May 1993

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TR-EE 93-19
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ABSTRACT

A redundant robot has more degrees of freedom than what is needed to uniquely position the robot end-effector. In practical applications the extra degrees of freedom increase the orientation and reach of the robot. The load carrying capacity of a single robot can be increased by cooperative manipulation of the load by two or more robots. In this paper we develop an adaptive control scheme for kinematically redundant multiple robots in cooperative motion.

In a usual robotic task, only the end-effector position trajectory is specified. The joint position trajectory will therefore be unknown, for a redundant multirobot system and it must be selected from a self-motion manifold for a specified end-effector or load motion. We show that the adaptive control of cooperative multiple redundant robots can be addressed as a reference velocity tracking problem in the joint space. A stable adaptive velocity control law is derived it ensures bounded parameter convergence, exponential convergence to zero of the load position error, the internal force error and the reference velocity error. The individual robot joint motions is shown to be stable by decomposing the joint coordinates into two variables one which is homeomorphic to the load coordinated, the other to the coordinates of the self-motion manifold. The dynamics on the self-motion manifold is directly shown to be related to the concept of ***zero-dynamics.*** It is shown that if the reference joint trajectory is selected to optimize a certain type of objective functions, then stable dynamics on the self-motion manifold results. The overall stability of the joint angle is established from the stability of two cascaded dynamic systems involving the two decomposed coordinates.

1. INTRODUCTION

Recently considerable amount of research has focused on the problem of cooperative control and coordination of multiple robots. Interest in multi-robot systems has arisen because several tasks require the use of two or more robots. Examples of such tasks include the joining and securing of large pipes for the construction of space structures, picking up and carrying heavy loads, and grasping odd shaped loads. Cooperative robots may be used in hazardous or unsafe environments such as in space, in deep waters and in radioactive environments. By using more than one robot the manipulation capability and the workspace of the system may be further increased. However cooperative multiple-robot systems are more difficult to control than single robots. Additional problems may arise in the control if the parameters of the robots and the manipulated load may not be known exactly.

Several control schemes, adaptive and non adaptive schemes have been proposed for cooperative multiple robots with rigid joints manipulating a common load. Zheng and Luh [32] considered the kinematic and dynamic model of the multi-robot system and developed an inverse dynamics schemes for load position control. Hsu et al [23] developed a control algorithm for the coordinated manipulation of multi-fingered robot hands. Tarn, et al [28] developed a robust nonlinear control scheme using nonlinear transformation techniques. Yun, et al [29] also used exact linearization and output decoupling techniques to control multiple robots. Yoshikawa and Zheng [31] also developed linearizing tracking control laws for multiple robots, experiments were also reported. Few adaptive control schemes for cooperative robots manipulating a common load have been proposed. Walker et al. [30] developed an adaptive algorithm for the control of two robots handling a common load of an unknown mass. Zribi and Ahmad [24] proposed a robust adaptive controller for the multi-robot system manipulating a rigid object cooperatively when subject to bounded disturbances. The problem of manipulating a load using multiple robots when the load makes contact with an environment was addressed by Hyati [33], Cole [34] and, Hu and Goldenberg [27]. Ahmad and Guo [26] addressed the problem of controlling multiple flexible-joint robots with linear dynamics. Ahmad [35] developed a feedback linearizing controller for multiple flexible joint robots.

There are a very few papers in the area of control of multiple redundant robots, these include the recent paper by Tarn et al. [36] which addressed the zero dynamics issue. The paper by Tao and Luh [37] also addressed multiple redundant robot control. The reason there is such a few works in the multiple redundant robot control is primarily because non-redundant robot control schemes cannot be easily extended to control redundant robot systems. This is because a redundant robot has more joints than what is required to position the end-effector. Usually the end-effector trajectory is known and thus the joint trajectory cannot be found uniquely. In fact, for a fixed end effector position there is a self-motion manifold on which joint motions could occur without effecting the end-effector position. In Figure. 1, we show a planar redundant robot with three prismatic axes, we see if the end-effector is stationary the joints may move in a straight line in the joint space without effecting the end-effector. Any arbitrary joint trajectory which ensures end effector position cannot be used as this may not result in stable joint motions on the self motion manifold and would therefore effect overall stability. These two problems have prevented the simple extension of non-redundant strategies being adopted for multiple redundant robots. We should note here that the extra joints are extremely useful in real applications as they can be used to configure the manipulator posture, to avoid obstacles in the workspace or to avoid joint singularities.

Initial interest in the control of redundant robots started with the work of Whitney [21] who devised a kinematical resolved motion rate control strategy. Since then a number of researchers have addressed the the joint coordination and control of redundant robots (see Nenchev [13] for a review of those developments). The tutorial review by Siciliano [17] and the tutorial workshop report on the theory and application of redundant robots at the 1989 IEEE robotics and automation conference [1] covered some more recent developments (see also [1-3,7,11-14,16,17]). In the area of redundant robot adaptive control, Seraji [16] presented an approach based on the model reference adaptive control theory. He resolved the redundancy problem by adding additional task dependent kinematic constraints to the end-effector kinematics. This effectively ensured the joint solutions were unique. Niemeyer and Slotine [14] applied sliding mode adaptive control to redundant manipulators. They used the passivity principle to prove the stability of the adaptive system. Niemeyer and Slotine also performed some experiments to demonstrate their control law. Colbaugh et al. [3] proposed an adaptive inverse kinematics algorithm that did not require the knowledge of the kinematics of the robots. However their algorithm required persistent excitation conditions; also their algorithm did not consider the dynamics of the robot. Luo, Ahmad and Zribi [38] developed an adaptive control law for redundant robots making use of weighted scaling functions and the concept of zero dynamics to show both the joint motions on the self motion manifold and the end-effector motions would be stable for their control law.

In this paper, we address the problem of controlling redundant multiple robots manipulating a load cooperatively. We assume the load mass/inertial parameters and the robot joints mass/inertial parameters are unknown. We first state the dynamic models of the robots and the load and give a few properties of the multi-robot system. Next the redundancy resolution problem is discussed, and a model for adaptive resolution of the redundancy is established. A controller that leads to the exponential tracking of the load position and the convergence of the internal forces to their desired values is then derived. The boundedness of the joint motion and control torques are proved next. The conclusions can be found in the final section of the paper.

2. MULTI-ROBOT SYSTEM MODEL

2.1 Dynamics Model

The general dynamic model for a cooperative multi-robot system has been investigated thoroughly in the literature, and is also described in the below for completeness. In Figure. 2 we depict the organization of the multiple robots grasping a common load which is to be manipulated cooperatively. We first state a few assumptions related to the robots grasp of the load and the reachability of the trajectory that will be used in the subsequent derivation.

Assumptions

(A1) The manipulators are rigidly grasping the load, (i.e., there is no motion between the contact point of the load and the robots end-effectors).

(A2) The desired trajectory is reachable and the end effector can be positioned at those workspace (of dimension six) positions without exceeding any joint motion limits.

The dynamic equation of the ith manipulator in cooperative manipulation can be written as,

$$D_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i) + J_{e_i}(q_i)^T F_{e_i} = \tau_i \qquad i=1,\ldots,k. \tag{1}$$

where, $q_i \in R^{n_i}$ is the vector of joint displacements, and $n_i>6$ is the number of joints of the ith robot The inertia matrix of the ith robot is $D_i(q_i) \in R^{n_i \times n_i}$, this is a positive &finite and symmetric matrix. The matrix of centrifugal and Coriolis forces is $C_i(q_i,\dot{q}_i) \in R^{n_i \times n_i}$; the vector of gravity forces is $G_i(q_i) \in R^{n_i}$, and the manipulator Jacobian is $J_{e_i}(q_i) \in R^{6\times n_i}$. The control input torque for the ith robot is $\tau_i \in R^{n_i}$. We will &fine the total number of joints of the k robots as n, $n = \sum_{i=1}^{i=k} n_i$.

The forces/moments applied by the ith manipulator on to the object at the point of contact are F_{e_i}. The contact forces/moments $F_{e_i} \in R^6$ can be written in terms of the contact forces $f_{e_i} \in R^3$ and contact moments $\eta_{e_i} \in R^3$, (where 6 represents the dimension of the Cartesian work space), such that,

$$F_{e_i} = \begin{bmatrix} f_{e_i}{}^T & \eta_{e_i}{}^T \end{bmatrix}^T \qquad i = 1,\ldots,k \,. \tag{2}$$

Now we will group the dynamics of the k-robots system to get,

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G_r(q) + J_e(q)^T F_e = \tau \,, \tag{3}$$

where D $\in R^{n\times n}$ is a block diagonal matrix whose diagonal elements are $D_i \in R^{n_i\times n_i}$. C $\in R^{n\times n}$ is a block diagonal matrix whose diagonal elements are $C_i \in R^{n_i x n_i}$ and J $\in R^{6k\times n}$ is a block diagonal matrix whose diagonal elements are $J_{e_i} \in R^{6\times n_i}$. Also we will define the following vectors as,

$$q=[q_1{}^T \ldots q_k{}^T]^T, G_r=[G_1{}^T \ldots G_k{}^T]^T, \tau=[\tau_1{}^T \ldots \tau_k{}^T]^T \text{ and, } F_e=[F_{e_1}^T \ldots F_{e_k}{}^T]^T \tag{4}$$

If we assume that the object is rigidly grasped, then the equations of motion of the object are obtained from the Newton-Euler mechanics as,

$$M_1 \ddot{x}_p + M_1 g_l = \sum_{i=1}^{i=k} f_{e_i} , \qquad (5)$$

$$and, \; I\dot{\omega} + \omega \times (I\omega) = \sum_{i=1}^{i=k} (\eta_{e_i} + r_i \times f_{e_i}) , \qquad (6)$$

where the position of the center of mass of the object expressed in the world coordinate frame is $x_p \in R^3$. The rotational velocity of the object expressed in the world coordinate frame is $\omega \in R^3$, and the gravity force vector of the object, expressed in the world coordinate reference frame is $g_l \in R^3$. The mass matrix $M_1 \in R^{3\times3}$ is a diagonal mamx whose diagonal elements are the mass of the load; the matrix $I \in R^{3\times3}$ is the inertia matrix of the load. The vector $r_i = [\, r_{ix}, r_{iy}, r_{iz}]^T \in R^3$ represents the translational displacements from the center of mass of the object to the contact point of the object and the ith manipulator.

