Simulation of Reciprocating Compressor Including Suction and Discharge Lines by Hybrid Computer

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1. INTRODUCTION

Development of the compressor model. - The compressor simulator, described hereafter, has been developed to serve as a tool for the valve designer. It is based on the following requirements:

(a) The practician usually cannot afford to make a substantial investment of time before he is able to make a fruitful use of the simulator. This goal can be reached when the model is simple and the man-machine communication is easy.
(b) The simulator must, nevertheless, be sufficiently accurate for practical design purposes. A model accounting for the pressure pulsations generated in the piping systems is necessary, as compressor performance is influenced by these pulsations as well as by the properties of the valves.
(c) It is considered essential that the simulator can be used for optimization studies, requiring large numbers of successive simulations with varying input parameter values.

An early version of hybrid*) simulator, designed for this purpose, is described in [1]. The necessary high speed of computation was achieved, in this case, by analog computer solution of the differential equations, while a digital computer performed the general control of the simulation process. By the construction of this simulator it was demonstrated that hybrid computer simulation was a feasible method, but the model had general disadvantages which hampered its practical application. The prediction of the motion of the valve plates was fairly accurate, but only when little or no pressure pulsations were present. The prediction of pressure pulsations, and, in relation, of performance figures such as volumetric efficiency was poor.

The present model is an improved and extended version of the early model. Moreover, in the course of the investigation, practical experience has been gained by using the simulator to assist in the optimization of a multiple reed valving system for an industrial refrigerating compressor which resulted in further improvements to the model. The main design criterion was a possibly low loss of energy by pressure drop. This valving system has been built and mounted in the test compressor used for the experimental validation of the simulator.

*) Analog/digital

The main improvements to the model are:

(a) The accessibility and ease of operation have been improved by using the digital computer as a comprehensive interface between the user and the compressor model, performing all routine operations, such as parameter in- and output, coefficient calculation and setting, etc.
(b) The model elements representing the suction and discharge piping system have been redesigned. These are "replacement systems", single degree of freedom oscillators, geometrically not similar to the real piping systems, but having the same (fundamental) natural frequency and offering comparable impedance loads to the compressor**). For more details of the piping model, see [2, 3]. The basic principle is described in this paper.
(c) To facilitate optimization studies, digital subroutines are developed for automatic stepwise variation of input parameter values during the simulation process. The results are given in the form of curves, showing the relative changes of certain criterion parameters as functions of varied input parameter.

2. THE PHYSICAL AND MATHEMATICAL MODELS

The physical model, for a single cylinder, single acting compressor, is outlined in fig. 1. The compressor model includes the following elements: a kinematical model of the crank/connection rod mechanism, thermodynamical models of the compressor cylinder and both plenum chambers, fluid flow and dynamic models of the valves and fluid flow models of the suction and discharge pipes. Each of the pipes and the adjacent plenum chamber are combined to form the single degree of freedom oscilllator models mentioned before. The valves are modeled as single degree of freedom mass/spring systems governed by Newton's second law; this model is adequate for the type of reed valve investigated. Provisions can be made for damping, sticktion and rebound.

**) Single degree of freedom replacement models are desirable when analog computation is envisaged, as they do not give rise to partial differential equations.

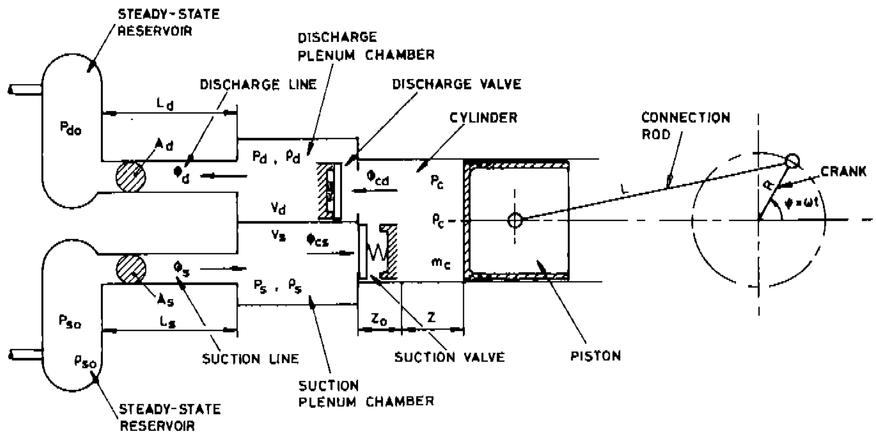

Fig. 1. Compressor model, showing the various system elements.

