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DYNAMIC BEHAVIOR OF SLIDING VANE IN SMALL ROTARY COMPRESSORS

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INTRODUCTION

We have often experienced pulse like noise caused by the chattering phenomena of the vane in the sliding vane type rotary compressors, under certain operating conditions such as in starting-up and low rotational speed operation. The chattering is the phenomenon that the tips of the vanes get detached from and spring back into contact with cylinder wall, while they should be kept in contact with and sliding on the cylinder wall in ideal operation. This chattering phenomena brings out not only the noise problems, but also the deterioration of durability and efficiency of the compressors.

This paper reveals the chattering phenomena both theoretical analysis of the vane motion based on the simple dynamical model of the compressor and vane motion observation experiments.

THEORETICAL ANALYSIS

We have simulated the gas-compression mechanism to analyze the vane motion with the simplified model. Figure 1 shows the schematic view of the rotary vane compressor having a circular rotor and cylinder. With the revolution of the rotor which is eccentric with the cylinder wall, the vanes rotate with their tips sliding on the cylinder wall, and move in and out along the guide slots machined in the rotor. The space surrounded by the cylinder wall, the rotor and any two adjacent vanes forms the compression chamber.

GEOMETRICAL RELATIONS

The basic quantity required for the derivation of the gas compression process is the vane extension, that is the distance between the rotor circle and the cylinder wall measured along the center line of the vane. As can be seen from Figure 1, this vane extension x is a function of the angle θ. We can obtain the following equation from simple geometrical consideration.

$$x = \left\{R_c{}^2 - a^2 - R_c e\cos\varphi\left(2-\frac{e^2}{4R_c{}^2}\right) + e^2\cos 2\varphi - \frac{e^3}{4R_c}\cos 3\varphi\right\}^{\frac{1}{2}} - (R_r{}^2 - a^2)^{\frac{1}{2}} \tag{1}$$

The area swept by the vane extension during the revolution of the rotor is given as

$$A_v = \frac{1}{2}\left\{R_c{}^2\psi - el\sin\varphi + R_r l\sin(\theta-\varphi) - \theta R_r{}^2\right\} \tag{2}$$

where $\cos\psi = \frac{1}{2R_c e}(R_c{}^2 + e^2 - l^2)$

By taking into account the vane thickness, from following equation we can get the volume of the compression chamber, whose cylinder length is h,

$$\begin{aligned} V &= h\left[A_v(\theta+\theta_v) - \tfrac{1}{2}tx(\theta+\theta_v)\right] \quad \cdots\cdots 0 \leqq \theta \leqq \theta_v \\ &= h\left[A_v(\theta+\theta_v) - A_v(\theta) - \tfrac{1}{2}t\left\{x(\theta+\theta_v)+x(\theta)\right\}\right] \\ &\qquad\qquad \cdots\cdots \theta_v \leqq \theta \leqq 2\pi-\theta_v \\ &= h\left[\pi(R_c{}^2 - R_r{}^2) - A(\theta) - \tfrac{1}{2}tx(\theta)\right] \\ &\qquad\qquad \cdots\cdots 2\pi-\theta_v \leqq \theta \leqq 2\pi \end{aligned} \tag{3}$$

where N_v: number of vanes

$$\theta_v = \frac{2\pi}{N_v}$$

The contact point of the vane with the cylinder wall shifts continuously with the revolution of the rotor as shown in Figure 2. If the vane tip configuration is assumed to be circular, we can calculate the contact angle γ of the vane tip with the cylinder wall, the angle between the centerline of the vane and the normal to the cylinder wall at the contact point as follows:

$$\gamma = \tan^{-1}\left[\frac{a-e_v-e\sin\theta'}{\sqrt{(R_c-R_v)^2-(a-e_v-e\sin\theta')^2}}\right] \qquad (4)$$

where R_v: radius of the vane tip contour

$$\theta' = \theta - \tan^{-1}\frac{a}{\sqrt{R_r^2-a^2}}$$

The cordinates of the contact point (x_t, y_t) is given by following equations

$$x_t = -e\cos\theta' + \sqrt{(R_c-R_v)^2-(a-e_v-e\sin\theta')^2} + R_v\cos\gamma \qquad (5)$$

$$y_t = a-e_v+R_v\sin\gamma \qquad (6)$$

PRESSURE CHANGE

We can derive the change of the state of the gas during the compression process as follows. To simplify the problem, we assume that the working fluid is ideal gas and that the leakage of pressurized gas from the compression chamber is negligible when the tip of the vane is kept in contact with cylinder.