If we let $X = [\, \dot{x}_p^T \;\; \omega^T \,]^T$, then the motion of the object expressed by equation (5) and (6) can be written as,

$$M\ddot{x} + N\dot{x} + G_l = GF_e = F_o , \qquad (7)$$

where $G \in R^{6\times6k}$ is the grasp mamx; G is defined as,

$$G = \begin{bmatrix} T_1 & T_2 & \dots & T_k \end{bmatrix} . \qquad (8)$$

The mamx $T_i \in R^{6\times6}$ is such,

$$T_i = \begin{bmatrix} I_{3\times3} & 0 \\ \Omega_i(r_i) & I_{3\times3} \end{bmatrix} \quad \text{and} , \quad \Omega_i(r_i) = \begin{bmatrix} 0 & -r_{iz} & r_{iy} \\ r_{iz} & 0 & -r_{ix} \\ -r_{iy} & r_{ix} & 0 \end{bmatrix} .$$

The mamx $I_{3\times3} \in R^{3\times3}$ is the identity mamx. Also we have,

$$M = \begin{bmatrix} M_1 & 0 \\ 0 & I \end{bmatrix} , \quad N\dot{x} = \begin{bmatrix} 0 \\ \omega \times (I\omega) \end{bmatrix} \quad \text{and} \quad G_l = \begin{bmatrix} M_1 g_l \\ 0 \end{bmatrix} . \qquad (9)$$

2.2 Kinematic Model

We are interested in controlling the manipulators in some predefined Cartesian task space such that,

$$x_{e_i} = K_{e_i}(q_i) \qquad i = 1, \dots, k , \qquad (10)$$

where $K_{e_i}(.) : R^{n_i} \rightarrow R^6$ is the transformation from the joint angle space of q_i to the task space containing x_{e_i}, and $x_{e_i} \in R^6$ is the position and orientation of the point of contact of the ith manipulator, with the load, expressed in the world coordinate frame. If we differentiate equation (10) with respect to time, and if we define $J_{e_i}(q_i)$ to be the differential map from the q_i space to x_{e_i} space, then we can write,

$$\dot{x}_{e_i} = J_{e_i}(q_i)\dot{q}_i \quad i = 1,\ldots,k \, . \tag{11}$$

If these equations are stacked into a single vector by forming the J_{e_i} into a block diagonal matrix, and concatenating the $\dot{q}_i$'s into one vector q, we get,

$$v_c = J_e \dot{q} \, , \tag{12}$$

where $v_c = \left[\dot{x}_{e_1}^T \ \dot{x}_{e_2}^T \ \ldots \ \dot{x}_{e_k}^T\right]^T$ is the velocity vector at the contact points and $J_e = diag(J_{e_1}, \ldots, J_{e_n})$.

Using equation (7), we can write,

$$F_o = \mathrm{G}F, \tag{13}$$

Now from the duality between the forces and the velocities, we can write,

$$\mathrm{G}^T \dot{x} = v_c \, , \tag{14}$$

where x is the velocity of the object. Thus for the k robots system, we can combine equations (12) and (14) to get,

$$G^T \dot{x} = J_e \dot{q} \, . \tag{15}$$

where G is the grasp matrix defined earlier.

2 3 Definition of Internal Forces and Internal Force Errors

The end-effector force of the ith manipulator, F_{e_i}, can be decomposed into two forces, the motion force and the internal grasping force. The internal grasping forces $F_I = [F_{Ie1}, .., F_{Iek}]^T \in R^{6k}$ do not cause any motion of the load. However we must control these end-effector internal forces, $F_{Iei} \in R^6$ *with* $i=1,..,k$, in order to prevent excessive compressive or expansive forces being applied to the load. 'We can calculate the internal force F_I from equation (7) if F_o is known and rank (G)= 6, then

$$F_e = G^+ F_o + F_I \, . \tag{16}$$

Here, $G^+ = G^T(GG^T)^{-1}$ and $GG^+ = I_6$, given I_6 is an 6×6 identity matrix. (For a discussion related to other choices of the inverse of the G^+ matrix see [36] and [39]. Notice that other choices of the inverse of G does not effect the derivations presented in this paper.) Therefore we see that $GF_I = 0$ and $GF_e = F_o$, i.e, the internal forces do not contribute to the motion of the load. The desired internal forces $F_{I,d} \in R^{6k}$ also satisfy $GF_{I,d} = 0$. The internal force error, $e_f = F_{I,d} - F_I$, also satisfies

$$G \, e_f = 0. \tag{17}$$

These internal force properties will be used to derive the control law.

2.4 A Few Properties of the Multi-Robot System

In the following we will state several properties which will be used in the derivations of the controller.

System Dynamics Properties

(P1) D and M are **symmetric** positive &finite matrices.

(P2) D − 2C is skew symmetric mamx or $\frac{1}{2}q^T(D-2C)q=0$. The proof of property **P2** can be found in **[19]** and **[40].**

(P3) $M-2N_2$ is skew symmemc mamx or, $\frac{1}{2}\dot{x}^T(\dot{M}-2N)\dot{x}=0$. This property can be seen from energy considerations. In general if M is not expressed in the object center of mass coordinate frame, M=M(x) and $N\dot{x}=N(x,\dot{x})\dot{x}$. The total energy of the load is given by $E_L=\frac{1}{2}\dot{x}^TMx+h(x)$ where $h(x)$ is the potential energy and $G_l=\frac{\partial}{\partial x}h(x)$. As the power input to the load is given by, $\frac{d}{dt}E_L=\dot{x}^TF_o=\dot{x}^T(M\ddot{x}+\frac{1}{2}\dot{M}\dot{x}+G_l)=\dot{x}^T(M\ddot{x}+N\dot{x}+G_l)$, thus we have the property, $\frac{1}{2}\dot{x}^T(\dot{M}-2N)\dot{x}=0$.

Two important properties of the **inertia** matrix, the **centrifugal/Coriolis** mamx and the gravity vector which will be used in the developments are now given.

(P4) Linear Parameterization of the Robot Dynamics

The linearity of D, C and G, with respect to the manipulators dynamic parameters P, $\in R^{s_r}$ **is** now stated, these parameters will be estimated by the proposed adaptive scheme. The robot dynamics can be linearly parameterized **[19]**, **[40]** and,

$$Da_r+Cv_r+G_r=Y_rP_r\ , \tag{17}$$

where a, $\in R^n$ and v, $\in R^n$ are vectors and we denote a, as the "reference acceleration of the robots" and also, v_r is the " reference velocity of the robots." The regressor matrix $Y_r(q,\dot{q},v_r,a_r)\in R^{n\times s_r}$ represents the **structure** of the robots dynamics, hence its elements are combinations of the nonlinear functions **present** in the inertia mamx, **centrifugal/Coriolis** matrix and the gravity vector.

(P5) Linear Parameterization of the Object Dynamics

The second property deals with the linearity of M, N_2 and G_l with respect to the load parameter vector P_o,

$$Ma_o+N_2v_o+G_l=Y_oP_o \tag{18}$$

where $a_o\in R^6$ and $v_o\in R^6$ are the "reference acceleration of the load and the "reference velocity of the load," respectively. We will denote by $P_o\in R^{s_o}$ the vector of s_o load parameters which are constants for a given load. These **parameters** will be estimated by the proposed adaptive scheme. The regressor matrix $Y_o(x,\dot{x},v_o,a_o)\in R^{6\times s_o}$ represents the structure of the load dynamics.

Remark 1

Let $\hat{P}_r$ be the vector of estimates of the parameters of the robots, then the error vector in the estimates of the robots **parameters** is $\tilde{P}_r = \hat{P}_r -$ P, . Similarly, we can write the parameter estimation error vector for the load as $\tilde{P}_o = \hat{P}_o -$ P , . Notice that we can write,

$$\hat{D}a, + \hat{C}v_r + \hat{G}, = Y_r\hat{P}_r \;, \qquad (19)$$

where $\hat{D}$ is the estimate of the inertia mamx D, $\hat{C}$ is the estimate of the **Coriolis/centrifugal** mamx and $\hat{G}_r$ is the estimate of the the gravity vector. Also notice that, $\tilde{D}a_r + \tilde{C}v_r + \tilde{G}_r = Y_r\tilde{P}_r = Y_r(\hat{P}_r - P_r)$, where $\tilde{D}$ is the **error** in the inertia mamx, $\tilde{C}$ is the **error** in the **Coriolis/centrifugal** mamx and $\tilde{G}_r$ is the error in the gravity vector.

Similarly we can write,

$$\hat{M}a, + \hat{N}_2 v_o + \hat{G}_l = Y_o\hat{P}_o \;, \qquad (20)$$

where M is the estimate of M, $\hat{N}_2$ is the estimate of N_2 and $\hat{G}_l$ is the estimate of G_l. Also notice that $\tilde{M}a_o + \tilde{N}_2 v_o + \tilde{G}_l = Y_o\tilde{P}_o$, where $\tilde{M}$, $\tilde{N}_2$ and $\tilde{G}_l$ **are** the differences between the estimates and the true values.

