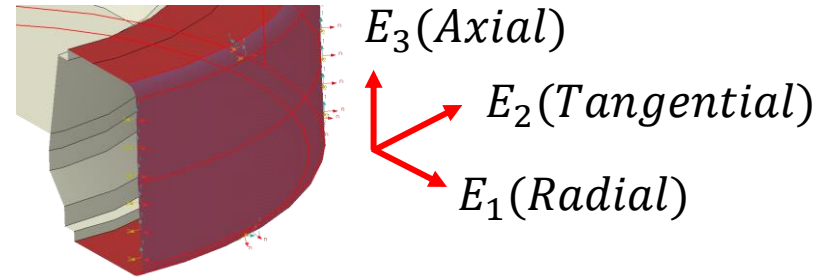


# FE Model Description

- Based on the experimental observations, **the material properties** in the tire structure such as Young's modulus, Poisson's ratio, and shear modulus, were **adjusted to** bring model into agreement with the **observed patterns** of the structural resonance modes
- **Macroscopic** and **orthotropic** material properties to reflect complexity of tire structure



Orthotropic material properties



Tire composition,  
courtesy of Wikipedia



# FE Model Description

- As an example of [material optimization](#), properties in structure and air cavity are given here for the modeled tire #1 (R18, 235/50/R18)

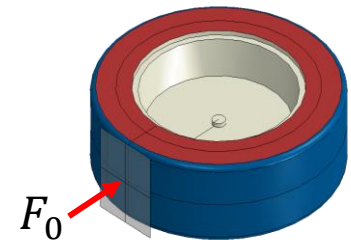
## Property in tire structure

ID	Part	Thickness, $t$	Density, $\rho$	Young's modulus, $E$ (Radial/Tangential/Axial)	Poisson's ratio, $\nu$	Shear modulus, $G$
18-4	Rim	5 mm	4387 kg/m <sup>3</sup>	70,000 MPa	0.3	N/A
	Treadband	12 mm	1520 kg/m <sup>3</sup>	200/650/200 MPa	0.5	50 MPa
	Sidewall	6 mm	1013 kg/m <sup>3</sup>	250/10/250 MPa	0.5	3 MPa

## Property in air cavity

Part	Density, $\rho_{air}$	Bulk modulus, $\beta = \gamma\beta_T$
Air	2.15 kg/m <sup>3</sup>	0.26 MPa

$$c = \sqrt{\frac{\gamma\beta_T}{\rho_{air}}}$$



# Empirical measurement

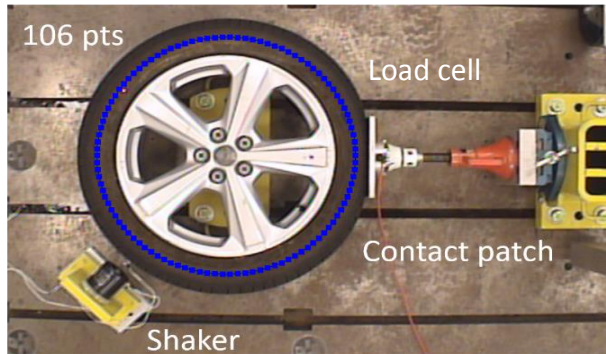
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- The objective is to observe **the split** in tire's cavity mode in an **experimental way** for a set of loaded tires under **static condition** [5]
- Secondly, **the relation** between **load level** and amount of **split** needs to be verified
- A single tire from **each rim size** is chosen to visualize test results with a different range of operation

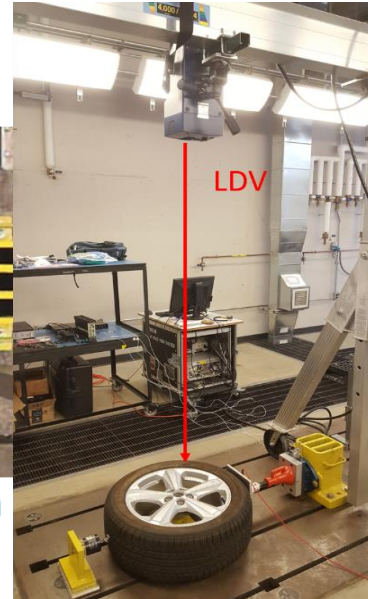
Model	R18	R20	R16	R17	R19
Rim [inch]	18	20	16	17	19
Width [mm]	235	265	205	215	255
Aspect ratio	50	35	65	55	40
Mass [kg]	12.7	13.2	10	10.3	13.4
Stiffness [N/mm]	306	349	245	274	295
Inflation [MPa]	0.24	0.24	0.23	0.23	0.22
Rated load [N]	5496	4812	4741	4913	4776

# Empirical measurement

- **Mobility** data can be obtained by measuring **surface velocity** ( $V$ ) and **input force** ( $f$ ) for the deformed tire for **106 points** along **sidewall** using **Laser Doppler Vibrometer**
- **Static force** ( $F$ ) (Max. 1700 lbs.) was applied through a **load cell**
- **Wavenumber** ( $k_\theta$ ) can be identified by applying **Discrete Fourier Transform** to the mobility spatially distributed along sidewall