The gas is assumed ideal. The effect of cylinder heat transfer has been investigated by the use of a digital computer model. It could be concluded, that cylinder heat transfer had very little effect on the predicted valve behaviour and pressure pulsations, but its influence on the volumetric efficiency and the indicated work was substantial. If, however, instead of these quantities, their relative changes as a function of input parameter variation were considered, the influence of heat transfer was not so important. As mathematical simplicity is essential in analog computation, the heat transfer option was omitted to save on computing elements. The results given in the next paragraphs permit an assessment of the accuracy obtained.

The essence of the piping model. - When the suction or the discharge piping system (pipe and adjacent plenum chamber) are replaced by a Helmholtz resonator model (fig. 2), the natural frequency of the model can be approximately be determined from [4].

$$\omega_h = a\sqrt{\frac{A}{LV}} \tag{1}$$

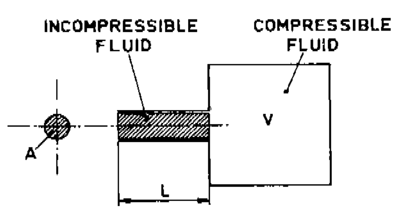

Fig. 2. Helmholtz resonator concept.

When, in another case, the gas in the pipe is assumed compressible (gas dynamical model), the fundamental natural frequency can be approximately determined by solving ω_g from the equation [4]:

$$tg(\omega_g L/a) = Aa/(V\omega_g) \tag{2}$$

for the interval $0 < \omega_g L/a < \pi/2$. For any set of values of a, A, L and V it is found that $\omega_h > \omega_g$. The difference can be reduced to zero by an appropriate change or "correction" to a, A, L or V in eq. 1. The basic idea of the pipe model is that, with such a modification, the Helmholtz resonator can be used as a replacement model for the piping system. Chosen is for correction of the chamber volume V by a certain amount V^*, as this correction restores not only the free oscillating frequency of the model, but also (approximately) the system impedance for the most relevant frequency range.
The volume correction V^* can be theoretically determined from

$$\omega_g = a\sqrt{\frac{A}{L(V+V^*)}} \tag{3}$$

When V^* is expressed as a fraction of the pipe volume

$$V^* = X\ A\ L \tag{4}$$

the volume correction factor X can be calculated from

$$X = \left(\frac{a}{\omega_g L}\right)^2 - \frac{V}{AL} \tag{5}$$

For practical purposes X is a function of the dimensionless parameter AL/V only.

X has limit values X = 0.333... if AL/V approaches zero (the theoretical no-pipe case) and $X = (2/\pi)^2$ = 0.40528... if AL/V approaches infinity (the theoretical no-plenum chamber case).
In equations D3 ... D6 of the mathematical model (table 1) the quantities V_s and V_d represent the corrected values. (The corrections are calculated by a digital subroutine.) Further corrections are the so-called "end corrections", accounting for the inert effect of the amount of gas in motion outside the pipe ends.

Mathematical model. - The mathematical model contains 12 coupled non-linear, first order differential equations (table 1). The equations contain variable coefficients, which are given by separate auxiliary equations*) (not shown in the table). For instance, the quantities Φ_{cs} and Φ_{cd}, the mass flows in each of the valves, are given by mass flow equations containing variable flow coefficients. Important functions of time are the piston displacement z and the piston velocity F_z, because all time dependent variables of the model depend on these functions. Fig. 3 permits an assessment of the accuracy

*) The model contains 21 auxiliary equations and 10 conditional equations (not listed in table 1).

with which the function z is generated.