3. REDUNDANCY RESOLUTION PROBLEM

3.1 Preliminaries

Consider a **kinematically** redundant manipulator with the **carried** load center of mass positioned at point x, and the joint position q_i. Then the differentiable **kinematic** mapping is K_i such that,

$$x = K_i(q_i) \qquad i = 1,\ldots,k \qquad (21)$$

where x $\in$ R ^{6} is the position of the load and $q_i \in R^{n_i}$ is the vector of joint positions of the ith manipulators and as $n_i > 6$, the degree of redundancy of the ith robot is $r_i = n_i - 6$. As a result of the joint redundancy at the end effector point x $= x_d$, there will exist a set of joint angles, the self motion manifold such that $Q\mathfrak{N}_i = \{q_i \,|\, x = x_d = K_i(q)\}$. (Recall the example in Figure. 1, the self motion manifold was the line in the joint space.) Thus, in order to find a unique joint angle q, additional requirements **are** necessary; these will be stated later. We will denote the Jacobian of the **kinematic** map (21) by, $J_i = T_i^{-T} J_{e_i} \in R^{6\times n_i}$. This relates differential map between the load position kinematics $K_i(\bullet)$ and the end effector kinematics $K_{e_i}(\bullet)$. The projection operator onto the null space of J_i is denoted by $P_{J_i}(q_i)$ $(i=1,\ldots,k)$. Also let all the columns of matrix N_{J_i} be the basis of ker (J_i), which is the null space of J_i. Hence we have,

$$J_i P_{J_i} = 0, \quad \text{and} \quad \ker(J_i) = \text{span}(N_{J_i}). \tag{22}$$

The matrix $N_{J_i} \in R^{n_i \times r_i}$ has the following properties that will be used in the text,

$$J_i N_{J_i} = 0 \in R^{6 \times r_i}, \; N_{J_i}^T J_i^T = 0 \in R^{r_i \times 6}, \; N_{J_i}^T J_i^{+} = 0 \in R^{r_i \times 6}, \tag{23}$$

$$N_{J_i}^T P_{J_i} = N_{J_i}^T \in R^{r_i \times n_i}, \; N_{J_i}^T N_{J_i} = I_{r_i \times r_i}, \; N_{J_i} N_{J_i}^T = P_{J_i} \in R^{n_i \times n_i}; \tag{24}$$

for any vector $\dot{q}_i \in R^{n_i}$ if $N_{J_i}^T \dot{q}_i = 0 \in R^{r_i}$ *then* $P_{J_i}\dot{q}_i = 0 \in R^{n_i}$. (25)

Notice also that $[\begin{smallmatrix} J_i \\ N_{J_i}^T \end{smallmatrix}]$ is a square matrix of full rank, thus we have,

$$\begin{bmatrix} J_i \\ N_{J_i}^T \end{bmatrix}^{-1} = [J_i^{+} \quad N_{J_i}]. \tag{26}$$

These properties show that the **pairs** $(J_i, N_{J_i}^T)$ and (J_i^{+}, N_{J_i}) are orthogonal complement operator pairs.

3 3 Statement of the Problem of the Redundancy Resolution

The redundancy is usually resolved by the constrained optimization of a performance index H_i (i=1,...,k), this function can be used to avoid joint limits, obstacles and **singularities (see** the review papers listed in the reference). The problem can be formulated as follows: given a desired position x_d, find the joint position q_i (i=1,..,k) such that,

$$\min_{q_i} \quad H_i(q_i) \quad \text{with} \quad x_d = K_i(q_i) \qquad i=1,\ldots,k. \tag{27}$$

We can conclude from the **Lagrange** multiplier method that the solution of the constrained optimization problem (27) necessarily satisfies the following set of constrained **differential** equations:

$$P_{J_i}\nabla H_i(q_i) = 0 \quad \text{and} \quad x_d = K_i(q_i) \qquad i=1,\ldots,k. \tag{28}$$

We **will define** the end-effector path **tracking** error e as,

$$e = K_i(q_i) - x_d \qquad i=1,\ldots,k. \tag{29}$$

***Our* goal** is to resolve the "asymptotic resolution of the redundancy problem" such that as t $\to \infty$, we have,

$$e \to 0, \qquad e \to 0, \quad \text{and,} \qquad P_{J_i}\nabla H_i(q_i) \to 0 \qquad i=1,\ldots,k. \tag{30}$$

We want to **optimize** H_i (i=1,...,k) by appropriate joint motion on the self-motion manifold, $Q_{N_i}^q$ (i=1,...,k). At the optimal point, we do not desire further motion on the self motion manifold. Therefore the projection of the joint velocity on the self-motion manifold must be zero, and $N_{J_i}^T \dot{q}_i + 0$ as, t $+ =$. Thus it is sufficient (not necessary) to write the **asymptotic** redundancy resolution as $t \to \infty$,

$$(\dot{e} + \gamma e \to 0) \; and, \; N_{J_i}^T (\dot{q}_i - \mu_i \nabla H_i) \to 0 \; , \; with \; N_{J_i}^T \dot{q}_i \to 0 \quad i=1,\ldots,k. \tag{31}$$

Here, $\gamma > 0$ and $\mu_i \neq 0$. The first equation can be written as $J_i \dot{q}_i - \dot{x}_d + \gamma e \to 0$. After grouping terms and using the matrix inversion expressed by (26), we find $\dot{q}_i$, and, as $t \to \infty$,

$$\dot{q}_i - \begin{bmatrix} J_i^{\dagger} & N_{J_i} \end{bmatrix} \begin{bmatrix} \dot{x}_d - \gamma e \\ \mu_i N_{J_i}^T \nabla H_i \end{bmatrix} \to 0 \; with \; q_i \to \{ q_i \mid N_{J_i}^T \nabla H_i = 0 \text{ and } x_d = K_i(q_i) \}. \tag{32}$$

Therefore the "asymptotic resolution of redundancy problem" can be expressed by the conditions given by (32). These conditions result in the joint velocities approaching their desired values, while the joint positions satisfy a set of constraint equations. Notice that the redundancy resolution problem is characterized by the fact that the desired joint positions are not known in advance. This fact prevents us from directly using the existing adaptive schemes that achieves joint position tracking.

We will denote by v_{r_i} $(i=1,\ldots,k)$ the joint reference velocity for the ith robot. We also will denote by v_o the load reference velocity. We will choose v_{r_i} $(i=1,\ldots,k)$ such that,

$$v_{r_i} = \begin{bmatrix} J_i^{\dagger} & N_{J_i} \end{bmatrix} \begin{bmatrix} \dot{x}_d - \gamma e \\ \mu_i N_{J_i}^T \nabla H_i \end{bmatrix} = J_i^{\dagger}(\dot{x}_d - \gamma e) + \mu_i P_{J_i} \nabla H_i \; . \tag{33}$$

We will group the v_{r_i} $(i=1,\ldots,k)$ into one vector v_r such that, $v_r = \begin{bmatrix} v_{r_1}^T & v_{r_2}^T & \ldots & v_{r_k}^T \end{bmatrix}^T$ is the joint reference velocity of the robots. We will choose v_o such that,

$$v_o = \dot{x}_d - \gamma e \; . \tag{34}$$

It should be noted that the choice of v_o guarantees that,

$$v_o = J_i v_{r_i} = T_i^{-T} J_{e_i} v_{r_i} \qquad i=1,\ldots,k. \tag{35}$$

The asymptotic resolution of redundancy problem can be solved by a mechanism that ensures $\dot{q}_i - v_{r_i} \to 0$, $(for \; i=1,\ldots,k)$, as $t \to \infty$.

In order to proceed further we will state a few more assumptions these will be needed in the control law development.

3.3 Assumptions - Continued

(A3) The desired paths $x_d(t)$, $\dot{x}_d(t)$ and $\ddot{x}_d(t)$ are bounded for all time t.

(A4) The Jacobian $J_i(q_i)$ $(i=1,\ldots,k)$ is a full rank continuously differentiable function matrix, that is, $J_i(q_i)$ is of class C^r, $r \geq 2$, (i.e., at least twice differentiable).

(A5) The cost function $H_i(q_i)$ $(i=1,\ldots,k)$ given in (27) is a twice differentiable real valued function.

In assumption (A4) the full **rank** restriction on $J_i(q_i)$ (i=1,...,k) requires that all possible joint motions q_i, do not pass through any singularity configuration of $J_i(q_i)$, this will be shown to be possible with the control law **derived** in this :paper, this will be addressed in the final section of the paper (see also [38]). If $J_i(q_i)$ is continuous and full **rank** in some subset $G_{J_i} \subset$ R'', then $J_i^{\dagger} = J_i^T(J_iJ_i^T)^{-1}$, $P_{J_i} = I_{n_i} - J_i^{\dagger}J_i$ and N_{J_i} are **continuous** in G_{J_i}. The mamces J_i, $J_i^{\dagger}$, P_{J_i} and N_{J_i} are shift varying linear operators. It is easy to show that any continuous linear operator is bounded, hence J_i, $J_i^{\dagger}$, P_{J_i} and N_{J_i} are bounded in G_{J_i}, (i.e. the induced norm of J_i, $J_i^{\dagger}$, P_{J_i} and N_{J_i} are finite in G_{J_i}). Furthermore, if $\dot{J}_i$ is continuous in G_{J_i}, then $\dfrac{dJ_i^{\dagger}}{dt}$ and $\dot{P}_{J_i}$ are continuous on any path with continuous $\dot{q}_i$ in G_{J_i}.