When the vane happens to be detached from the cylinder wall, gas leaks from the high pressure side to the low pressure side through the clearance between the vane tip and the cylinder wall. The leakage flow rate can be approximated with the following equation.

$$\frac{dG_m}{d\tau} = A_p c_m \sqrt{2g\frac{K}{K-1}\frac{P_1}{v_1}\left(\phi^{\frac{2}{K}}-\phi^{\frac{K+1}{K}}\right)} \qquad (7)$$

where $\frac{P_2}{P_1} \geq \phi_c \qquad \phi = \frac{P_2}{P_1}$

$\frac{P_2}{P_1} \leq \phi_c \qquad \phi = \phi_c = \left(\frac{2}{K+1}\right)^{\frac{K}{K-1}}$

A_p: leakage area between vane tip and cylinder

c_m: coefficient of the leakage flow rate

Subscript 1 and 2 refer to the upstream and the downstream condition of the leakage flow respectively.
The pressure change in the compression chamber is obtained by applying the first low of thermodynamics

$$\frac{dp}{d\tau} = PK\left(\frac{T_1}{T}\frac{1}{G}\frac{dG_{mi}}{d\tau} - \frac{1}{G}\frac{dG_{mo}}{d\tau} - \frac{1}{V}\frac{dV}{d\tau}\right) \qquad (8)$$

$$\frac{dT}{d\tau} = T\left\{\left(K\frac{T_1}{T}-1\right)\frac{1}{G}\frac{dG_{mi}}{d\tau} - (K-1)\frac{1}{G}\frac{dG_{mo}}{d\tau} - (K-1)\frac{1}{V}\frac{dV}{d\tau}\right\} \qquad (9)$$

EQUATION OF MOTION FOR SLIDING VANE

To establish the mathematical model to estimate the behavior of the sliding vane, we have considered the forces acting on the vane as shown in figure 3. These forces are summarized in Table 1. We have evaluated these forces based on the following assumptions:

1) The friction forces acting on both side-ends of the vane are caused by the oil film between the vane and the side wall. They are obedient to the hydrodynamical lubrication theory.
2) Other frictional forces are obedient to Coulomb's law.
3) Pressure forces are acting on leading and trailing surfaces of the vane at the middle point of the distance between the rotor and the cylinder, that is, the leakage of the pressur fluid into the rotor-vane slots is ignored.

The directions of the frictional forces depend on the motion of the vane. we obtain three basic equations to describe the vane motion from above considerations.

$$M\ddot{x}_v + F_{vt} + \frac{\dot{x}_v}{|\dot{x}_v|}\mu_r(|R_1|+|R_2|) - (F_{pb}+F_{ex}+F_{rx}-F_{pf}) = 0 \qquad (10)$$

$$R_1-R_2+F_{ey}+F_c-F_{ry}+(F_{p1}-F_{p2})+F_{vn} = 0 \qquad (11)$$

$$\frac{1}{2}l_v(F_{ey}+F_c)+l_vF_{vn}+F_{p1}\left\{l_v-\frac{1}{2}\left(x_v+\frac{\delta}{2}\right)\right\} - \left[\frac{1}{2}l_vF_{ry}+F_{p2}\left\{l_v-\frac{1}{2}\left(x_v-\frac{\delta}{2}\right)\right\}+R_2\left(l_v-x_v+\frac{\delta}{2}\right)\right] = 0 \qquad (12)$$

Concerning the reaction force on the vane tip from the cylinder wall, following relations must be satisfied.

$$F_{vn} = F_{vt}\tan\theta_d \qquad (13)$$

$$\theta_d = \gamma + \tan^{-1}\mu_v \qquad (14)$$

Where μ_v is the coefficient of sliding friction between the vane and the cylinder wall.