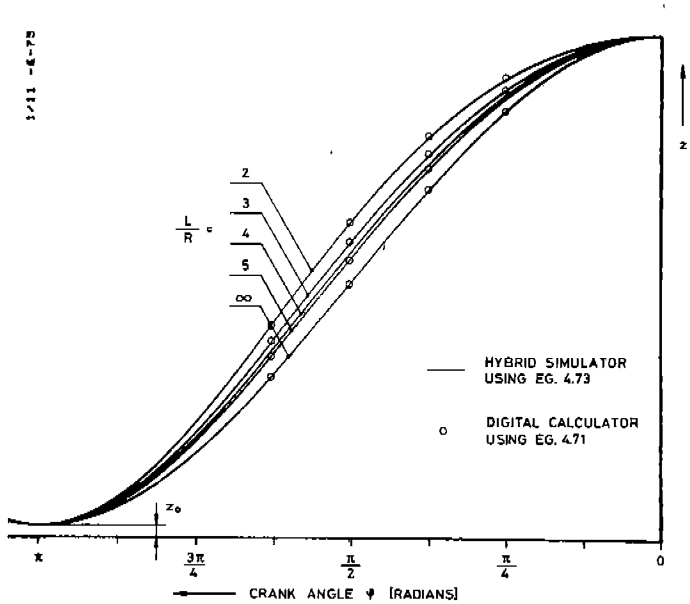

Fig. 3. Output of analog piston displacement generator, compared with some exact values of z.

The compressor cycle is subdivided into 5 phases. For each phase the equations may have different forms (variable equation model). The sequence of the phases is not a fixed one but is governed by conditional equations (valve logics). Possible changes from one phase into another are given by fig. 4.

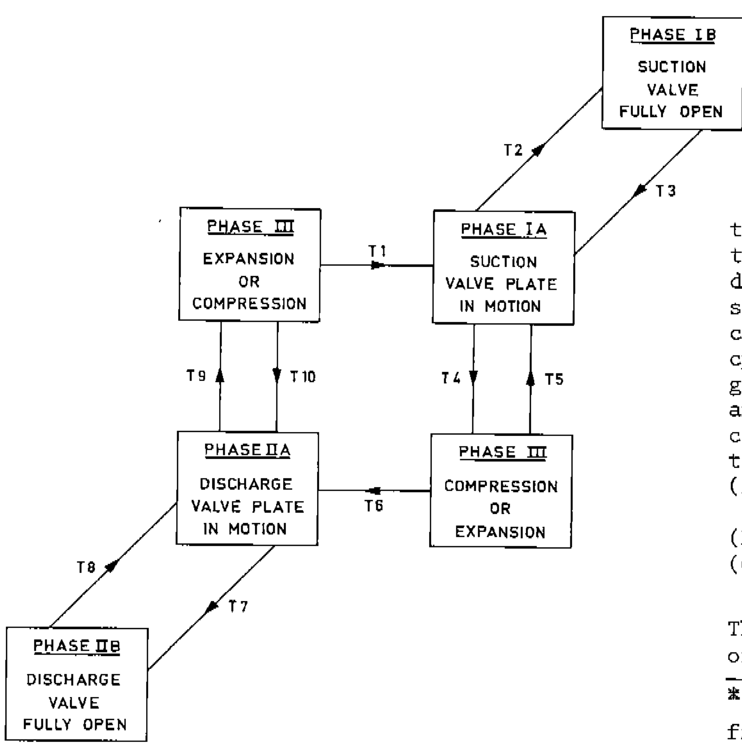

Fig. 4. Possible changes of one phase of operation into another. The symbols T1 ... T10 refer to conditional equations (not listed here).

3. IMPLEMENTATION

As the single degree of freedom replacement models for the piping systems do not give rise to partial differential equations, full advantage could be taken of the exceptionally high speed of computation obtainable with modern analog computers. A large analog computer is required because the non-linear equations necessitate a large number of computing elements*). For component economy, the valve model is used alternatively for the suction and discharge valve. This is not possible with the suction and discharge piping systems because in the normal case the flow processes are continued when the valves are closed. Fig. 5 gives a simplified analog functions diagram, showing the analog computing elements and their (patchboard) interconnections. The symbols D1, D2 ... correspond with the equation identifications used in table 1. The implementation of equations D11 and D12 is not shown. Although many logic functions are accomplished by the simulator, the logic functions diagram is not given as it is not well feasible to reduce this diagram to a simplified form which can be reproduced here.

The digital computer**) performs the general control of the simulation process and acts as a comprehensive interface between the user and the analog computer. In this function, it performs parameter input (by the use of a typewriter, which also gives messages to the user: the simulator is of the interactive type and can be used in a conversational way, (output of numerical results, coefficient calculation and setting, and control of the operation of the various output facilities (oscilloscope, line printer, and others). The software includes a number of subroutines for automatic operation of the simulator. One of these is described below.