4. DESIGN OF THE CONTROL AND UPDATE LAWS

4.1 Design of the Control Law

Our goal is to design an adaptive controller that guarantees the asymptotic convergence of the load tracking **error** to zero, the convergence of the internal forces to their desired values and the redundancy resolution. We will start by defining a few **variables** needed for the development. The weighted reference velocity error for the ith robot is defined as,

$$\rho_{r_i} = w_t(\dot{q}_i - v_{r_i}) \qquad i=1,\ldots,k \,. \tag{36}$$

The scalar weighting function w_t will be chosen as, $w_t = e^{\lambda t}$, where λ is a positive constant (see [18] for the use of **weighting** functions in the adaptive control of single **rigid robots**). We **will** group the ρ_{r_i} (i=1,...,k) into one vector ρ_r such that, $\rho_r = \begin{bmatrix}\rho_{r_1}^T & \rho_{r_2}^T & \cdots & \rho_{r_k}^T\end{bmatrix}^T$. Also the weighted reference velocity error for the load is **defined** as,

$$\rho_o = w_t(\dot{x} - v_o) \,. \tag{37}$$

It is easy to show that,

$$\rho_o = J_i\rho_{r_i} \qquad i=1,\ldots,k \,. \tag{38}$$

We **will** choose $\dot{\rho}_r$ and $\dot{\rho}_o$ such that,

$$\rho_{r_i} = w_t(\ddot{q}_i - a_{r_i}) \qquad i=1,\ldots,k \,, \tag{39}$$

$$\text{and, } \rho_o = w_t(\ddot{x} - a_o) \,. \tag{40}$$

The choice of ρ_{r_i} given by equation (36), and the choice of $\dot{\rho}_{r_i}$ given by equation (39) will result in the following value for a,. ,

$$a_{r_i} = \dot{v}_{r_i} + \lambda(v_{r_i} - \dot{q}_i) \qquad i=1,\ldots,k \,. \tag{41}$$

We can group the a $(i=1,\ldots,k)$ into one vector a, such that, a, $= \left[a_{r_1}^T \;\; a_{r_2}^T \;\; \ldots \;\; a_{r_k}^T\right]^T$. The choice of ρ_o given by equation (37), and the choice of $\dot{\rho}_o$ given by equation (40) will result in the following value for a_o,

$$a_o = \dot{v}_o + \lambda(v_o - \dot{x}) \,. \tag{42}$$

Notice that v, and v_o are **independent** of q and $\dot{x}$, hence a, and a, are not functions of q and $\ddot{x}$. Therefore the proposed adaptive scheme does not require the measurements of 'the accelerations q and x.

Theorem 1

Given that the matrices K_o, K_r, Γ_r and Γ_o are positive definite mamces, K_f is a positive **semidefinite** diagonal mamx, the control law given by (43) and the parameter update laws given by (45) and (46) ensure that ρ_r , $\rho_o \in L_2 \cap L_\infty$ and that P, , $P_o \in L_\infty$.

$$\tau = \hat{D}a_r + \hat{C}v_r + \hat{G}_r - K_r(\dot{q}-v_r) + J_e^T G^+[\hat{M}a_o + \hat{N}_2 v_o + \hat{G}_l - K_o(\dot{x}-v_o)] + J_e^T \tau_f$$

$$= Y_r \hat{P}_r - K_r(\dot{q}-v_r) + J_e^T G^+[Y_o \hat{P}_o - K_o(\dot{x}-v_o)] + J_e^T \tau_f \,, \tag{43}$$

The force torque τ_f is given by,

$$\tau_f = F_{el_d} - K_f \int e_f \,, \tag{44}$$

The parameters update laws are such,

$$\dot{\hat{P}}_r = -\Gamma_r^{-1} Y_r^T \rho_r w_t \,, \tag{45}$$

$$and, \;\; \dot{\hat{P}}_o = -\Gamma_o^{-1} Y_o^T \rho_o w_t \,. \tag{46}$$

Preliminaries to the Proof.

Before proving theorem 1, we will derive the equation of the closed loop system. We can solve for the force from equation (7), thus we get,

$$F_e = G^+(M\ddot{x} + N_2\dot{x} + G_l) + F_{el} \,. \tag{47}$$

If we combine equations (3) and (47), we get,

$$D\ddot{q} + C\dot{q} + \mathrm{G,} \; + J_e^T G^+(M\ddot{x} + N_2\dot{x} + G_l) + J_e^T F_{el} = \tau \,. \tag{48}$$

Now we will multiply both sides of equation (48) by $G(J_e^T)^+$, we get,

$$\mathrm{G}(J_e^T)^+[\,\mathrm{Dq} + \mathrm{Cq} + \mathrm{G,}\;] + M\ddot{x} + N_2\dot{x} + G_l = G(J_e^T)^+\tau \,. \tag{49}$$

Here we used the fact that $GF_{el} = 0$.

Replacing τ by its value from equation (43), and using the fact that $G\tau_f = 0$, we get,

$$G(J_e^T)^+[\, D\ddot{q} + C\dot{q} + G_r \,] + M\ddot{x} + N_2\dot{x} + G_l$$
$$= G(J_e^T)^+[\, Y_r\hat{P}_r - K_r(\dot{q}-v_r) \,] + Y_o\hat{P}_o - K_o(\dot{x}-v_o)$$
$$= G(J_e^T)^+[\, Y_r\tilde{P}_r + Y_rP_r - K_r(\dot{q}-v_r) \,] + Y_o\tilde{P}_o + Y_oP_o - K_o(\dot{x}-v_o)\,. \qquad (50)$$

Replacing Y_rP_r and Y_oP_o by their values from equations **(17)** and **(6)**, we obtain,

$$G(J_e^T)^+[\, D(\ddot{q}-a_r) + C(\dot{q}-v_r) + K_r(\dot{q}-v_r) \,] + M(\ddot{x} - a_o) + N_2(\dot{x}-v_o) + K_o(\dot{x}-v_o)$$
$$= G(J_e^T)^+Y_r\tilde{P}_r + Y_o\tilde{P}_o\,. \qquad (51)$$

Thus the composite system can be written as,

$$G(J_e^T)^+[D\dot{\rho}_r + C\rho_r + K_r\rho_r] + M\dot{\rho}_o + N_2\rho_o + K_o\rho_o = G(J_e^T)^+Y_r\tilde{P}_r w_t + Y_o\tilde{P}_o w_t\,. \qquad (52)$$

Proof of Theorem 1:

Consider the following Lyapunov function candidate

$$V = \frac{1}{2}\,\rho_r^T D\rho_r + \frac{1}{2}\,\tilde{P}_r^T\Gamma_r\tilde{P}_r + \frac{1}{2}\,\rho_o^T M\rho_o + \frac{1}{2}\,\tilde{P}_o^T\Gamma_o\tilde{P}_o\,. \qquad (53)$$

Now if we differentiate V with respect to time and use propemes **P1 - P3**, we get,

$$\dot{V} = \rho_r^T[\, D\dot{\rho}_r + C\rho_r \,] + \tilde{P}_r^T\Gamma_r\dot{\hat{P}}_r + \rho_o^T[\, M\dot{\rho}_o + N_2\rho_o \,] + \tilde{P}_o^T\Gamma_o\dot{\hat{P}}_o\,, \qquad (54)$$

using the fact that $\rho_r = J_e^+ G^T\rho_o$, **V** becomes,

$$\dot{V} = \rho_o^T[\, G(J_e^T)^+(D\dot{\rho}_r + C\rho_r) + M\dot{\rho}_o + N_2\rho_o \,] + \tilde{P}_r^T\Gamma_r\dot{\hat{P}}_r + \tilde{P}_o^T\Gamma_o\dot{\hat{P}}_o\,. \qquad (55)$$

Using equation **(52)**, we get,

$$\dot{V} = \rho_o^T[\, -G(J_e^T)^+K_r\rho_r - K_o\rho_o + G(J_e^T)^+Y_r\tilde{P}_r w_t + Y_o\tilde{P}_o w_t \,] + \tilde{P}_r^T\Gamma_r\dot{\hat{P}}_r + \tilde{P}_o^T\Gamma_o\dot{\hat{P}}_o$$
$$= -\,\rho_o^T G(J_e^T)^+K_r J_e^+ G^T\rho_o - \rho_o^T K_o\rho_o + \rho_o^T G(J_e^T)^+Y_r\tilde{P}_r w_t + \rho_o^T Y_o\tilde{P}_o w_t$$
$$+ \tilde{P}_r^T\Gamma_r\dot{\hat{P}}_r + \tilde{P}_o^T\Gamma_o\dot{\hat{P}}_o\,. \qquad (56)$$

Using the update laws given by equations **(45)** and **(46)**, we get,

$$\dot{V} = -\,\rho_o^T G(J_e^T)^+K_r J_e^+ G^T\rho_o - \rho_o^T K_o\rho_o + \rho_o^T G(J_e^T)^+Y_r\tilde{P}_r w_t - \tilde{P}_r^T Y_r^T\rho_r w_t$$
$$+ \rho_o^T Y_o\tilde{P}_o w_t \;\; - \tilde{P}_o^T Y_o^T\rho_o w_t\,. \qquad (57)$$

Thus,

$$\dot{V} = -\,\rho_o^T G(J_e^T)^+K_r J_e^+ G^T\rho_o - \rho_o^T K_o\rho_o = -\,\rho_r^T K_r\rho_r - \rho_o^T K_o\rho_o\,. \qquad (58)$$

Hence V > 0 and $\dot{V} \leq 0$. Thus we can conclude that $\rho_r \in L_2 \cap L$, , $\rho_o \in L_2 \cap L$, , $\tilde{P}_r \in L_\infty$ and $\tilde{P}_o \in L$,.

□

corollary 1

$\dot{q}_i - v_{r_i} \rightarrow 0$ (i=1,...,k) and $\dot{x} - v_o \rightarrow 0$ at the rate of $e^{-\lambda t}$.

Proof:

Using equation (36), we can write $\dot{q}_i - v_{r_i} = \rho_{r_i} e^{-\lambda t}$. Hence $\dot{q}_i - v_{r_i} \rightarrow 0$ (i=1,...,k) at the **rate** of $e^{-\lambda t}$. **Similarly**, from equation (37), we can write, $\dot{x} - v_o = \rho_o e^{-\lambda t}$. Hence, $\dot{x} - v_o \rightarrow 0$, at the rate of $e^{-\lambda t}$.