BOUNDARY CONDITION

The vanes rotate with the rotor and slide in and out along the guide slots in the rotor. When the vane is kept in contact with and sliding on the cylinder wall vane extension x_v is obtained from equation (1).

$$x_v = x \tag{15}$$

Velocity and acceleration relative to the rotor are

$$\dot{x}_v = \dot{x} \qquad \ddot{x}_v = \ddot{x} \tag{16}$$

In order to obtain the reaction forces acting on the vane at a given vane position, the pressures P_1 and P_2 in the neighboring compression chambers and the inertia forces must be computed in advance. Then we can calculate various forces actig on the vane from three dynamic equilibrium equations (10),(11), and (12) with given frictional coefficients.
Figure 6 shows an example of the computation result of the reaction forces on a vane.

Table 1. Forces Acting on Vane

Force	Description
F_{p1}, F_{p2}	pressure force on leading surface and trailing surface
F_{pf1}, F_{pf2}	pressure force on vane tip
F_{pb}	pressure force on bottom end of vane
F_{rx}, F_{ry}	body force due to centrifugal acceleration of vane
F_{ex}, F_{ey}	friction force on both side-end of vane
F_{vt}, F_{vn}	reaction force on vane tip
F_c	body force due to Coriolis acceleration arising from combined sliding and rotating motion of vane
R_1, R_2	force of rotor slot on leading surface and trailing surface

When the reaction force F_{vt} on the vane tip becomes zero, vane tip takes off from the cylinder surface and the vane slide into the slot. The vane may slide into the slot so deep that it runs against the bottom end of the slot. The vane slides out later to get again in contact with the cylinder surface. In the cource of this vane motion there occur vane collisions with the vane-slot bottom end and/or cylinder surface, but the duration of impact of these collisions is so extremely short compared with the total time of the vane motion as to be negligible. The deformations of the impact surfaces caused by the collisions are also extremely small compared with the total distance of the vane motion and are ignored. Hence we have estimated the vane velocity relative to the rotor after the collision from the one before the collision by means of the coefficient of restitution. We can thus determine the vane velocity relative to the rotor after the collision using following relations.
At the bottom end of the vane slot:

$$x_v(\tau_i) = l_v - l_{vo} \tag{17}$$

$$\dot{x}_v(\tau_{i+0}) = -\varepsilon_b \dot{x}_v(\tau_{i-0}) \tag{18}$$

At the surface of the cylinder wall:

$$x_v(\tau_j) = x(\tau_j) \tag{19}$$

$$\dot{x}_v(\tau_{j+0}) = -\varepsilon_f \left\{ \dot{x}_v(\tau_{j-0}) - \dot{x}(\tau_{j-0}) \right\} + \dot{x}(\tau_{j+0}) \tag{20}$$

where $x_v(\tau_{-0})$: vane velocity before impact

$x_v(\tau_{+0})$: vane verocity after impact

$\varepsilon_b, \varepsilon_f$: coefficient of restitution

RESULT AND DISCUSSION

We have performed experiments to observe the behavior of sliding vanes in an actual compressor. Tests were conducted using a three-vane type compressor with transparent housing made of acrylic resion.
Figure 4 shows an example of the vane behavior in chattering phenomena. The vane gets detached from cylinder surface before it passes across the minimum extension point, which may be named "axial seal part", and is sucked down into the slot in the rotor. At the bottom of the slot the vane hits the rotor. After passing across the axial seal part, the vane slides back out again and runs against the cylinder surface abruptly.
Figure 5 shows the motion of the vane tip measured from the photograph recorded by the high-speed camera. In this case, we could recognize the pulse like noise

caused by the vane chattering.
Figure 6, 7, and 8 show the results computed with this model. Figure 6 shows computed reaction forces on the vane under chattering condition. Figure 7 and 8 show the vane extension and the vane velocity.