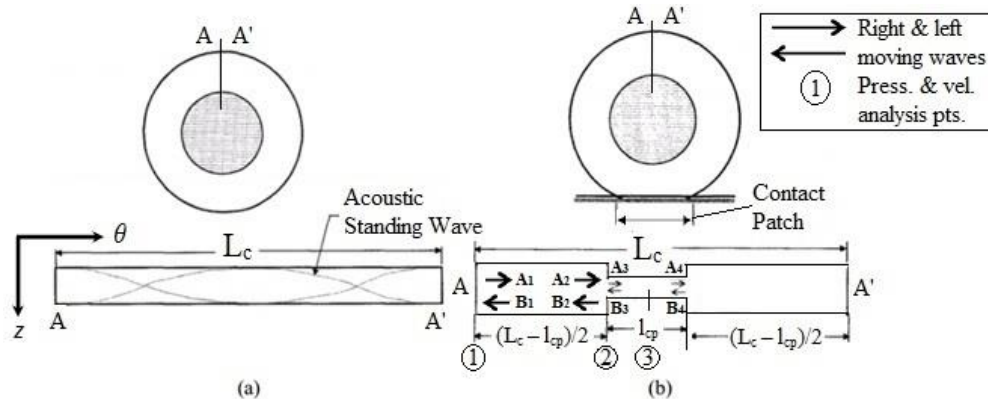


Shaker at 135° from contact patch



# Simulation results : Analytical model for frequency split

- Analytical formula is adopted from Thompson's model
- Difficulty in obtaining  $m$  value experimentally; It was obtained from FE static simulation
- Investigate the variation of frequency split ( $\Delta = f_V - f_H$ ) depending on applied load



Loaded tire model adopted from Thomson [6]