□

Hence we can conclude from equation (51) that,

$$G(J_e^T)^+ D\dot{\rho}_r e^{-\lambda t} + M\dot{\rho}_o e^{-\lambda t} - G(J_e^T)^+ Y_r \tilde{P}_r - Y_o \tilde{P}_o \rightarrow 0 \qquad as\ t \rightarrow \infty. \qquad (59)$$

provided that the joint angles q are bounded and $v_r(q)$ is bounded. We can show through the analysis of the perturbed dynamical systems q $- v_r(q) = w_t^{-1} \rho_r \rightarrow 0$, as, r $\rightarrow \infty$ that q for an appropriate choice of $v_r(q)$ will be bounded and **stable.** This will be shown next (see also [38]). In fact the **boundedness** q and the boundedness $J_e^+ G^T$ (**i.e,** robot trajectories do not pass through singular configuration) both depend on the stability of $q = v_r(q)$ and therefore on the choice of $v_r(q)$, this will be seen in the final sections (see also [38]). If $J_e^+ G^T$ is bounded (J_e nonsingular and q is bounded), we can write,

$$D\dot{\rho}_r e^{-\lambda t} + J_e^T G^+ M\dot{\rho}_o e^{-\lambda t} - Y_r \tilde{P}_r - J_e^T G^+ Y_o \tilde{P}_o \rightarrow 0 \qquad as\ t \rightarrow \infty. \qquad (60)$$

4.2 Boundedness of the Internal Forces

Theorem 2

The control law given by (43) and the parameter update laws given by (45) and (46) ensure the convergence of the internal forces to their desired trajectories. (**i.e.,** $e_f \rightarrow 0\ as$ r $\rightarrow \infty$) .

Proof:

If we combine equations (48) and (43), we get,

$$D\ddot{q} + C\dot{q} + G_r + J_e^T G^+ (M\ddot{x} + N_2\dot{x} + G_l) + J_e^T F_{el} = Y_r \hat{P}_r - K_r(\dot{q} - v_r)$$
$$+ J_e^T G^+ [Y_o \hat{P}_o - K_o(\dot{x} - v_o)] + J_e^T \tau_f. \qquad (61)$$

Using the facts that $\hat{P}_r = \tilde{P}_r +$ P,, and $\hat{P}_o = \tilde{P}_o + P_o$, and replacing $Y_r P_r$ and $Y_o P_o$ by their values from equations (17) and (18), we obtain,

$$D(\ddot{q}-a_r)+C(\dot{q}-v_r)+K_r(\dot{q}-v_r)+J_e^T G^+[\,M(\ddot{x}-a_o)\ +N_2(\dot{x}-v_o)+K_o(\dot{x}-v_o)\,]$$
$$=Y_r\tilde{P}_r+J_e^T G^+ Y_o\tilde{P}_o+J_e^T(\tau_f-F_{el})\,. \tag{62}$$

or,

$$D\dot{\rho}_r e^{-\lambda t}+C\rho_r e^{-\lambda t}+K_r\rho_r e^{-\lambda t}+J_e^T G^+[\,M\dot{\rho}_o e^{-\lambda t}+N_2\rho_o e^{-\lambda t}+K_o\rho_o e^{-\lambda t}\,]$$
$$=Y_r\tilde{P}_r+J_e^T G^+ Y_o\tilde{P}_o-J_e^T(e_f+K_f\textstyle\int e_f). \tag{63}$$

Using corollary 1 and equation (60) (assuming $J_e^+ G^T$ and q is bounded), we can conclude that,

$$J_e^T(e_f+K_f\textstyle\int e_f)\rightarrow 0 \quad as \quad t\rightarrow\infty\,. \tag{64}$$

The mamx J_e^T is not a singular matrix, and it is a full rank mamx, thus we can conclude from equation (64) and with appropriate choice of K_f that, $e_f \rightarrow 0$, as, $t \rightarrow \infty$. Notice that K_f can be set to zero if the internal forces are not measurable.

□

5. BOUNDEDNESS OF THE JOINT MOTIONS AND CONTROL TORQUES

In this section we will show the boundedness of q, q, and the control torque τ based on a perturbation model. We notice that equation (32) can be written as a decayed perturbation system,

$$\dot{q}_i=v_{r_i}(q_i)+\delta_{v_i}(q_i,t) \qquad i=1,\ldots,k. \tag{65}$$

Recall from Corollary 1 that $\|\delta_{v_i}(q_i,t)\|\rightarrow 0$, as, $t\rightarrow\infty$, thus the perturbation $\delta_{v_i}=w_t^{-1}\rho_{ri}\ (i=1,\ldots,k)$ is bounded and tends to zero as $t\rightarrow\infty$.

We will prove the boundedness of q_i in the perturbed system, described by equation (65), by ensuring the boundedness of q_i in the unperturbed system $\dot{q}_i=v_{r_i}(q_i)$. In the following, we will consider several Lemmas that establish the relationship between the boundedness of the perturbed and unperturbed systems. The first important lemma which is stated without proof is the result of Markus and Opial (see [5] pp. 282). Recall that the set S is said to be invariant if each solution starting in S remains in S for all t [5]. Specifically, for a continuous time system, S is said to be an invariant set under the vector field $\dot{z}=f(z)$ if for any $z(0)=z^0\in$ S, we have $z(t)\in$ S for all $t\in R^+$ (with $z\in R^n$ and $f: R^n \rightarrow R^n$).

Lemma 1 (Stability of the perturbed system) [5]

Consider the perturbed differential equation with $z_\delta \in R^n$ and $f: R^n \rightarrow R^n$.

$$\dot{z}_\delta = f(z_\delta) + \delta(z_\delta, t) \qquad \text{with} \qquad z_\delta(0) = z^0. \tag{66}$$

This system is called "asymptotically autonomous" if:
(1) $\delta(z_\delta, t) \rightarrow 0$ as $t \rightarrow \infty$ uniformly for z_δ in an arbitrary compact set Ω in R^n, or, (2) $\delta(z_\delta, t) \in L_1$ for all $z_\delta(t)$ which are bounded and continuous on Ω for $t \geq 0$.

Then, the positive limit sets (i.e., the set with $t \in R^+$ and $t \rightarrow \infty$) of the solutions of (66) are invariant sets of the original differential equation,

$$\dot{z} = f(z) \qquad \text{with} \qquad z(0) = z^0 . \tag{67}$$

Notice that because of the choice of w_t, the redundancy resolution equation (65) modeled as a perturbed system is indeed asymptotically autonomous, since the perturbed term δ_{v_i} is absolutely integrable as,

$$\int_0^\infty \|\delta_{v_i}\| dt \leq \frac{B_{\rho_i}}{\lambda}, \tag{68}$$

where B_{ρ_i} is a positive constant.

Lemma 2 (Asymptotic stability of the perturbed system)

Assume that the perturbed system (66) is an asymptotically autonomous system. Then the limit solution set of (66) is the limit solution set of (67). If the positive limit set of (67) is bounded, then $\|z_\delta - z\|$ is bounded as $t \rightarrow \infty$.

Proof:

Let V be a continuous Lyapunov function defined on the set G_s which is a subset of R^n. We define E to be the set of all points in the closure [15] of G_s, (the closure of G_s will be denoted by $\bar{G}_s$), where $\dot{V}(z) = 0$, that is,

$$E = \{ z \mid \dot{V}(z) = 0, z \in \bar{G}_s \} . \tag{69}$$

Let M_s be the largest invariant set in E, then LaSalle's theorem [10] asserts that every solution of (67) approaches M_s as $t \rightarrow \infty$. Thus the result of Lemma 1 yields that the positive limit set of (66) is the positive limit set of (67), hence z_δ tends to some limit points of the unperturbed system in (67). Moreover, if the positive limit set of (67) is bounded, then $\|z_\delta - z\|$ is bounded as $t \rightarrow \infty$.

□

We should note that the asymptotic convergence to the positive limit set is a local behavior. Lemma 2 tells us that if Δ_h is the measure of the limit set of (67) (i.e, $\|z_\delta - z\| < \Delta_h$ as $t \rightarrow \infty$). then given any number $h > \Delta_h$, we can always find a time t_h such that for $t > t_h$ we have $\|z_\delta(t) - z(t)\| < h$. The next lemma enables us to show that the trajectory of (66) is bounded in $t \in [0, t_h]$.

Lemma 3 (Boundedness of the perturbed system)

Consider the **perturbed** differential equation (66) and suppose that the mapping $f : R^n \rightarrow R^n$ has a Lipschitz constant $C_L > 0$, and suppose that the perturbation $\delta(z_\delta, t)$ along the trajectory z_δ has bounded L_1 norm. Then the trajectory $z_\delta(t)$ is bounded up to a given time t_h if the original differential equation

$$\dot{z} = f(z) \qquad \text{with} \qquad z(0) = z^0 \tag{70}$$

is stable.

Proof:

It is sufficient to show that $\|z_\delta - z\|$ is **bounded** for all $t \in [0,\infty)$, since z (t) is bounded by the assumption of the stability of (70).

The solution curve of (66) can be written as, $z_\delta(t) - z^0 = \int_{u=0}^{t} f(z_\delta)du + \int_{u=0}^{t} \delta(z_\delta, u)du$.

Similarly for unperturbed system (70), we have, $z(t) - z^0 = \int_{u=0}^{t} f(z)du$. Combining these two equations, we get, $z_\delta(t) - z(t) = \int_{u=0}^{t} \delta(z_\delta, u)du + \int_{u=0}^{t} (f(z_\delta) - f(z))\, du$. As f (.) is **Lipschitz** by assumption, hence, $\|z_\delta - z\| \le B_\delta + \int_{u=0}^{t} C_L \|z_\delta - z\| du$. Recall that the norm in Banach space is always a continuous and nonnegative function (Banach spaces are complete normed spaces). Hence this allows us to use the **Bellman-Gronwall's** lemma **(see [6],** p. **169),** thus we have

$$\|z_\delta - z\| \le B_\delta e^{C_L t_h}\,, \tag{71}$$

for $t = t_h$. Hence the stability of the unperturbed system (70) ensures the boundedness of z, then z_δ is bounded in $t \in [0, t_h]$ for any given $t_h \ge 0$.

□

Using Lemma 2 and Lemma 3 to solve the asymptotic redundancy resolution problem, we arrive at the following propositions.

Proposition 1 (Boundedness of joints and parameters)

If we assume that the function v_{r_i} $(i=1,\ldots,k)$ in (33) is Lipschitz, then we can find a set $R_{q_i^0}$ (the set of the initial q_i), such that the solutions of the adaptive control system **(i.e.** the parameters and the joint positions) are bounded for any time. Therefore with the adaptive control law given by **(43), (45)-(46),** the solution of (36) is bounded for any time, if the solution of the unperturbed system,

$$J_i \dot{q}_i = \dot{x}_d + ye, \qquad N_{J_i}^T \dot{q}_i = \mu_i N_{J_i}^T \nabla H_i \qquad i=1,\ldots,k \tag{72}$$

is bounded in $R_{q_i^0}$.