The reaction force F_{yt} disappears a little before the vane passing the axial seal part. At this moment the vane starts taking off from the cylinder surface and accelerates gradually thereafter to run against the bottom of the slot.
In this simulation the pressure in the bottom chamber of the vane-slot, which acts on the bottom end of the vane and pushes it out, is assumed to be constant during the compression process.
As seen from Figure 5 and 7, this simulation is able to give good explanation to the chattering motion of the vane.

CONCLUSIONS

We studied the dynamic behavior of sliding vane in small rotary compressors theoretically and experimentally. In the theoretical analysis we have approximated the compressor with a simplified dynamic model. By the experiments we observed the vane behavior in a actual compressor through the transparent housing wall.
We obtained good qualitative agreement between the results of simulation analysis and the experimental results.
From this analysis, we can estimate the effects of relevant parameters on the generation of the chattering noise of the sliding vane type rotary compressors.

NOMENCLATURE

A_v	area swept by the vane extension
a	distance between center of rotor and center line of vane
A_p	leakage area between vane tip and cylinder surface
C_m	coefficient of leakage flow rate
e	eccentricity $(=R_c-R_r)$
e_v	distance between center of the vane tip circle and centerline of vane
G	weight of gas in compression chamber
G_m	weight of leakage gas
g	acceleration of gravity
h	cylinder length
l	distance between center of rotor and vane tip
l_v	vane length
l_{vo}	vane-slot length
M	mass of vane
N_v	number of vane
P_1, P_2	pressure in compression chamber
P_b	pressure in bottom chamber of vane-slot
P_d	discharge pressure
P_s	suction pressure
R_c	cylinder radius
R_r	rotor radius
R_v	radius of vane tip circle
T	temperature in compression chamber
t	vane thickness
V	volume of compression chamber
v	specific volume
x	extension of vane in ideal operation
x_v	extension of vane
x_t, y_t	contact point of vane tip
θ	angular vane position
φ	angular vane tip position
γ	contact angle of vane tip
$\varepsilon_t, \varepsilon_b$	coefficient of restitution
κ	specific heat ratio
μ_t, μ_v	coefficient of sliding friction
τ	time
ω	angular velocity of rotor

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2. Edwards, T.C. and McDonald, A.T., "Analysis of Mechanical Friction in Rotary Vane Machines," Purdue Compressor Technology Conference Proceedings, 1972