## 1. Horizontal mode

$$f_H = \frac{c}{L_c + (1 - m)l_{cp}} \quad (1)$$

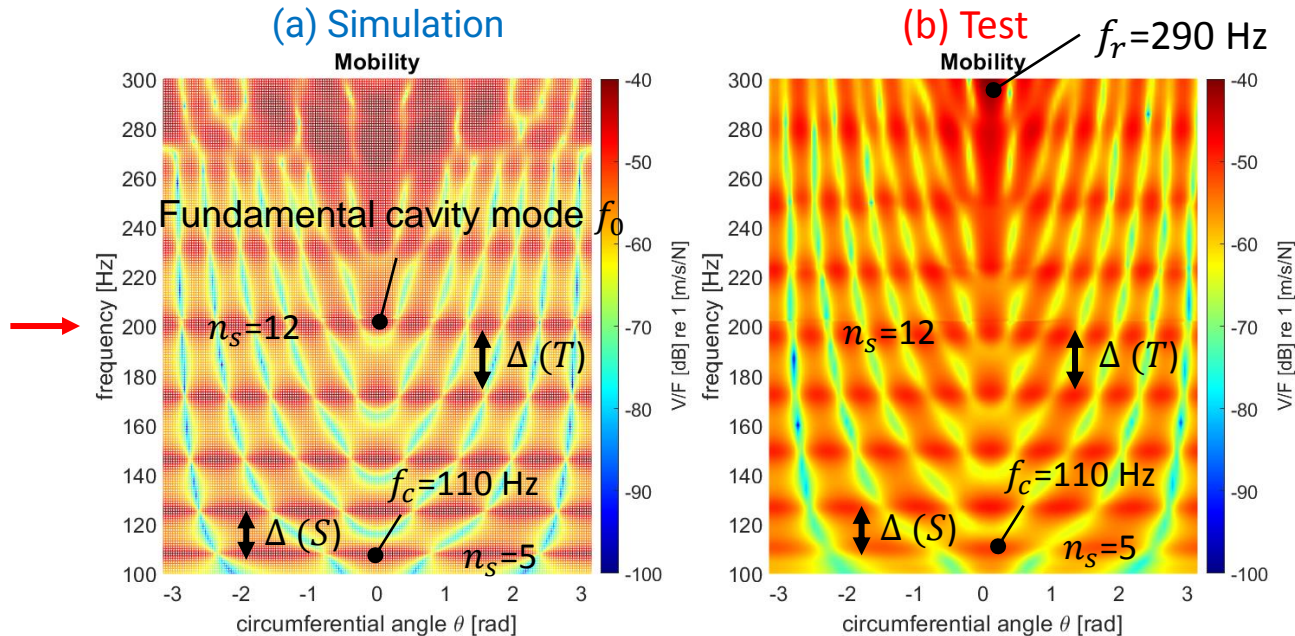
## 2. Vertical mode

$$f_V = \frac{c}{L_c - (1 - m)l_{cp}} \quad (2)$$

$$m = \frac{A_{Loaded}}{A_{Unloaded}}$$

# Simulation results : Mobility , R18 (unloaded tire)

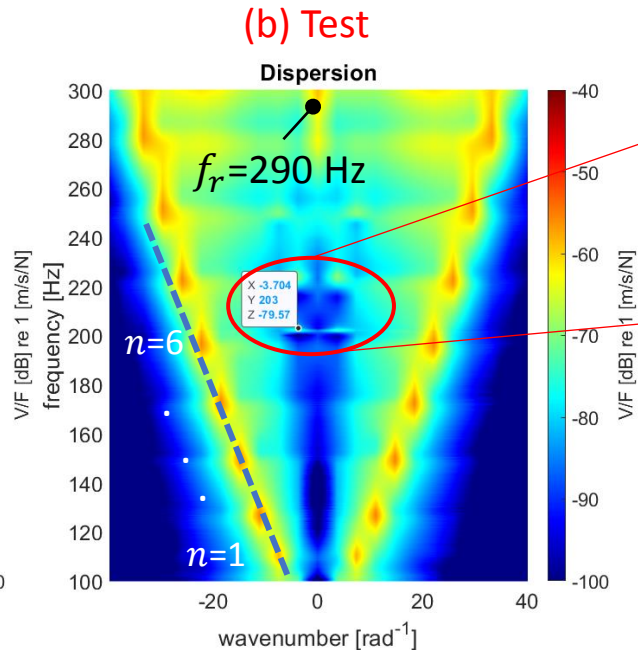
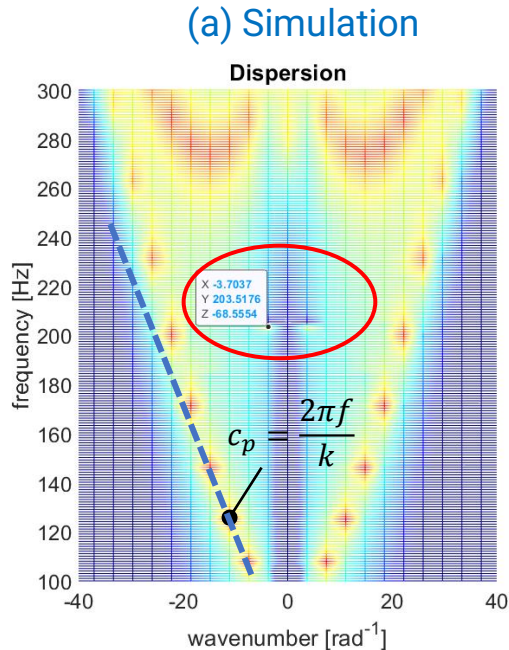
- The FE simulation and measured **mobility plots** for the **unloaded tire** (i.e., normal surface velocity divided by input force) are compared
- The results confirm that the **two patterns** are in a **good agreement**, especially near 200 Hz, close to the **acoustic cavity mode**



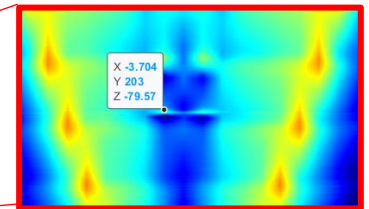
$f_0$  : fundamental cavity mode  
 $f_r$  : ring frequency  
 $f_c$  : cut-on frequency  
 $n_s$  : mode number on sidewall  
 $\Delta(S)$  : mode interval by sidewall  
 $\Delta(T)$  : mode interval by tread

# Simulation results : Dispersion, R18 (unloaded tire)

- All of the structural modes are placed at **equivalent positions** with **the same phase speed**, and also the acoustic cavity mode appears at the same frequency and wavenumber in both cases
- Cross-sectional mode above 280 Hz is different due to the complexity of tire structure



Fundamental cavity mode  
 $f_0 = 203 \text{ Hz @ } k = 3.7 \text{ [rad/m]}$



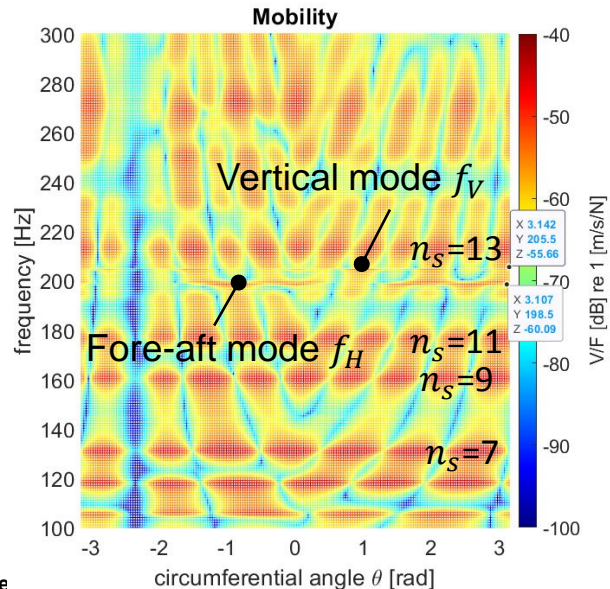
$n$  : mode number on tread  
 $c_p$  : phase speed of wave



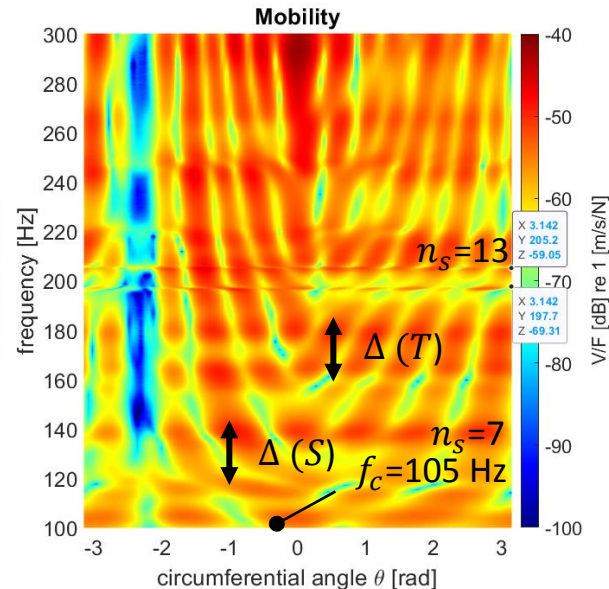
# Simulation results : Mobility, R18 (loaded tire)

