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SUMMARY

A general parsing technique is presented which is, at worst, as fast as the LR(k) technique, from which is derived and, at best, considerably faster. The higher speed is obtained through reduction of stack traffic.

A practical construction algorithm for any LR(k) grammar (and for all the subclasses, SLR(k) and LALR(k), etc.) is provided, as well as formal proofs and discussion of side effects, the most interesting of which is that the Fast LR(k) machine adjusts itself to the complexity of the grammar it has to parse, reducing, for regular grammars, to a deterministic finite state automaton. This fact allows the practical embodiement of lexical rules into syntactic ones, thus avoiding their artificial separation.

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Entia non sunt multiplicanda
praeter necessitatem.

Occam's Razor

1-INTRODUCTION

The key idea to this paper is that many states that are stacked during LR(k) parsing [4] [1], actually don't need to be.
This is evident when we parse a regular grammar with an LR(k) parser. We know that regular grammars are recognized by finite state automata, yet, using an LR(k) parsing, we use the full strenght of a deterministic pushdown automaton to analyze it.
Which states need to be stacked, then?
Intuitively, each state in which a nonterminal input is allowed must be stacked. This because such a state is a state in which the recognition of a rule p_1 is discontinued, in order to parse a rule p_2, whose left part symbol appears in the right part of p_1. After the recognition of rule p_2, parsing for p_1 must be resumed at the state in which it was discontinued, and we have to know which state that is.

States in which no nonterminal input is allowed appear therefore to be uselessly stacked. In fact, since only terminal inputs are found, we'll never discontinue the recognition of rule p_1 in these states and therefore we don't need the resume address on the stack. This is however not true, without qualification, because the number of states that are to be unstacked upon the recognition of rule p_1 will not generally be unique in this scheme, and our machine would be incorrect.

The reason is that the recognition of rule p_1 may start in different states of the machine and follow therefore different control sequences. As the reader is aware, this happens because the left hand side symbol of rule p_1 appears in the right hand side of other rules, say p_2, p_3, ... Let A be the left hand side symbol of p_1 and p_2: $B \rightarrow \gamma A \omega$. If $FIRST_j(A) = FIRST_j(\omega)$, $1 \leq j < k$, then one of the control sequences for p_1 and one of the control sequences for p_2 will have j states in common. Suppose that in one of these states, say s, there is a nonterminal input caused by rule p_2. This state becomes relevant for the parsing of p_1 in only one of its control sequences. Therefore reductions for rule p_1, with different control sequences must pop out of the stack a different number of states. This machine is then ambiguous because, upon recognition of rule p_1, we don't know how many states are to be unstacked.

We will prove, however, that a class of grammars (FLR(k) grammars) exists, for which we can avoid stacking states in which no nonterminal input is allowed and still have consistent machines.

We will develop a general algorithm for solving FLR(k) conflicts, which can be applied to any LR grammar [3] (SLR(k), LALR(k) and LR(k) grammars). The resulting machine will generally be a modified FLR(k) machine, which we call a Tentative FLR(k) (or TFLR(k)) machine.

Philosophically, our approach will be opposite to the SLR(k), LALR(k) one: while those methods start from a simple machine and modify it in order to increase its power, the FLR(k) method starts from a machine and modifies it in order to reduce it to the minimum one needed to parse its grammar. It is conceivable that 'power increasing' methods could be attempted in constructing FLR(k) machines: this approach has not yet been studied, however.

From our method an interesting formal and practical result arises: for regular grammars in the form $A \rightarrow Bx|y$, $x,y \in VT^*$, FLR(k) machines reduce to deterministic finite state automata. Thus FLR(k) parsers can self adjust to the complexity of the grammar they have to parse. This result allows us to treat regular grammars and a subclass of context-free grammars with a unique formal device and in the same time allows compiler writers to embody lexical into syntactic analysis and be sure that lexical analysis is not slowed down by the cost of an overpowerful parser.

2-CONSTRUCTION OF FLR(k) PARSING MACHINES

We introduce here a class of machines, called FLR(k), closely related to LR(k) machines (we follow the LR(k) machine definition of [3] [8]) and derived from them by inhibiting the stacking of states in which no nonterminal input is allowed. We are following the intuitive (and, as we discussed, in general wrong) idea that the stacking of these states is irrelevant to the progress of the analysis. We shall introduce in the following sections consistency conditions for these machines and the class of grammars that produce consistent FLR(k) machines.