Proof:

The adaptive system given by equations (3-6), (43), (45) and (46) is an asymptotically autonomous system because we have shown that the perturbation term is uniformly bounded time decreasing function. The set $\{q_i \mid \ \|\dot{q}_i - v_{r_i}\| \le B_{p_i}\}$ can be taken as the compact set Ω in Lemma 1. Thus Lemma 2 and Lemma 3 guarantee the boundedness of the adaptive system for all time if q_i $(i=1,\ldots,k)$, the solution of (72), is bounded.

The boundedness of the unperturbed system will be studied in the next section. To show the boundedness of the control torque we will make use of the assumptions stated earlier.

Proposition 2 (**Boundedness of $\dot{q}_i$**)

Based on assumptions (A3), (A4) and (A5), the boundedness of the joint motion q_i $(i=1,\ldots,k)$ ensures the boundededness of the joint velocity $\dot{q}_i$ $(i=1,\ldots,k)$.

Proof:

The joint reference velocity v_{r_i} (i=1,...,k) given by (33) is a function of x_d, $\dot{x}_d$ and q_i. By Assumption (A5), the boundedness of q_i yields the boundedness of $\dot{x}_d$ −ye. By Assumptions (A4) and (A5), the boundedness of q_i yields the boundedness of $J_i^{\dagger}(q_i)$, $P_{J_i}(q_i)$ and $\nabla H_i(q_i)$ (for $i=1,\ldots,k$), hence $v_{r_i}(q_i)$ (for i=1,...,k) is bounded for all bounded q_i (for $i=1,\ldots,k$). Therefore the boundedness of $\|\dot{q}_i - v_{r_i}\|$ in the adaptive system leads us to the boundedness of $\dot{q}_i$, provided that q_i is bounded.

Proposition 3 (Boundedness of the control torque)

Based on assumptions (A3) - (A5), if q_i and $\dot{q}_i$ $(i=1,\ldots,k)$ are bounded then the adaptive control torque defined by (43) is bounded.

Proof:

Based on assumptions (A3) - (A5) and the boundedness of q_i and $\dot{q}_i$, the reference velocity v_{r_i} and acceleration a_{r_i} expressed by (33) and (41) respectively are bounded. Therefore the control torque is bounded.

6. THE STABILITY OF THE UNPERTURBED SYSTEM

The trajectories q_i $(i=1,\ldots,k)$ of the unperturbed system are bounded if q_i $(i=1,\ldots,k)$ of the self motion manifold is bounded. The dynamics of q_i on the self motion manifold have to be shown to result into joint angle q_i which is bounded. We will show that the quadratic form cost function $H_i(q_i)$ (i=1,...,k) is a special choice which guarantees the boundedness of q_i (i=1,...,k).

Below, we will examine the boundedness of the unperturbed system by using a homeomorphic transformation of the coordinates. A homeomorphism is a continuous mapping between two topological spaces if its inverse mapping is also continuous. A homeomorphism also maps a continuous function to another continuous function. A homeomorphism preserves the topological properties such as the openness, connectedness, and the convergence of a set. We will find a homeomorphism which transforms the coordinates of the configuration q_i (i=1,...,k) into a decomposable coordinates ξ_i and ζ_i $(i=1,\ldots,k)$, where ξ_i is homeomorphic to the workspace coordinates x. The variable ζ_i will be used to represent the dynamics on the self motion manifold. Hence the unperturbed system $\dot{q}_i = v_{r_i}(q_i)$ $(i=1,\ldots,k)$ is transformed into a cascaded system,

$$\dot{\zeta}_i = v_{\zeta_i}(\zeta_i, \xi_i)\,, \qquad \dot{\xi}_i = v_{\xi_i}(\xi_i) \qquad i=1,\ldots,k\,. \tag{73}$$

The boundedness of q_i $(i=1,\ldots,k)$ will be deduced from the boundedness of ξ_i and ζ_i. We will adopt the method used to prove the sufficiency of the Frobenius' theorem [9], to find the homeomorphism. We will construct the diffeomorphism based on the self-motion manifold. For any given x, all the points q_i such that $x = K_i(q_i)$ lie on the leaf of the self-motion manifold $Q_N^{q_i^0}$. The leaf of the self-motion manifold will be denoted by $Q_N^{q_i^0}$. This manifold is a connected region. By assumption $N_{J_i}(q_i)$ is nonsingular, then the distribution $\Delta_i = \ker(J_i) = \text{span}(N_{J_i})$ is nonsingular. The null space of a Jacobian matrix is always completely integrable, hence Δ_i is involutive. The distribution $\Delta_i = ker(J_i)$ has an annihilator $\Delta_i^{\perp}$ which is spanned by J_i which is the exact differential of the kinematic map K_i. The integrability of Δ_i allows us to construct the integral manifold by piecewise integrating every column of N_{J_i}.

Let $\Phi_t^{f_i}$ &note the flow of the vector field f_i, such that $q_i(t) = \Phi_t^{f_i}(q_i^0)$ solves the ordinary differential equation $\dot{q}_i = f_i(q_i)$ with initial condition q_i^0. The transition mapping $\Phi_t^{f_i}$ which maps q_i^0 to $q_i(t)$ is a diffeomorphism, and has the property $\frac{\partial \Phi_t^{f_i}(q_i^0)}{at} = f_i(q_i(t))$ [6,9]. The flow of each vector field represented by a column of Nj. is the solution of the following differential equations,

$$\dot{q}_i = N_{J_i^l}(q_i) \quad \text{with} \quad q_i(0) = q_i^0, \quad \text{for} \quad l=1,\ldots,r_i; \tag{74}$$

and can be written as,

$$q_i(t=\zeta_i^l) = \Phi_{\zeta_i^l}^{N_{J_i^l}}(q_i^0) \qquad l=1,\ldots,r_i; \tag{75}$$

Thus we have,

$$\frac{\partial \Phi_{\zeta_i^l}^{N_{J_i^l}}(q_i^0)}{\partial \zeta_i} = N_{J_i^l}(q_i) \qquad l=1,\ldots,r_i. \tag{76}$$

Lemma 4 (The parameterized equation of the self-motion manifold)

Given a kinematic mapping $x = K_i(q_i)$. The composite mapping $Q_{\zeta_i} : R^{r_i} \rightarrow Q^{q}_{N_i}$ such that,

$$(\zeta_i^1, \ldots, \zeta_i^{r_i}) \rightarrow q_i(t) = \Phi^{N_{J_i^{r_i}}}_{\zeta_i^{r_i}} o \ldots o \Phi^{N_{J_i^1}}_{\zeta_i^1}(q_i^0) \quad and \quad t = \zeta_i^1 + \ldots + \zeta_i^{r_i} . \tag{77}$$

is a locally parametrized equation of the manifold $Q_N^{q_i^0} = \{q_i \in C(q_i^0)$ *such that* $x_0 = K_i(q_i) = K_i(q_i^0)\}$, which passes through q_i^0. Here $C(q_i^0)$ is used to denote the connected regions of the self-motion manifold and $C(q_i^0)$ passes through the initial joint configuration q_i^0.

Proof:

We shall show that for $t = \zeta_i^1 + \ldots + \zeta_i^{r_i}$, we have $K_i(q_i(t)) = K_i(q_i^0)$. Since $x = K_i(q_i)$, it suffices to show that x is unchanged whenever ζ_i varies locally, i.e. $\frac{\partial x}{\partial \zeta_i^l} = 0$ for $l=1,\ldots,r_i$.

First. consider the rightmost integral $\Phi^{N_{J_i^1}}_{\zeta_i^1}$ in *(77)*.Let $q_{\zeta_i^1} = \Phi^{N_{J_i^1}}_{\zeta_i^1}(q_i^0)$. Then

$$\frac{\partial x}{\partial \zeta_i^1} = \frac{\partial K_i}{\partial \zeta_i^1}\Big|_{q_{\zeta_i^1}} = \frac{\partial K_i}{\partial q_i}\Big|_{q_{\zeta_i^1}} \frac{\partial q_{\zeta_i^1}}{\partial \zeta_i^1} = \frac{\partial K_i}{\partial q_i}\Big|_{q_{\zeta_i^1}} \frac{\partial \Phi^{N_{J_i^1}}_{\zeta_i^1}(q_i^0)}{\partial \zeta_i^1}$$

$$= J_i(q_{\zeta_i^1}) N_{J_{i1}}(q_{\zeta_i^1}) = 0 . \tag{78}$$

Hence $q_{\zeta_i^1} \in Q_N^{q_i^0}$ when $q_i^0 \in Q_N^{q_i^0}$. Similarly, we have

$$\frac{\partial x}{\partial \zeta_i^l} = 0 \qquad for \ l = 2,\ldots,r_i . \tag{79}$$

Then for the lth transition, we have, $q_i(t=\zeta_i^1+\ldots+\zeta_i^l) = \Phi^{N_{J_i^l}}_{\zeta_i^l}(q_i(t=\zeta_i^1+\ldots+\zeta_i^{l-1}))$ for $l=1,\ldots,r_i$. Hence $q(t=\zeta_i^1+\ldots+\zeta_i^l) \in Q_N^{q_i^0}$. Moreover these q_i*'s are* connected since $\Phi^{N_{J_i^l}}_{\zeta_i^l}$ for $l=1,\ldots,r_i$. are continuous mapping with respect to ζ_i^l $(l=1,\ldots,r_i)$. Therefore *(77)* maps ζ_i to $q_i(t) \in Q_N^{q_i^0}$. This mapping is a diffeomorphism because it is a composition of the diffeomorphisms $\Phi^{N_{J_i^l}}_{\zeta_i^l}$.