Automatic repetitive mode. - For the purpose of optimization, it may be required to observe the change of certain criterion functions resulting from the variation of a chosen input parameter. For this purpose a subroutine has been designed which changes the chosen input parameter by small steps over a certain interval. After introduction of the necessary data (parameter name, interval limits, number of steps), the simulator will complete four successive compressor cycles for each parameter value (one cycle is generally insufficient to obtain a convergent solution). The results of each fourth cycle are storaged. When the process of simulation is completed, four quantities are recorded as a function of the chosen input parameter:

(A) The relative change of the volumetric efficiency.
(B) The relative change of the indicated work.
(C) and (D). Two criterion functions, related to the valve plate impact loads.

These criterion functions are formed by the product of the valve plate mass and the impact velocity,

*) The Applied Dynamics Four computer used, was sufficiently large for the single cylinder model. For a two-cylinder model, the aid of a second AD-4 computer is required.

**) An I.B.M. type 1800 has been employed.

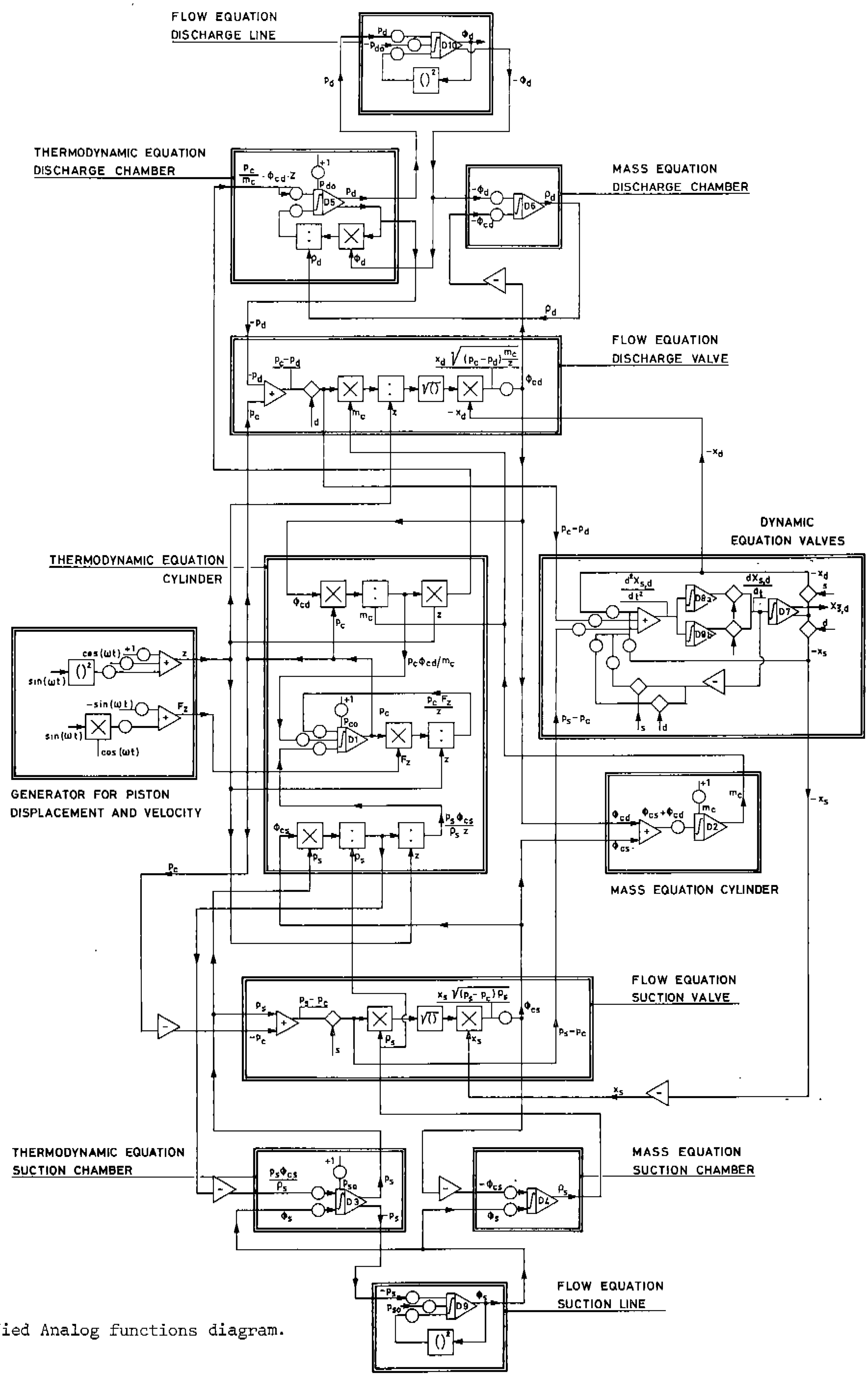

Fig. 5.
Simplified Analog functions diagram.

summarized for all impacts taking place during one compressor cycle. These functions are determined separately for the suction and the discharge valves. For the suction valve:

$$C = M_{vs} \sum_{1\,\mathrm{cycle}} |F_{xsi}| \tag{6}$$

where F_{xsi} is the impact velocity.