DEF 2.1- An LR(k) state in which no nonterminal input is allowed is called a transfer state.■

DEF 2.2- An LR(k) state in which nonterminal input is allowed is called a shift state.■

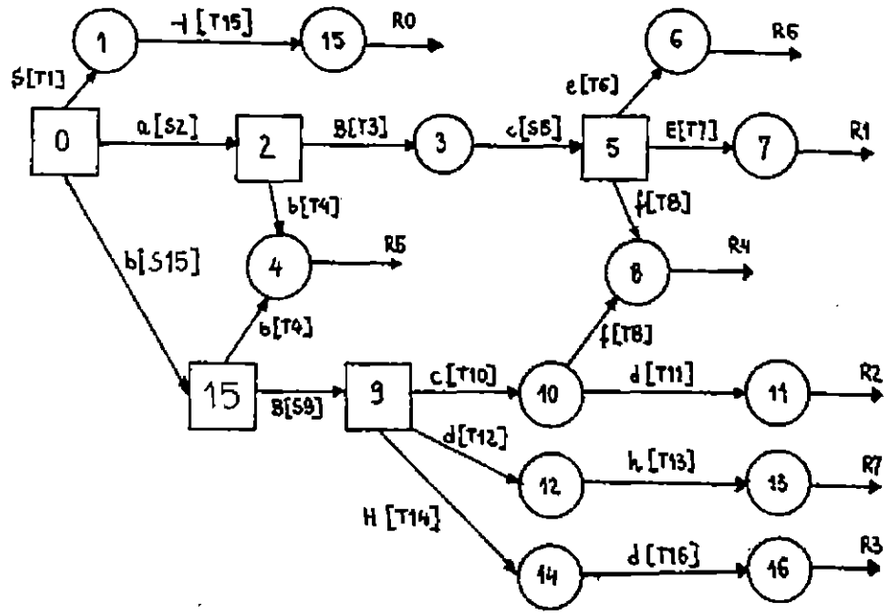
The algorithm to construct the FLR(k) parsing machine for a grammar G is as follows:

ALGOR 2.1- Apply the following steps:
1- Construct the LR(k) parsing machine for grammar G.
2- for each transfer state s_i:
   2.1- every γ-transition from the state set
   $\{ s_j \mid s_j$ is a γ-predecessor of state $s_i \}$
   to a state s_i is labelled with T s_i.■

The action T corresponds to a transfer of control without pushing the current state onto the stack.

EXAMPLE 2.1
Define the grammar $G_1=(VN,VT,P,S)$ as
1- S → a B c E |
2- b B c d |
3- b B H d
4- B → B c f |
5- b
6- E → e
7- H → d h
We have the following FLR(0) parsing machine (shift states are represented as squares, and actions are enclosed in brackets). Remember that, to construct an LR(k) machine, you have to augment the grammar, by adding rule 0 in the form $S' \rightarrow S\ \ddagger^{k+1}$, where $S' \notin V$ is the new

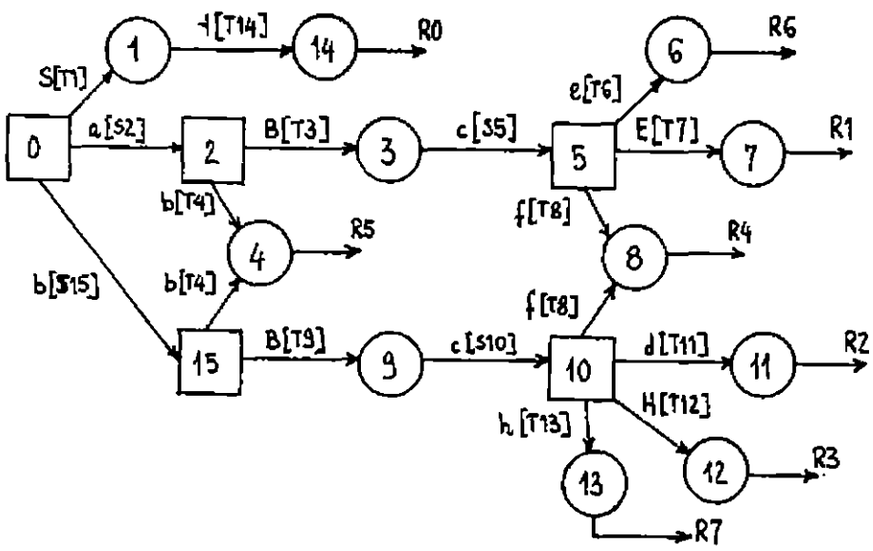
start symbol, and ‡∈VT is the terminator:



EXAMPLE 2.2
Define the grammar G_2=(VN,VT,P,S):

1- S → a B c E |
2- b B c d |
3- b B c H
4- B → B c f |
5- b
6- E → e
7- H → h

We have the following FLR(0) parsing machine:



3-CONDITIONS FOR CONSISTENT FLR(k) PARSING MACHINES

Intuitively, an FLR(k) parser is consistent iff the number of states that must be unstacked is unique for each reduction. We formally define this concept.