- Fore-aft acoustic mode at 198 Hz and the vertical acoustic mode at 205 Hz are well predicted even considering the difference in frequency resolution: 0.5 Hz (simulation) versus 0.15 Hz (test)
- The modal cut-on frequencies and the resonance frequencies of the structure are also in good agreement in the current simulation

(a) Simulation



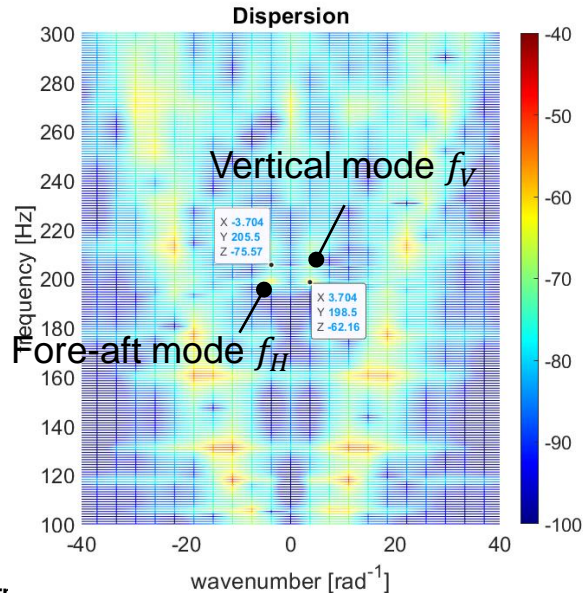
(b) Test



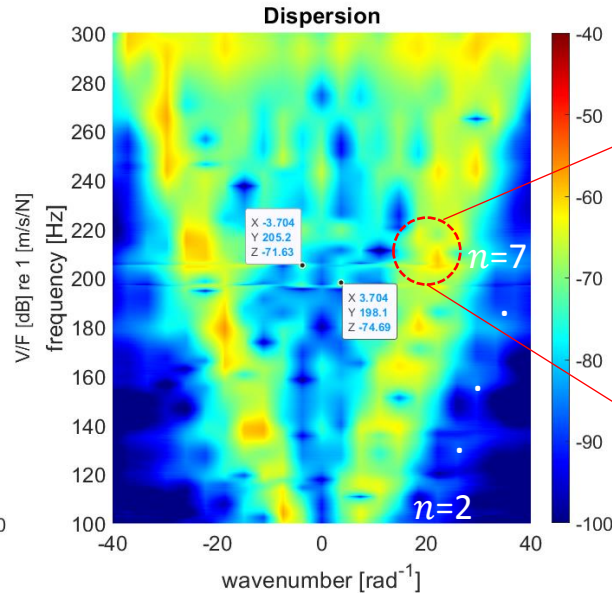
# Simulation results : Dispersion, R18 (loaded tire)

- **Strong coupling** between the 7th structural mode and the cavity resonance at vertical mode can be observed, indicating **an amplified force response** in the vertical direction at the hub
- **Deformation** plays the role of a spatial low-pass filter that **suppresses high wavenumber components**, thus making the plot “blurrier”