Fig. 6 gives an example of the output obtained for the case of a variation of the suction pipe length from 0 to 10 metres in 50 equal steps.

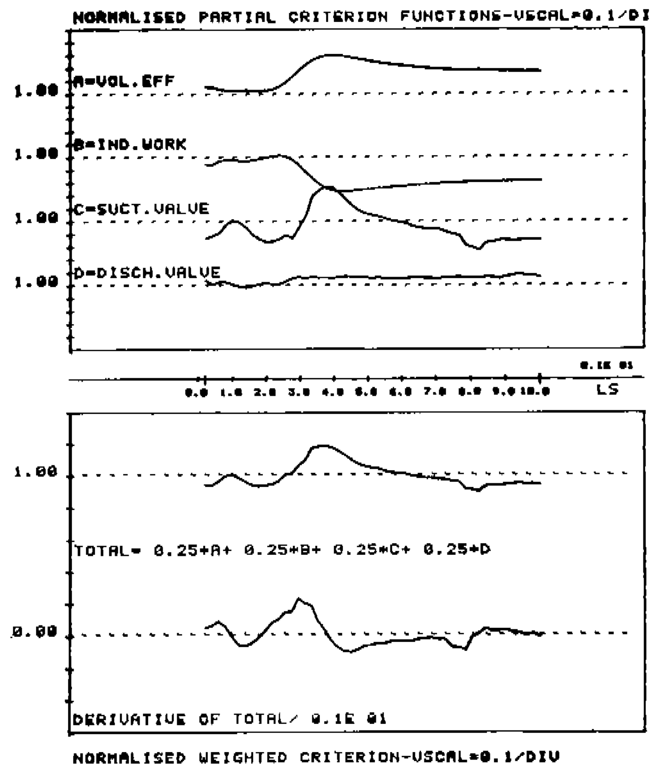

Fig. 6. Typical output (automatic repetitive mode).

The upper diagram gives the functions A, B, C and D. The lower diagram gives a weighted criterion function, derived from the functions A to D, and its derivative, showing the points where compressor performance is most sensitive to changes of the chosen input parameter. Such graphs can be made for any of the (about 45) input parameters.

4. MODEL VALIDATION

An important element of the development of a compressor model is the experimental validation. Only a fragmentary record of the results can be given here*).

Figs. 7a and 7b give a selection of the comparative results obtained. In order to permit an assessment of the systematic errors introduced by the strongly simplified pipe model, theoretical results obtained by the use of the single degree of freedom

*) Further results can be found in [2, 3].