Lemma 5 (Decomposition of the coordinates)

Given a kinematic mapping $x = K_i(q_i)$ $(i=1,\ldots,k)$, and let U_i be the image of the joint space Q_i. At any point $q_i \in Q_i$, there exists a diffeomorphism F_i^{-1}, which decomposes q_i into $\zeta_i \in R^{r_i}$ and $\xi_i \in R^{m_i}$ (here m_i=6 for all $i=1,\ldots,k$), such that $[\begin{smallmatrix}\zeta_i\\ \xi_i\end{smallmatrix}] = F_i^{-1}(q_i)$. The mapping $\zeta_i(q_i)$ maps a point q_i on the corresponding self-motion manifold $Q^{q}_{N_i}$ into ζ_i.

Proof:

We will construct the desired diffeomorphism on the given leaf of the self-motion **manifold.** Recall that N_{J_i} is the orthogonal complement of $J_i^{\dagger}$. The **matrix** J_i is assumed to be full rank and has the right inverse $J_i^{\dagger}$, $J_i^{\dagger} = J_i^T (J_i J_i^T)^{-1}$. 'Then the range space of $J_i^{\dagger}$ and the range space of J_i^T are equal. The domain space of any matrix is the direct-sum of its row space and its null space, hence the domain of J_i is R^{n_i}. Thus we have,

$$rank([N_{J_i}\ ,\ J_i^{\dagger}]) = n_i\ . \tag{80}$$

Consider the composite mapping $F_i : R^{n_i} \rightarrow Q_i$

$$(\zeta_i^1,\ldots,\zeta_i^{r_i},\xi_i^1,\ldots,\xi_i^{m_i}) \rightarrow q_i(t) = \Phi_{\xi_i^{m_i}}^{J_i^{\dagger}} o \ldots o \Phi_{\xi_i^1}^{J_i^{\dagger}} o \Phi_{\zeta_i^{r_i}}^{N_{J_i}} o \ldots o \Phi_{\zeta_i^1}^{N_{J_i}} (q_i^0). \tag{81}$$

The mapping F_i is a diffeoinorphism, since the composition of diffeomorphisms is a diffeomorphism. Hence the inverse of F_i , F_i^{-1}, exists and it is a smooth mapping. Thus,

$$\begin{bmatrix} \zeta_i \\ \xi_i \end{bmatrix} = F_i^{-1}(q_i) \tag{82}$$

where $\zeta_i = (\zeta_i^1,\ldots,\zeta_i^{r_i})^T$ and $\xi_i = (\xi_i^1,\ldots,\xi_i^{m_i})^T$ are real functions.
We have,

$$(\zeta_i\ ,\ \xi_i) = F_i^{-1} \circ F_i(\zeta_i,\xi_i)\ , \tag{83}$$

then the Jacobian matrices F_i^{-1} and F_i should satisfy the following equation,

$$\begin{bmatrix} \dfrac{\partial \zeta_i}{\partial q_i} \\ \dfrac{\partial \xi_i}{\partial q_i} \end{bmatrix} \begin{bmatrix} \dfrac{\partial F_i}{\partial \zeta_i} & \dfrac{\partial F_i}{\partial \xi_i} \end{bmatrix} = I_{n_i} \tag{84}$$

In the next lemma we can find the relationships between the derivatives of (ζ_i,ξ_i) and q_i.

As the **distribution** $\Delta_i = \ker(J_i)$ is involutive, the diffeomorphism F_i has the property, ([4] pp. 27) that for every q_i, the r_i columns of the Jacobian **matrix** $\frac{\partial F_i}{\partial \zeta_i}$ are linearly independent vectors in the distribution Δ_i.

Lemma 6 (The time derivatives of the **transformed** coordinates)

The transformation F_i given in Lemma 5 allows us to write,

$$\dot{\xi}_i = M_{J_i}\ J_i\ \dot{q}_i \tag{85}$$

$$\dot{\zeta}_i = M_{N_i}^{-1}\ N_{J_i}^T\ \dot{q}_i \tag{86}$$

Proof:

We can always find a **nonsingular** $r_i \times r_i$ matrix M_{N_i}, which expresses $\frac{\partial F_i}{\partial \zeta_i}$ as a linear combination of the columns in N_{J_i},

$$\frac{\partial F_i}{\partial \zeta_i} = N_{J_i} M_{N_i} . \tag{87}$$

From (84) we have $\frac{\partial \xi_i}{\partial q_i} \frac{\partial F_i}{\partial \zeta_i} = 0$, thus,

$$\frac{\partial \xi_i}{\partial q_i} N_{J_i} M_{N_i} = 0 \in R^{m_i \times r_i} . \tag{88}$$

Hence N_{J_i} **annihilates** $\frac{\partial \xi_i}{\partial q_i}$. Recall **that** $J_i N_{J_i} = 0$, thus each row of $\frac{\partial \xi_i}{\partial q_i}$ must be a **linear** combination of the rows of J_i. Hence,

$$\frac{\partial \xi_i}{\partial q_i} = M_{J_i} J_i \tag{89}$$

where M_{J_i} is a **nonsingular** $m_i \times m_i$ matrix. Therefore $\xi_i = \frac{\partial \xi_i}{\partial q_i} \dot{q}_i = M_{J_i} J_i \dot{q}_i$ **yields** (85). From (84) we have, $\frac{\partial \xi_i}{\partial q_i} \frac{\partial F_i}{\partial \xi_i} = I_{m_i}$; combining this equation with (89) yields,

$$\frac{\partial F_i}{\partial \xi_i} = J_i^{+} M_{J_i}^{-1} , \tag{90}$$

because the **nonsingular** mamx J_i has a unique pseudo-inverse J_i^{+} such that $J_i J_i^{+} = I$. We can write,

$$\dot{q}_i = \frac{\partial F_i}{\partial \zeta_i} \dot{\zeta}_i + \frac{\partial F_i}{\partial \xi_i} \dot{\xi}_i . \tag{91}$$

Thus we have,

$$\frac{\partial F_i}{\partial \zeta_i} \dot{\zeta}_i = (I_{n_i} - \frac{\partial F_i}{\partial \xi_i} M_{J_i} J_i) \dot{q}_i = (I_{n_i} - J_i^{+} J_i) \dot{q}_i = P_{J_i} \dot{q}_i . \tag{92}$$

To **obtain (86),** we substitute (87) into the above equation and **premultiply** both sides by $N_{J_i}^T$. Notice that $N_{J_i}^T N_{J_i} = I_{r_i}$, since each column of N_{J_i} is a normalized basis vector.

Remark 2

Equation (85) implies that $\dot{\xi}_i = M_{J_i} \dot{x}$ and $\frac{\partial \xi_i}{\partial x} = M_{J_i}$. From the implicit mapping theorem, the non singularity of M_{J_i} ensures that ξ_i is homeomorphic to x.

Lemma 7 (The decomposition of the unperturbed system)

Using the transformation F_i given by lemma 5, we can write the unperturbed system $\dot{q}_i = v_{r_i}(q)$, (v_{r_i} is expressed by (33)) as a cascaded system in the following form,

$$\dot{\zeta}_i = \mu_i M_{N_i}^{-1} (N_{J_i}^T \nabla H_i) (q(\zeta_i, e)), \tag{93}$$

$$\dot{e} + \gamma e = 0. \tag{94}$$

The notation used in (93) means that $N_{J_i}^T$ and ∇H_i are functions of (ζ_i, e) through dependency on the joint variable q_i.

Proof:

The unperturbed system is now given by,

$$\dot{q}_i = J_i^\dagger(\dot{x}_d - \gamma e) + \mu_i P_{J_i} \nabla H_i \qquad i=1,\ldots,k\ . \tag{95}$$

Equation (94) is obtained by premultiplying both sides of (95) by J_i and recalling that $J_i P_{J_i} = 0$. Similarly, equation (93) is obtained by premultiplying both sides of (95) by $M_{N_i}^{-1} N_{J_i}^T$ and recalling that $N_{J_i}^T J_i^\dagger = 0$. Notice that q_i can be decomposed into (ζ_i, ξ_i) by F_i^{-1} given by (82). Also notice that ξ_i is homeomorphic to x. Thus ξ_i is homeomorphic to e because there is a one to one mapping between x and e. Then e is independent of ζ_i, so q_i can be decomposed into (ζ_i, e).

Lemma 8 (The stability of a cascaded system)

Consider the system (93) and (94) in hierarchical form,

$$\dot{\zeta}_i = f_i(\zeta_i, \xi_i) \quad \textit{and,} \qquad \dot{\xi}_i = g_i(\xi_i)\ . \tag{96}$$

If the functions f_i and g_i are continuously differentiable, then $(\zeta_i, \xi_i) = (0,0)$ is a locally asymptotically stable equilibrium of the system, if and only if $\xi_i = 0$ is a locally asymptotically stable equilibrium of $g_i(\xi_i)$ and $\zeta_i = 0$ is a locally asymptotically stable equilibrium of $f_i(\zeta_i, 0)$.

The proof of this lemma can be found in Vidyasagar [9].