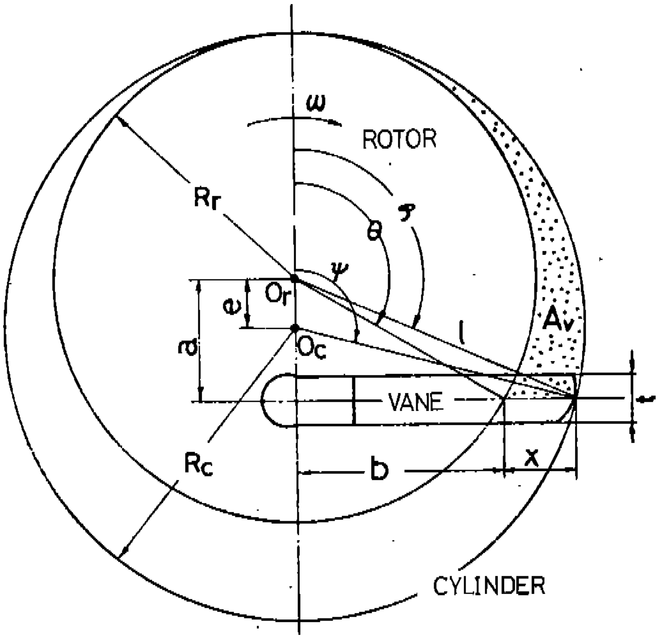

Figure 1. Geometry

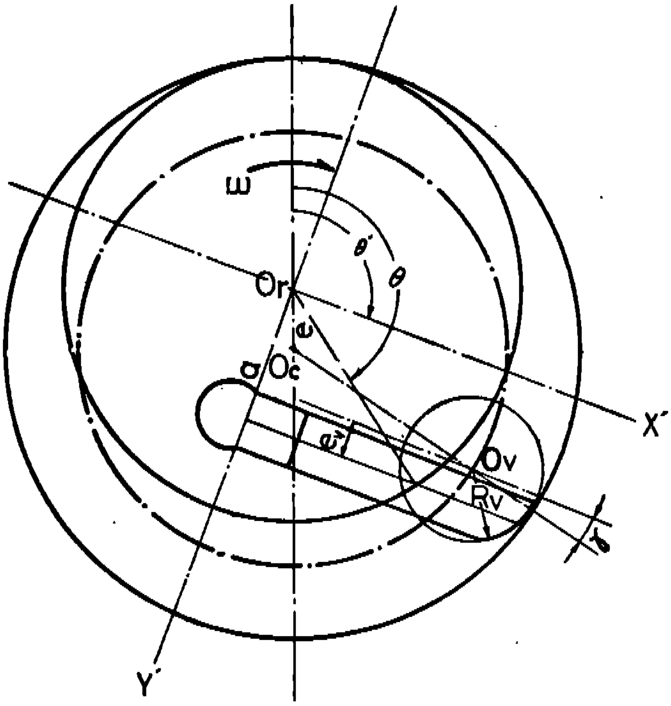

Figure 2. Contact point of the vane tip with the cylinder wall

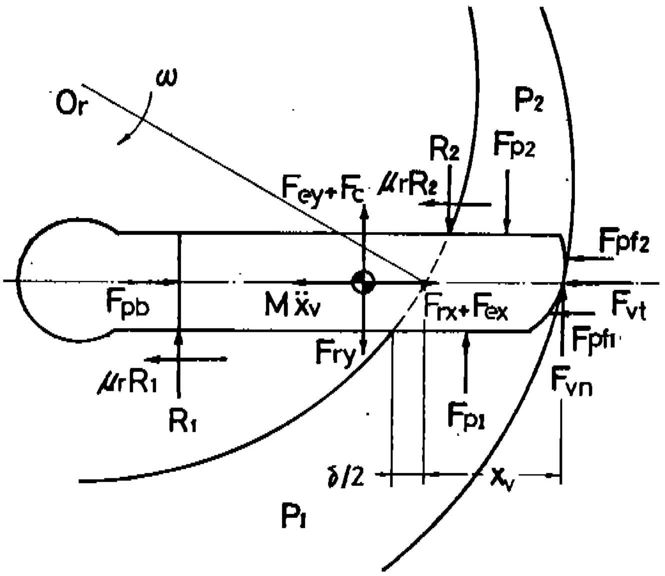

Figure 3. Forces acting on the vane