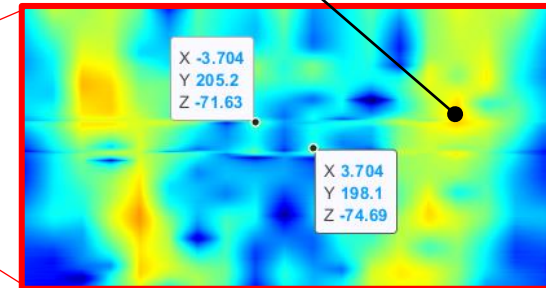
(a) Simulation



(b) Test



Strong coupling

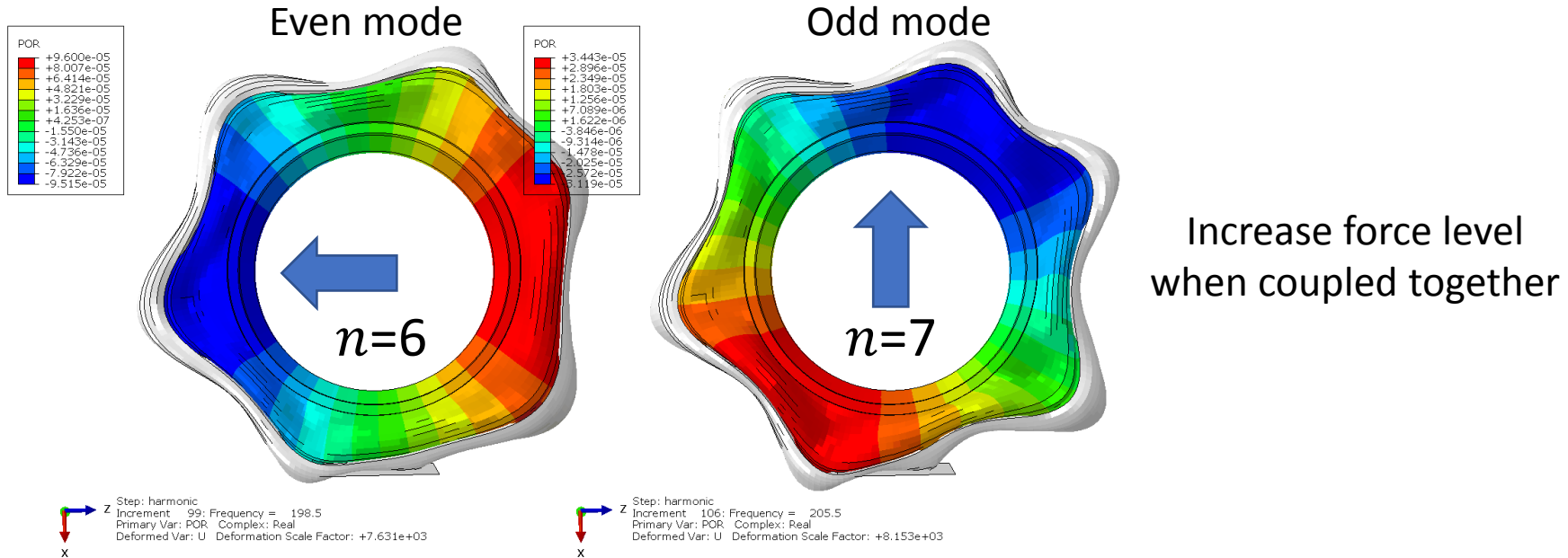


Increase force level



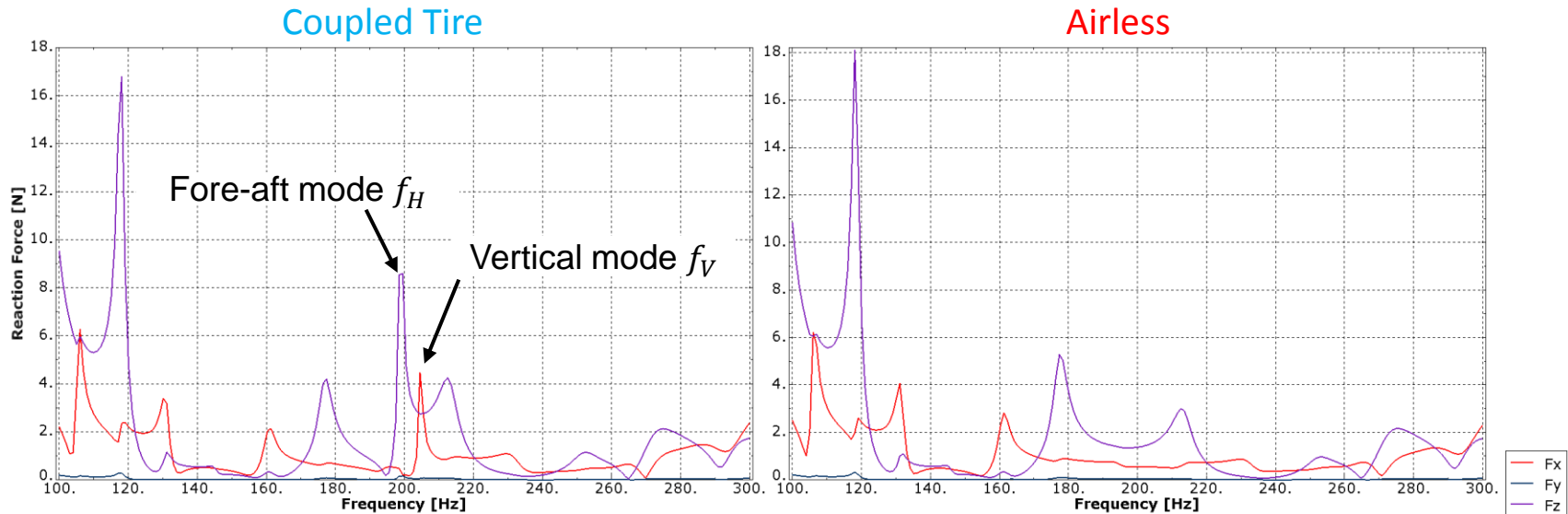
# Simulation results : Acoustic Pressure, R18 (loaded tire)

- Mode number of the structural vibration is an **odd number** ( $n = 7$ ) at the **vertical mode** frequency, so the net structural vibration is directed **vertically**
- **Even mode** couples with **fore-aft mode** to create **net horizontal vibration**



# Simulation results : Reaction force, R18 (loaded tire)

- Two distinct force peaks appear that are associated with the two acoustic modes
- Force level is determined by the interaction between the structural vibration and the acoustic mode