DEF 3.1-A state s_i is an immediate path predecessor state of state s_j, on level m, according to rule p (whose right part is n symbol long), which is written

$$s_i = IPPS(s_j, m, p)$$

iff one of the following conditions holds:

1- if $m=n$, then s_j must be a p-reduce state (i.e. a state in which a reduce action for rule p is defined) and there must be a transition from s_i to s_j, involving the reading of a symbol $\gamma \epsilon V$, and γ must be the n-th symbol in the right part of rule p.

2- if $0<m<n$, then there must be a transition from s_i to s_j, involving the reading of a symbol $\gamma \epsilon V$, and γ must be the m-th symbol in the right part of rule p.■

EXAMPLE 3.1- Referring to the machine for grammar G_2:
state 5 is an IPPS(8, 3, 4)
state 3 is an IPPS(5, 2, 4)

DEF 3.2- A reduce path for rule p, RP(p), where rule p has an n symbol long right part, is a sequence of states

$$\{ s_{p0}, \ldots, s_{pn} \}$$

of length n+1, where

- s_{p0} is a p-reduce state
- $s_{p(i+1)} \epsilon \{ s_j = IPPS(s_{pi}, n-i, p) \}$.■

EXAMPLE 3.2- Referring to the machine for grammar G_2:
{ 8, 5, 3, 2 } is an RP(4)

As a machine can have several distinct reduce paths for a rule p, we'll arbitrarily number all the different reduce paths and we write $RP_i(p)$ to mean the i-th reduce path for rule p.

DEF 3.3- The value of an $RP_i(p)$, $VAL_i(p)$, is the number of shift state in $RP_i(p)$ minus 1.■

EXAMPLE 3.3- Referring to the G_2 machine, if $RP_1(4)=\{ 8,5,3,2 \}$, then $VAL_1(4)=1$.

Therefore an FLR(k) machine is consistent if, for each rule p, the value of p is unique. No ambiguity will then rise when we apply a reduce action. In relation to our starting point, LR(k) machines, we note that consistent FLR(k) machines are machines for which the intuitive idea, that states in which no nonterminal input is allowed, need not to be stacked, proves true. These machines show that the LR(k) method is more powerful than required by the generating grammar.

DEF 3.4- An FLR(k) machine is said to have an FLR(k) conflict for rule p, iff $i,j: VAL_i(p) \neq VAL_j(p)$.■

DEF 3.5- An FLR(k) machine for a grammar G is said to be consistent iff

- the LR(k) machine for G has no conflicts and
- the FLR(k) machine for G has no FLR(k) conflicts.■

4-FLR(k) PARSING

We now introduce the FLR(k) parsing algorithm for FLR(k) consistent machines.

ALGOR 4.1- Apply the following steps:
Initialization:
1- current state is state s_0
2- push s_0 onto the pushdown
Running:
3- read a symbol $\gamma \epsilon V$ from the input tape and lookahead a k symbol long string w on VT in order to determine which action is to be taken in the current state
4-if no action is defined in the current state for $\gamma\{w\}$, then:
- output(error)
- halt the parsing
else
apply the action
4- goto 3.

Actions are defined as follows:
1- S s_i (shift s_i) :
i- move ahead the input head by 1 cell
ii- push s_i onto the pushdown
iii- current state is s_i
2- T s_i (transfer s_i):
i- move ahead the input head by 1 cell
ii- current state is s_i
3- R p (reduce p):
i- output(p)
ii- if p=0 then halt the parsing, else
iii- pop VAL(p) items from the stack
iv- current state is the state at the top of the stack
v- force the input of the left part symbol of rule p.■

We note that the differences between FLR(k) and LR(k) parsing are very slight, namely the new action T and the number of items to be unstacked by the reduce action.

EXAMPLE 4.1- Refer to G_1 (example 2.1).
We have
VAL(1),VAL(2),VAL(3)=2
VAL(4)=1
VAL(5),VAL(6),VAL(7)=0
therefore $M(G_1)$ is consistent.