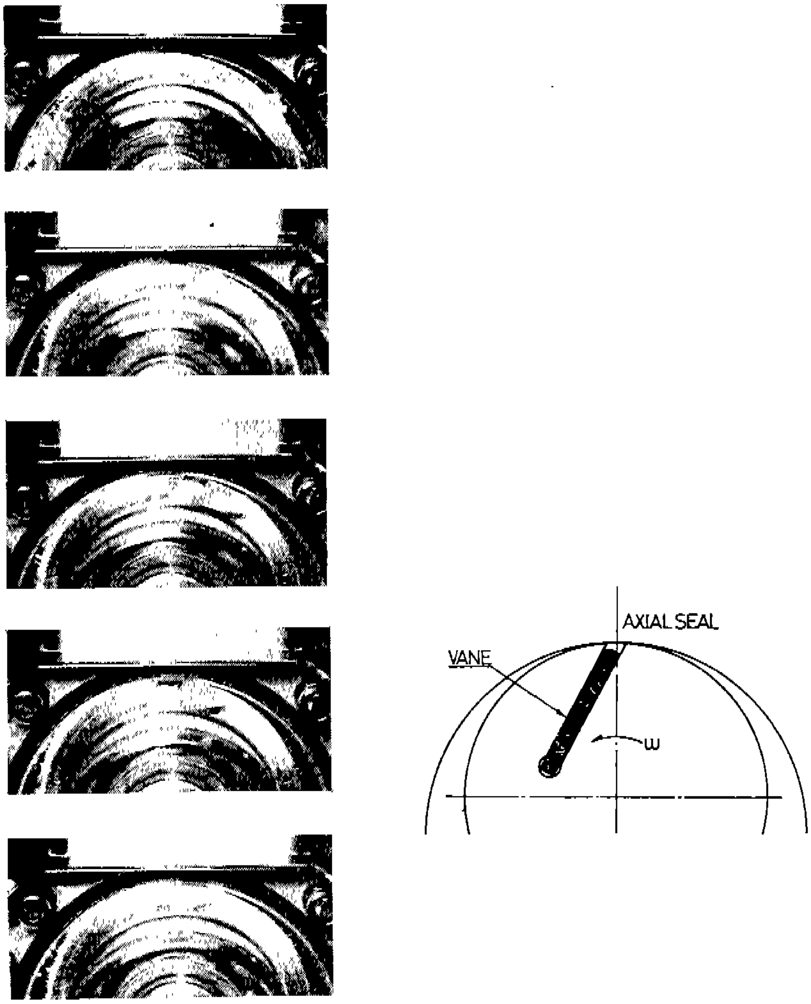

Figure 4. Chattering of the sliding vane

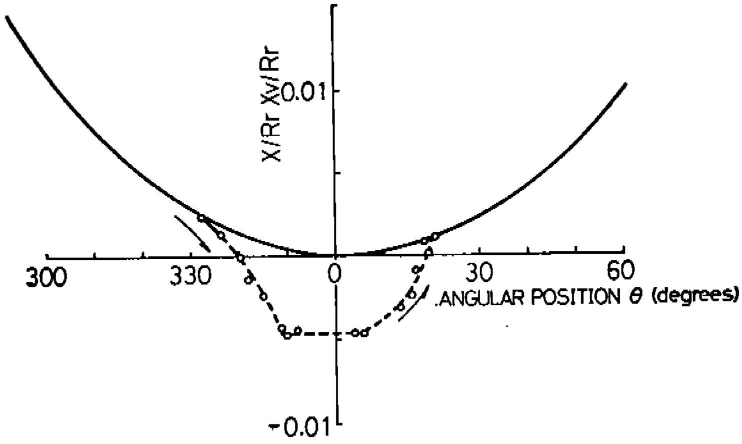

Figure 5. Extension of vane in chattering phenomena

$\left[P_d/P_s=2.5,\ P_b/P_s=2.0,\ \omega=125.7(\mathrm{rad/s}) \right]$