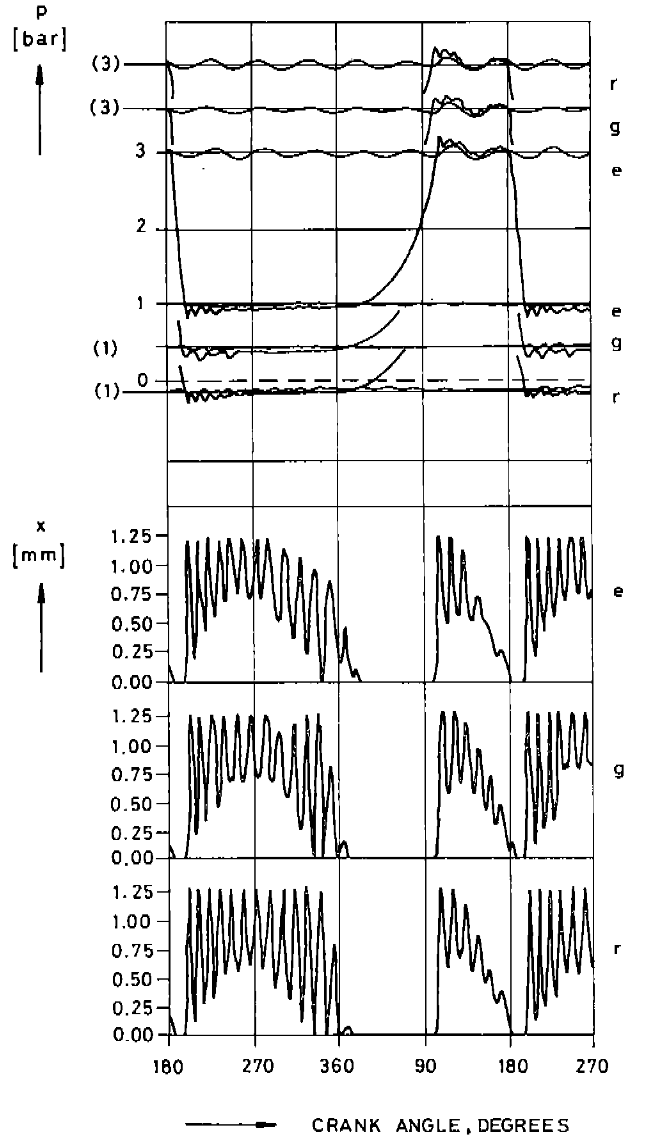

Fig. 7.a. Comparison of theoretical and experimental recordings of discharge plenum pressure p_d, cylinder pressure p_c, suction pressure p_s, suction valve plate lift x_s and discharge valve plate lift x_d.

r) theoretical results obtained by the use of the single degree of freedom replacement model for the piping systems.

g) theoretical results botained by the use of the gas dynamical pipe model.

e) experimental results.

Special test compressor with multiple reed valving system, operated with air, N = 600 r.p.m., L_s = 0.225 m L_d = 0.4 m.

The pressure scales of the theoretical recordings r) and g) are shifted with respect to the coordinate system used for the experimental recordings e). Corresponding pressure values are given in parentheses.

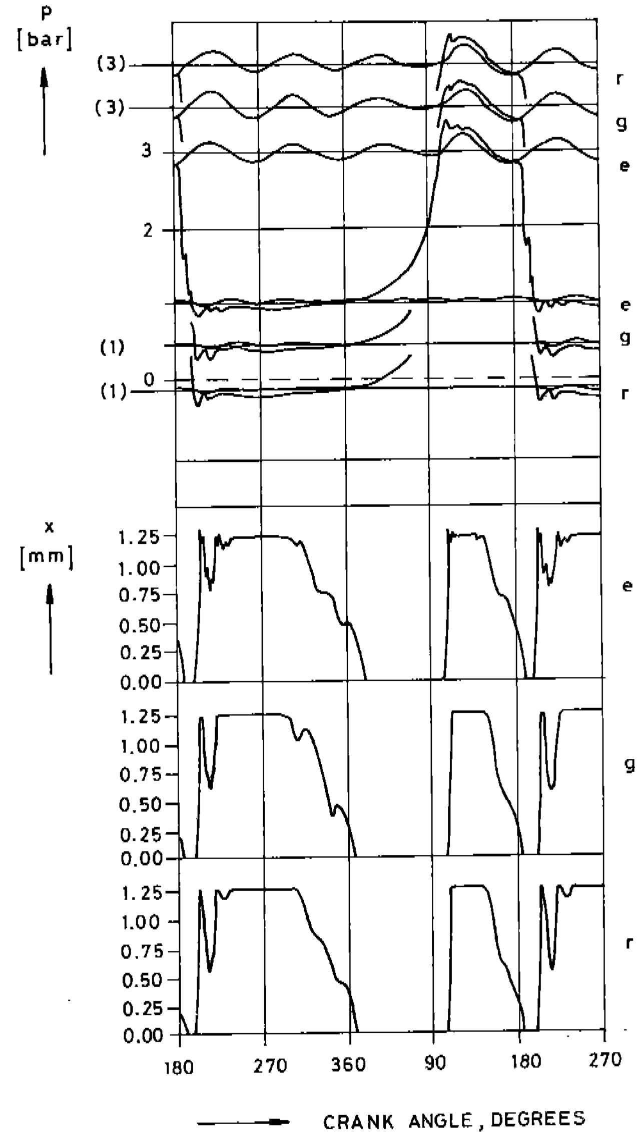

Fig. 7b. Similar to fig. 7a, but for N = 1000 r.p.m.

replacement model (r) are given as well as theoretical results obtained by the use of the gas dynamical pipe model developed by Benson and Uçer [5, 6]*). Such comparisons have been made for different operating conditions, for pipe lengths varying from 0 to 5 m, and compressor speeds varying from 600 to 1000 r.p.m.