The equilibrium point of the cascaded system given in lemma 7 is e = 0, $\zeta_i = \zeta_i^*$ (for i=1,...,k). Here ζ_i^* is the coordinates such that,

$$(N_{J_i}^T \nabla H_i)(q_i(\zeta_i^*, 0)) = 0 \qquad i=1,\ldots,k. \tag{97}$$

The equilibrium joint position q_i^* is then,

$$q_i^* = F_i(\zeta_i^*, 0) \qquad i=1,\ldots,k. \tag{98}$$

Remark 3

Setting e = 0 in (93) gives us the zero-dynamics [4],

$$\dot{\zeta}_i = \mu_i M_{N_i}^{-1} (N_{J_i}^T \ \nabla H_i)\ (q_i(\zeta_i, 0)) \qquad i=1,\ldots,k, \tag{99}$$

of the unperturbed system. The zero dynamics is defined on the manifold R^{r_i}. Equations (86) and (99) lead to,

$$N_{J_i}^T \dot{q}_i = \mu_i (N_{J_i}^T \ \nabla H_i)(q_i(\zeta_i, 0)) \ \ or \ \ P_{J_i} \dot{q}_i = \mu_i (P_{J_i} \ \nabla H_i)(q_i(\zeta_i, 0)) \quad i=1,\ldots,k. \tag{100}$$

Notice that $q_i(\zeta_i, 0) \in Q_{N_i}^0$. Equation (100) is defined on the manifold of $\{q_i = F_i(\zeta_i, \xi_i)$ *such that* $\zeta_i \in R^{r_i}$ and e = 0$\}$. This manifold is also expressed by,

$$Q^q_{N_i} = \{q_i \in Q_i \ \textit{such that} \ x_d = K_i(q_i) \text{ and } J_i\dot{q}_i = 0\} \text{ for } i=1,\ldots,k, \quad (101)$$

and it is **indeed** the self-motion manifold over x_d. We observe that the identity $\dot{q}_i = (J_i^+ J_i + P_{J_i})\dot{q}_i$ is satisfied on any $q_i \in Q_i$. However for motions on the self-motion manifold $\dot{x} = J_i(q_i)\dot{q}_i = 0$, and thus for motions on the self motion manifold we also have $\dot{q}_i = P_{J_i}\dot{q}_i$. Equation (100) can be rewritten as,

$$q_i = \mu_i(P_{J_i} \nabla H_i)(q_i) \quad \textit{for all } q_i \in Q^q_{N_i} . \quad (102)$$

Equation (102) will be **called** the "equivalent zero dynamics" expressed in the joint space and defined on the manifold $Q^q_{N_i}$.

Proposition 4 (**Boundedness** of the unperturbed system)

The equilibrium points q_i^* $(i=1,\ldots,k)$ of the unperturbed system is asymptotically stable if the equilibrium point $(\zeta_i^*,0)$ of the zero-dynamics (102) is asymptotically stable. The trajectory $q_i(t)$ of the unperturbed system **starting** from any finite initial configuration q_i^0 is **bounded** if the solution trajectory of the zero dynamics defined on the self-motion manifold $Q_N^{q_i^0} = \{q_i \in C(q_i^0) \ \textit{such} \text{ that } K_i(q_i) = K_i(q_i^0) = x_0\}$ is **bounded.**

Proof:

Lemma 7 asserts that the **unperturbed** system given by (102)) can be decomposed into a **cascaded** system, then the asymptotic stability results are obtained immediately from Lemma 8.

Proposition 5 (Boundedness is guaranteed with the choice of H_i)

Let the cost function $H_i(q_i)$ be a quadratic of the form :

$$H_i(q_i) = \frac{1}{2}(q_i - q_{C_i})^T M_{h_i}(q_i - q_{C_i}) \qquad i=1,\ldots,k \quad (103)$$

where q_{C_i} $(i=1,\ldots,k)$ is **fixed,** and M_{h_i} is a **symmetric** positive definite mamx. **Further** let q_{C_i} be given in a set of **isolated** points. Consider the zero-dynamics,

$$\dot{q}_i = \mu_i P_{J_i} \nabla H_i(q_i) = \mu_i P_{J_i} M_{h_i}(q_i - q_{C_i}) \ \text{ with } . \ q_i \in Q_{N_i}{}^q . \quad (104)$$

The vector q_i is bounded and $q_i \to q_i^*$ as $t \to \infty$ for every fixed q_{C_i}. Where q_i^* (i=1,...,k) is the optimal solution of the problem given by (27).

Proof:

Let the Lyapunov function candidate V_i be,

$$V_i = \frac{1}{2}(q_i - q_{C_i})^T M_{h_i}(q_i - q_{C_i}) \qquad q_i \in Q^q_{N_i} , \quad (105)$$

The derivative of V_i is,

$$\dot{V}_i = \mu_i(q_i - q_{C_i})^T M_{h_i} P_{J_i} M_{h_i}(q_i - q_{C_i}) = \mu_i \| P_{J_i} M_{h_i}(q_i - q_{C_i}) \|^2 \leq 0, \; for \; \mu_i < 0. \qquad (106)$$

Here the fact that P_{J_i} is a projector was used. Hence $q_i - q_{C_i} \in$ L,,, in addition, because of the boundedness of q_{C_i} we have $q_i \in$ L,. Notice that the set $E_i = \{q_i \mid \dot{V}_i = 0\}$ is the the set of equilibrium points of (108), and is therefore an invariant set. From LaSalle's extension of Lyapunov direct method [5], $q_i(t) \rightarrow q_i^*$ $(i = 1, \ldots, k)$ as t $\rightarrow \infty$ because q_i is in a bounded set.

Remark 4

Thus we see from the last proposition the choice of q_{C_i} and M_{h_i} for i=1,...,k can be used to ensure that point q_i^* is far from singular configurations. Thus ensuring that the robot joints do not go through the singular configuration this was assumed in A3 for the purpose of the development of the control law at the beginning of the paper. We should note the exact value of the joint angles $q_i^* \in Q\mathcal{Y}_i$ for all i=1,...,k can be obtained by simulation of the equation (104).

Remark 5

The quadratic performance function defined in (103) ensures that the function v_{r_i} (i=1,...,k) is locally Lipschitz.

$$v_{r_i} = J_{i^+}(\dot{x}_d - \gamma e) + \mu_i P_{J_i} M_{h_i}(q_i - q_{C_i}) \qquad i = 1, \ldots, k. \qquad (107)$$

The mamces $J_i^\dagger$ and P_{J_i} are differentiable because of assumption (A4). A continuously differentiable function is locally Lipschitz. Also notice that M_{h_i} is a constant mamx. Hence the function given in (107) is differentiable with respect to q_i, and is therefore Lipschitz.

7. CONCLUSION

In this paper, we addressed the problem of controlling redundant multiple robots manipulating a load cooperatively. We proposed an adaptive controller that ensures the exponential tracking of the load position to its desired value and the: convergence of the internal forces to their desired values. The controller also guaranteed that the parameters errors remained bounded, and that the redundancy resolution error was asymptotically stable. Measurements of the joint or load accelerations were not required. The concepts of zero dynamics and stability of perturbed nonlinear dynamical systems were used to prove the stability of the adaptive system, particularly the stability of the joint motions on the self motion manifold. The overall stability of the adaptive system was established for a certain class of optimization functions used for redundancy resolution.

Further work can be done to simplify the control law calculations, as the control law is rather complex. Other possible areas of future developments can address actuator dynamics, the effects of joint flexibility and effects of bounded actuator power or torques. At this stage experimental work should be carried out to verify the effectiveness of the control law proposed in this paper. In such an experiment the workspace trajectory must be selected which is reachable and the actuator power/torque capacities must also be sufficient to ensure the &sired load trajectories are feasible. If such a desired trajectory is found then the collisions between the robots and the singularities may be avoided by an appropriate selection of $H(q)$.

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Robot **Kinematics**

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \hat{q}_1 + \hat{q}_3 \\ \hat{q}_2 \end{pmatrix}.$$

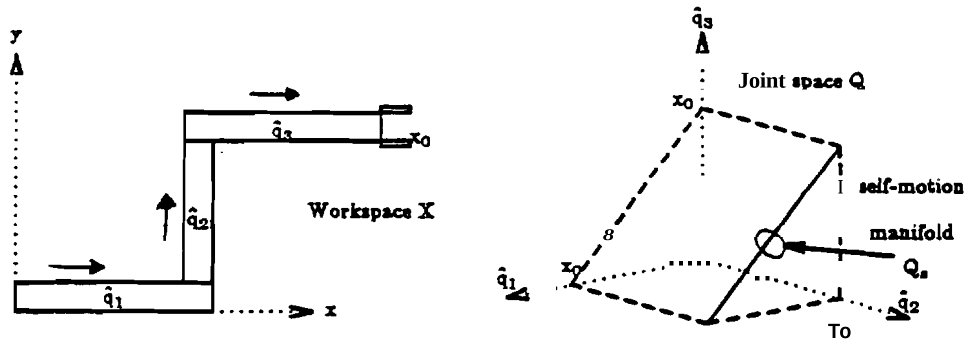

Figure 1. Self motion manifold for a three link prismatic joint PPP robot

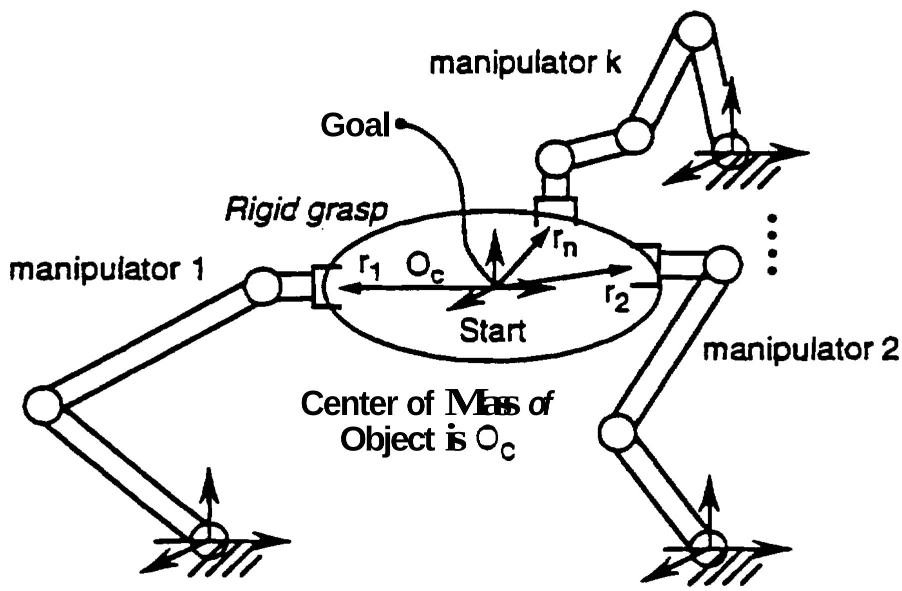

Figure 2 Multirobot system **organization. with desired trajectory**