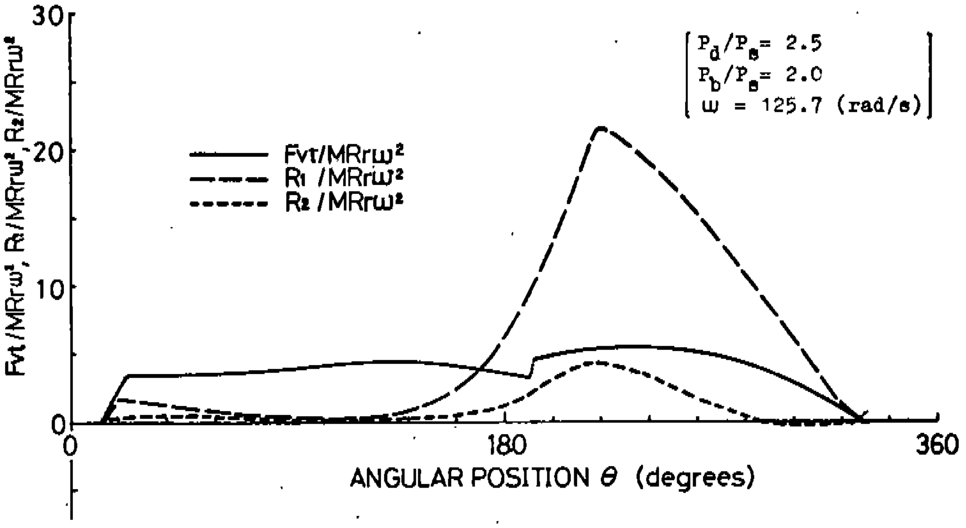

Figure 6. Reaction forces acting on the vane

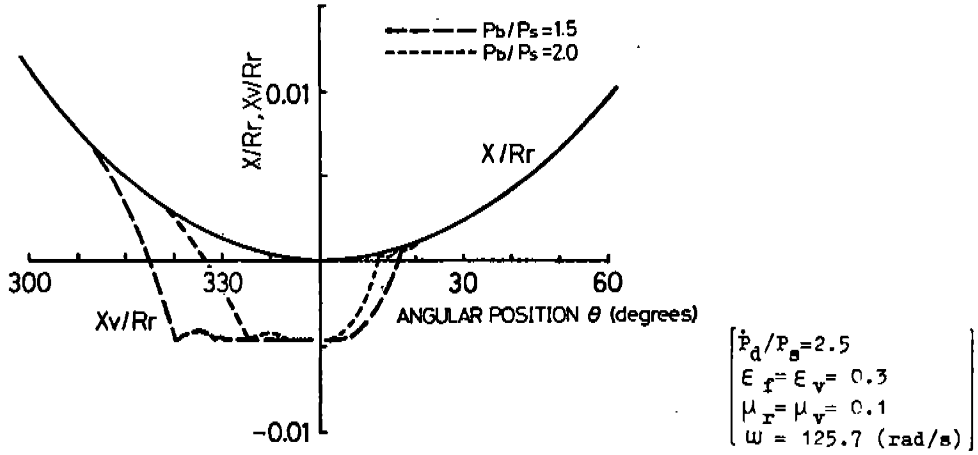

Figure 7. Extension of vane

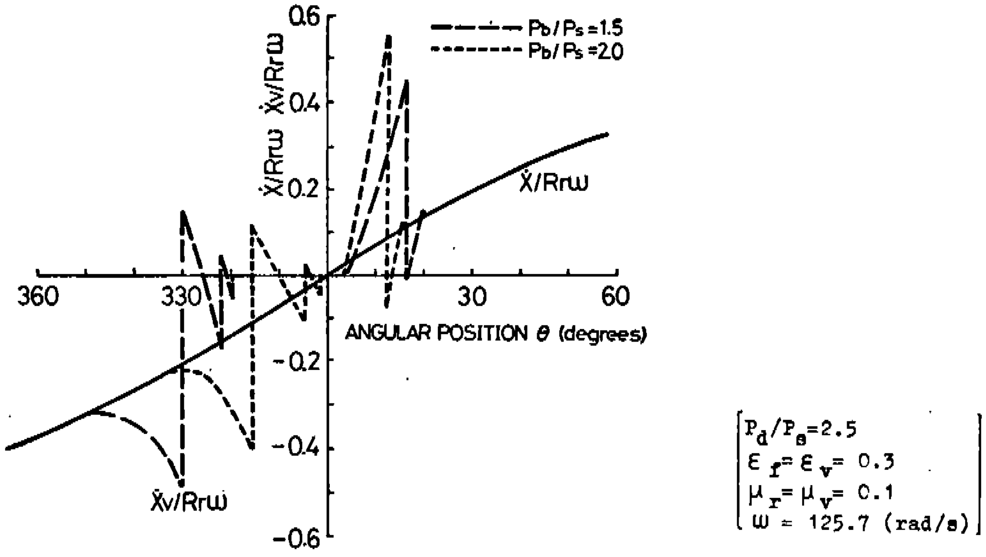

Figure 8. Velocity of vane