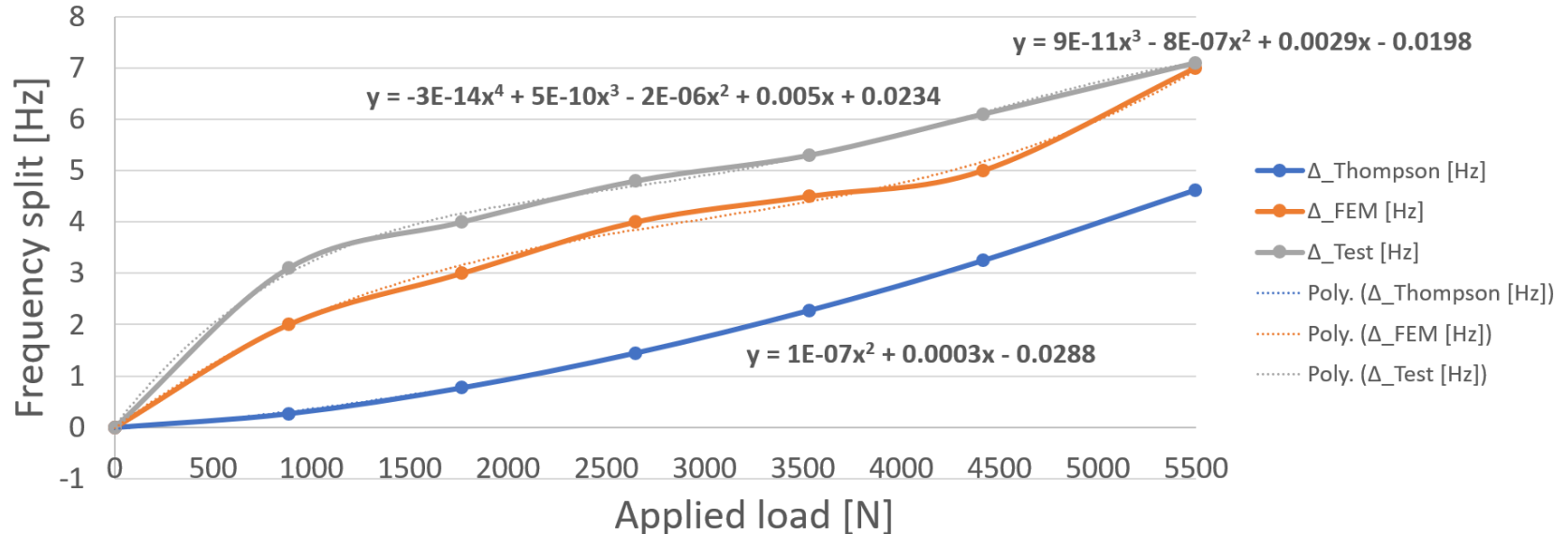


- Fx, Vertical force
- Fz, Horizontal force

## Simulation results : Frequency split vs. Applied load, R18 (loaded tire)

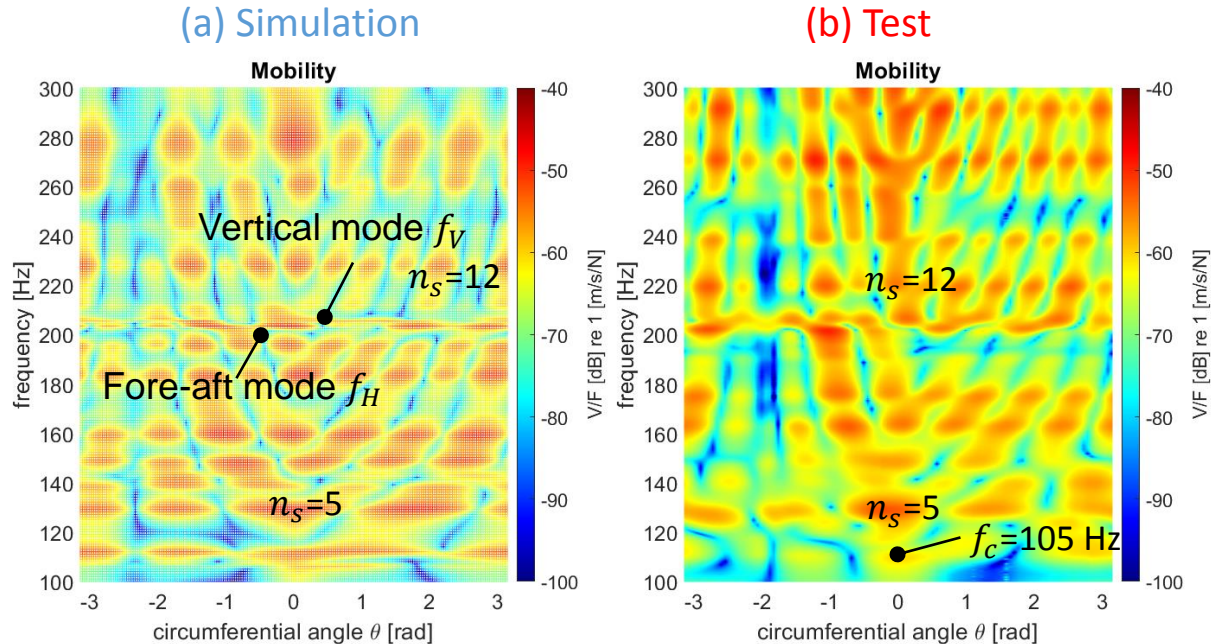
- The **split in the cavity mode** was examined when changing the **applied load** from 0 N to 5496 N (the rated load for this tire)
- **FE simulation** is much closer to the trend in test than **Thompson theory** as a function of applied load

Frequency split vs. applied load (R18)