Fig. 8 gives a comparison of theoretical results for suction pipe tuning, obtained by Bráblík [7, 8] by the use of a gas dynamical pipe model, and by the model developed in this investigation, respectively.

*) Dr. A.M. Bredesen, at Trondheim University, Norway, was so kind as to perform the necessary simulations with the gas dynamical model, using a digital computer.

To permit an assessment of the advantages of hybrid simulation over numerical solution of the equations using a digital computer, the same mathematical model was implemented also by using a digital computer (I.B.M. 370), and simulated results were produced for the same compressor using identical input parameters. This was repeated for different operating conditions. Some conclusions drawn from this experiment are given in table 2.

NOMENCLATURE

(Symbols explained in the text are not listed.)

- A area
- a velocity of sound
- F velocity (F_x = valve plate velocity, F_z = piston velocity)
- F force (F_g = gas force, F_s = spring force, F_f = friction force)
- L length, without index: connection rod length
- M_v valve plate mass
- p pressure
- R crank radius
- V volume
- X volume correction factor
- x valve plate lift
- z piston displacement
- ζ overall coefficient for pipe flow resistance
- κ ratio of specific heats c_p/c_v
- ρ specific mass
- Φ mass flow

INDICES

- s suction valve or suction piping system
- d discharge valve or discharge piping system
- c cylinder
- so constant conditions at entry of suction pipe
- do ibid., at outlet of discharge pipe

