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Continuum mechanics beyond the second law of thermodynamics

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ABSTRACT

The entire field of continuum mechanics has been developed subject to the second law of thermodynamics, requiring a non-negative rate of irreversible entropy production. Given the results in physics over the past two decades, this law is a restriction to macroscales because on very small scales and for very short times, the entropy production rate may become negative. This has been demonstrated by experiments, theory, and simulations. For example, the time-averaged shear stress for a system undergoing Couette flow has fluctuations ranging into the negative values. These results suggest that an extension of continuum mechanics should be made, wherein the second law axiom is replaced by the transient fluctuation theorem (actually, a set of such theorems). The approach we take is facilitated by these observations: (i) Just like the dissipation function describes irreversible phenomena, the free energy function describes quasi-conservative phenomena both, in statistical mechanics and in continuum thermomechanics. Hence, a continuum theory is made possible in terms of these two functions (free energy and dissipation) appearing as stochastic functionals. On macroscales, they simplify to the well-known deterministic functionals. (ii) There is a natural mathematical model for a dissipation function which, on average, tends to exceed its earlier value, but may randomly (every so often) go below it: a submartingale. Then, the Doob decomposition theorem (splitting the submartingale into a martingale plus a weakly increasing process) allows one to classify all the physical phenomena as they fall within these limiting cases: reversible processes on large scales; dissipative processes on large scales; reversible processes on very small scales; dissipative processes on very small scales. (iii) There is a very wide range of scalar random field models that can be employed for the martingale, even allowing one to grasp the fractal and Hurst effects. (iv) The departures from the second law are relevant for phenomena exhibiting strong spatial and/or temporal gradients. This is illustrated on the example of acceleration fronts in general dissipative/nonlinear-elastic continua: when the wavefront thickness dV is nanoscale level, the possibly negative random fluctuations in the dissipation function change the standard growth-and-decay behaviors of the wavefront.