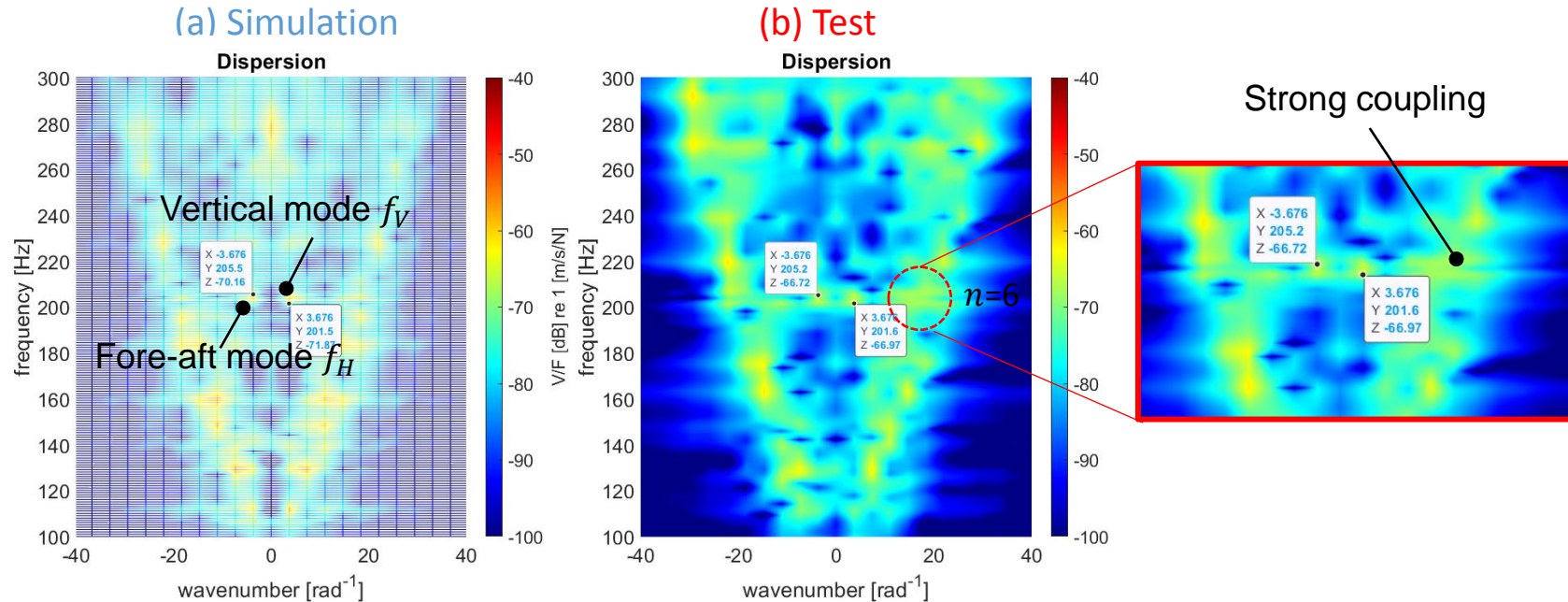
# Simulation results : Mobility, R20 (loaded tire)

- The frequency split agrees closely with the experimental result
- Two mobility plots showed good consistency in the structural pattern, including the modal cut-on frequency at 105 Hz, and the natural frequency of the 5th structural mode, for instance



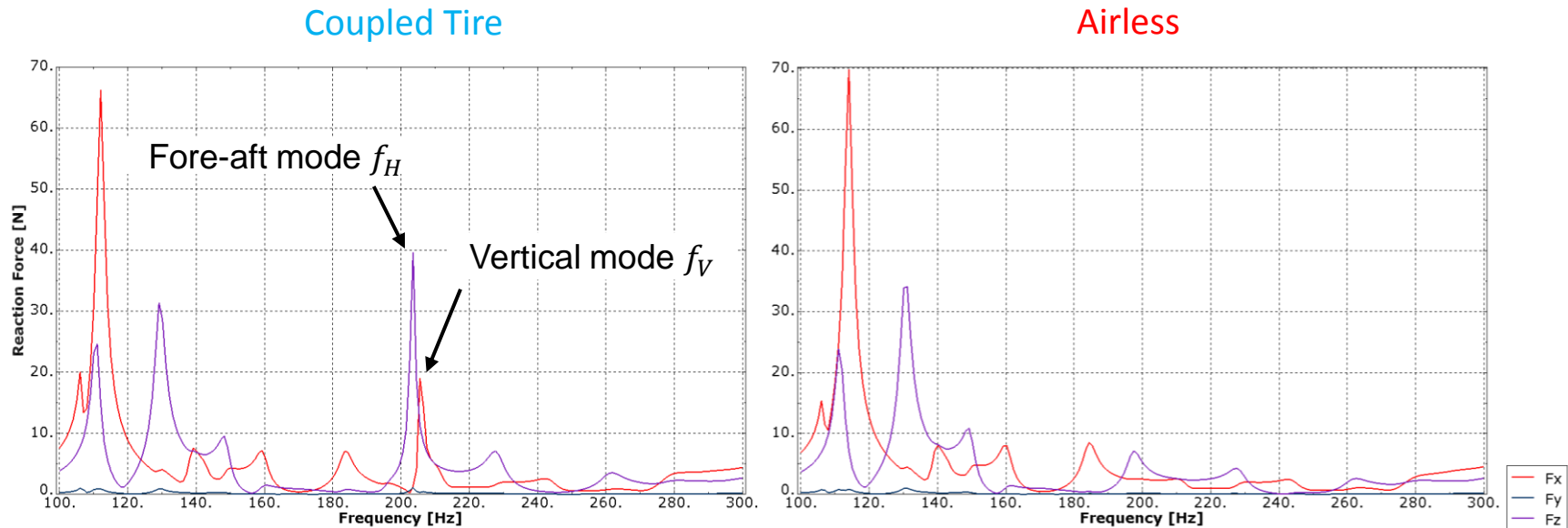
# Simulation results : Dispersion, R20 (loaded tire)