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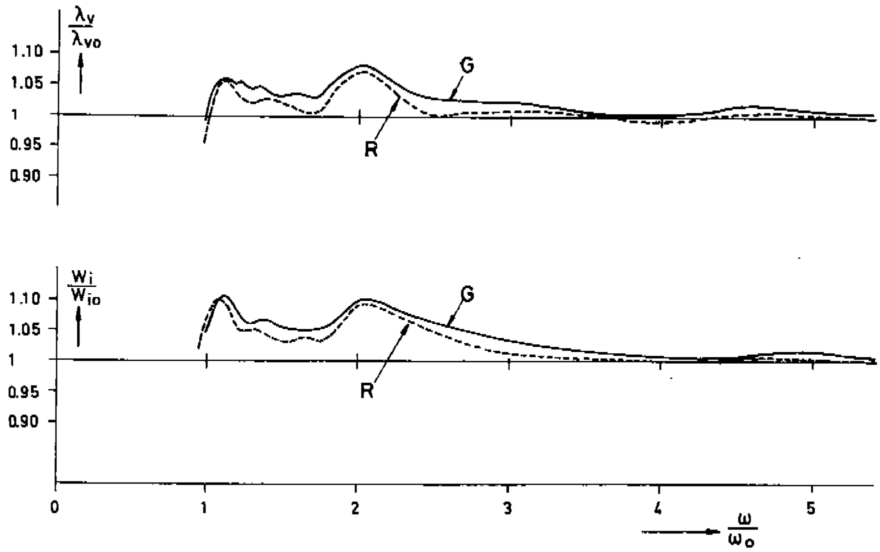
No.		PHASE I: SUCTION IA: valve plate in motion IB: valve fully open	PHASE II: DISCHARGE IIA: valve plate in motion IIB: valve fully open	PHASE III Compression or expansion
		Energy equation for gas in cylinder		
D1	$\frac{dp_c}{dt} =$	$-\kappa \frac{p_c F_z}{z} + \frac{\kappa}{A_c} \cdot \frac{p_s}{\rho_s z} \Phi_{cs} + \frac{\kappa}{A_c} \cdot \frac{\Phi_h}{z}$	$-\kappa \frac{p_c F_z}{z} - \frac{\kappa}{A_c} \cdot \frac{p_c}{\rho_c z} \Phi_{cd} + \frac{\kappa}{A_c} \cdot \frac{\Phi_h}{z}$	$-\kappa \frac{p_c F_z}{z} + \frac{\kappa}{A_c} \cdot \frac{\Phi_h}{z}$
		Mass equation for gas in cylinder		
D2	$\frac{dm_c}{dt} =$	Φ_{cs}	$-\Phi_{cd}$	0
		Energy equation for gas in suction chamber		
D3	$\frac{dp_s}{dt} =$	$\frac{\kappa}{V_s} \left(\frac{p_{so}}{\rho_{so}} \Phi_s - \frac{p_s}{\rho_s} \Phi_{cs}\right)$	$\frac{\kappa}{V_s} \cdot \frac{p_{so}}{\rho_{so}} \Phi_s$	$\frac{\kappa}{V_s} \cdot \frac{p_{so}}{\rho_{so}} \Phi_s$
		Mass equation for gas in suction chamber		
D4	$\frac{d\rho_s}{dt} =$	$\frac{1}{V_s} (\Phi_s - \Phi_{cs})$	$\frac{1}{V_s} \Phi_s$	$\frac{1}{V_s} \Phi_s$
		Energy equation for gas in discharge chamber		
D5	$\frac{dp_d}{dt} =$	$-\frac{\kappa}{V_d} \cdot \frac{p_d}{\rho_d} \Phi_d$	$-\frac{\kappa}{V_d} \left(\frac{p_d}{\rho_d} \Phi_d - \frac{p_c}{\rho_c} \Phi_{cd}\right)$	$-\frac{\kappa}{V_d} \cdot \frac{p_d}{\rho_d} \Phi_d$
		Mass equation for gas in discharge chamber		
D6	$\frac{d\rho_d}{dt} =$	$-\frac{1}{V_d} \Phi_d$	$-\frac{1}{V_d} (\Phi_d - \Phi_{cd})$	$-\frac{1}{V_d} \Phi_d$
		Valve dynamics equations (valid for phases IA and IIA only)		
D7	$\frac{dx}{dt} =$	F_{xs}	F_{xd}	
D8	$\frac{dF_x}{dt} =$	$\frac{1}{M_{vs}} (F_{gs} - F_{ss} - F_{fs})$	$\frac{1}{M_{vd}} (F_{gd} - F_{sd} - F_{fd})$	
		Discharge and suction line dynamics equation (valid for all phases)		
D9	$\frac{d\Phi_s}{dt} =$	$\frac{A_s}{L_s} (p_{so} - p_s) - \frac{\zeta_s}{2\rho_{so} L_s A_s} \Phi_s^2 \frac{\lvert\Phi_s\rvert}{\Phi_s}$		
D10	$\frac{d\Phi_d}{dt} =$	$\frac{A_d}{L_d} (p_d - p_{do}) - \frac{\zeta_d}{2\rho_d L_d A_d} \Phi_d^2 \frac{\lvert\Phi_d\rvert}{\Phi_d}$		
		Equation for indicated work		
D11	$\frac{dW_i}{dt} =$	$A_c p_c F_z$ (valid for all phases)		
D12	$\frac{dW_{iv}}{dt} =$	$(p_s - p_c) \lvert F_z \rvert A_c$	$(p_c - p_d) \lvert F_z \rvert A_c$	0

Table 1. Mathematical model.
Differential equations.



Influence of suction pipe length on compressor performance.

λ_v/λ_{vo} = relative change of volumetric efficiency
W_i/W_{io} = ibid., of indicated work
ω/ω_o = dimensionless parameter, representing piping geometry
ω = natural frequency of piping system
ω_o = angular speed of compressor shaft

Fig. 8. Comparison of gas dynamical model (G) and single degree of freedom replacement model (R).

	Digital simulator	Hybrid simulator
main advantages	versatility; accepts almost any compressor model. digital computer available to most investigators. no amplitude and time scale problems.	best possible man-machine communication. interactive way of operation. very high speed of computation. direct response. permits extensive optimization programs.
computation times and costs [1)]	actual computation time of 1 simulation run of 4 cycles 200 seconds, Dfl. 140,- Automatic repetitive mode not available, but would take 10.000 seconds Dfl. 7000,-	actual computation time negligible. Payed time = time needed to operate the controls. Time of operation of the single simulation mode: 120 seconds, Dfl. 10,- Time of operation of the automatic repetitive mode (50 steps): 180 seconds, Dfl. 15,-
accuracy	good	good
skills required by the operator	general knowledge of digital programming and numerical methods	for routine work: study of user's handbook + 1 week practical experience. for advanced work, requiring changes to the model: hybrid computer course.

[1)] approximate costs based on rates of Summer 1975 (Computer centre of Delft University of Technology, The Netherlands)

Table 2. Comparison of digital and hybrid simulator.