- The frequency split agrees closely with the experimental result
- The two acoustic modes interact with the 6th structural mode strongly near 200 Hz, contributing to an increased force response at the hub in both horizontal and vertical directions



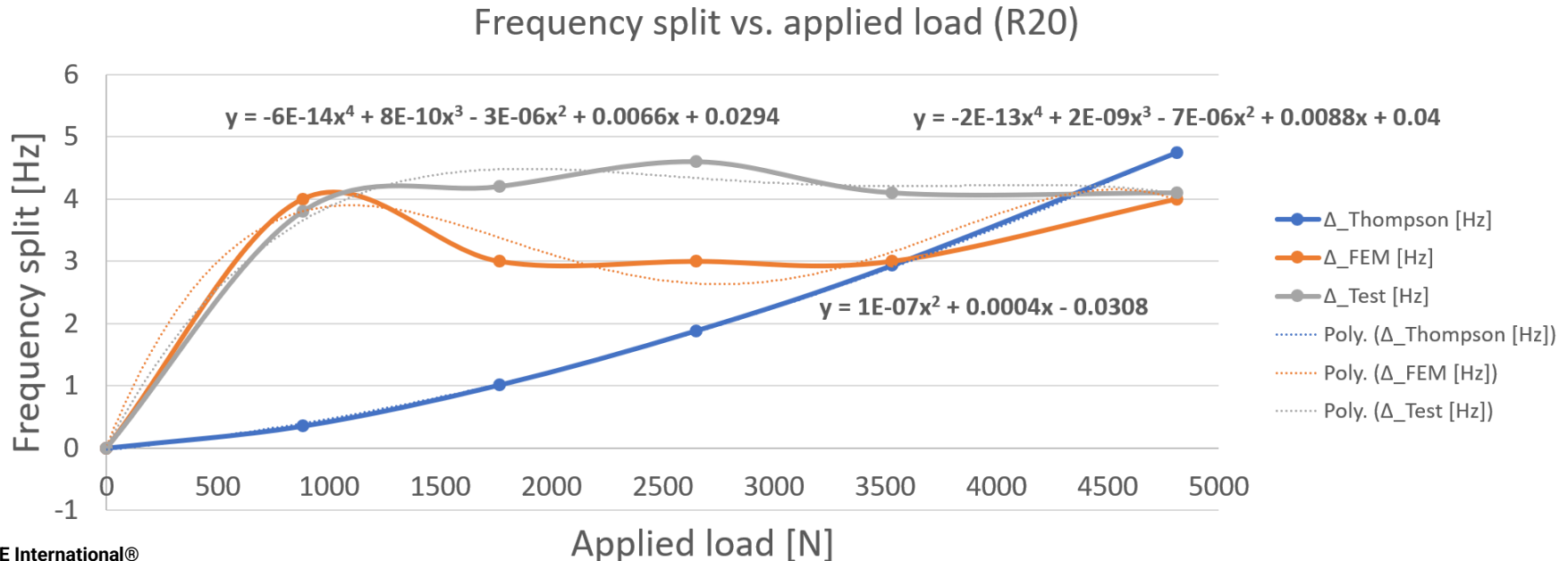
## Simulation results : Reaction force, R20 (loaded tire)

- Two distinct force peaks appear that are associated with the two acoustic modes
- Frequency split is reduced compared to the previously presented R18 tire



## Simulation results : Frequency split vs. Applied load, R20 (loaded tire)

- The **split** in the cavity mode was examined when changing the **applied load** from 0 N to 4812 N (the rated load for this tire)
- The estimation from the FE simulation **matches the trend** of the experimental results quite well, and follows **a fourth order polynomial fit**



# Summary

- The frequency split of the fundamental acoustic mode changes from 4 to 7 Hz depending on **tire's specifications**, such as rim size, stiffness, and applied load

Model	R18	R20	R16	R17	R19
$f_0$ [Hz] 1st cavity mode	203.5	200.9	215.6	212.6	207.5
$f_H$ [Hz] Fore-aft mode	198.5	201.5	211.6	206.5	204.5
$f_V$ [Hz] Vertical mode	205.5	205.5	217.6	213.5	208.5
$\Delta f$ [Hz] Frequency split	7	4	6	7	4



# Conclusions

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- **A FE model** was developed to make it possible to predict **the split of the fundamental air-cavity mode** of loaded tires in a static condition for **five commercial tires**
- **The observed split** varied from a minimum of **4 Hz to a maximum of 7 Hz** at rated load in the five cases described here
- **A large dynamic force** at the hub resulting from the acoustic mode might be anticipated when the **air-cavity modes** are **close in frequency** to spatially-matched **structural modes**
- The relation between the degree of the split and contact load showed a nearly **fourth-order variation** in the test and the **FE simulation reproduced that trend**
- In the next stage of this work, **hub force data** measured in the laboratory **for rolling tires** will be quantitatively compared with the simulation results to demonstrate their reliability

# Acknowledgement

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- Ford Motor Company – Financial support / Tire & wheel sample provider
- Matthew Black – Program manager and logistical support
- Dan Haakenson – Technical advice and industrial feedback

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