Let's parse the string abcfce with $M(G_1)$:

Current state	Input string	Action	Stack situation before action (stack top is at right)
0	abcfce‡	S2	0
2	bcfce‡	T4	0 2
4	cfce‡	R5	0 2
2	Bcfce‡	T3	0 2
3	cfce‡	S5	0 2
5	fce‡	T8	0 2 5
8	ce‡	R4	0 2 5
2	Bce‡	T3	0 2
3	ce‡	S5	0 2
5	e‡	T6	0 2 5
6	‡	R6	0 2 5
5	E‡	T7	0 2 5
7	‡	R1	0 2 5
0	S‡	T1	0
1	‡	T15	0
15		R0	0

In the LR(0) machine for G_1 there are 15 shift states, while in the FLR(0) machine for the same grammar the number of shift states is 5. Assuming each state is entered with equal probability, on the average the FLR(0) machine should have 1/3 of the stack traffic of the LR(0) one.
We note that in our parsing only 3 states needed to be stacked, while the LR(0) parsing would have required 11 pushes.
Moreover the stack alphabet Γ(FLR) for an FLR(k) machine is a subset of the stack alphabet Γ(LR) for an LR(k) machine. In our example $\Gamma(FLR)=\{0,2,5,8,10\}$.
It's then obvious that an FLR(k) machine has less stack traffic and requires a smaller stack.

5-FLR(k) GRAMMARS

So far, we have examined FLR(k) construction and parsing, but we haven't yet determined which class of grammars produces consistent FLR(k) machines.

We will prove that FLR(k) grammars, whose definition follows, achieve this. The FLR(k) condition is cumbersome and restrictive, but it will be used to develop a tentative construction algorithm for any LR(k) grammar.

DEF 5.1-An LR(k) grammar G=(VN,VT,P,S) is FLR(k) iff at least one of the following conditions holds:

1- no pair of derivations in the form
   $S \Rightarrow^* \delta\ \alpha\ A\ \sigma\ B\ w_1$
   $S \Rightarrow^* \beta\ \alpha\ A\ \sigma\ c\ w_2$
exists, where
$\delta \neq \beta$, $|\alpha|$, $|w_1|$, $|w_2| \geq 0$, $|\sigma| \geq k$
$B, A \in VN$, $c \in VT$
2- there is in P no rule p in the form
   $A \rightarrow A\ \sigma\ \lambda$
such that $FIRST_1(\lambda) \in VT$
3- if $S \Rightarrow^* A\ w_3$ then
   $FIRST_k(\lambda w_3) \neq FIRST_k(B w_1)$
4- there is at least 1 derivation in the form
   $S \Rightarrow^* \beta\ \alpha\ A\ \sigma\ D\ w_4$
such that $FIRST_k(D w_4) = FIRST_k(c w_2)$
5-defining
   $SD(\mu, S)$
as the number of different *-derivations in the form
   $S \Rightarrow^* \mu\ \alpha\ A\ \theta_i\ E_i\ w_1$
where
 $E_i \in VN$
 θ_i varies between ϵ and $\delta\ \lambda$
 $FIRST_k(\sigma\lambda - \theta_i) = FIRST_k(E_i w_1)$
then
   $SD(\delta, S) = SD(\beta, S)$■

FLR(k) grammars are a proper subclass of LR(k) grammars.

DEF 5.2- An FLR(k) grammar is <u>strong</u> iff at least one out of the first four conditions in def 5.1 holds. Otherwise it is <u>weak</u>.■

It is obvious that if one of the conditions ranging from 1 to 4 is true, then condition 5 is true, while the opposite doesn't hold. It should be understood that the function SD is equivalent to the value of a reduce path for rule $A \rightarrow A\sigma\lambda$, and that condition 5 simply states that this value must be unique. Conditions 1 to 4, on the other hand, require that, for each reduce path of that rule, all states with the same sequence number are to be of the same type (i.e. all shift or all transfer states). If it is so, the value of all the reduce paths will also be unique, and, moreover, if we construct all the reduce

paths for that rule in parallel, at any time the value of the incomplete reduce paths will be unique: this is obviously stronger than what needed by condition 5.
Therefore the following relation holds:

STRONG FLR(k)⊂ WEAK FLR(k)≡FLR(k)⊂ LR(k).

In analogy to Strong and Weak FLR(k) grammars, we define Strongly and Weakly consistent FLR(k) machines.

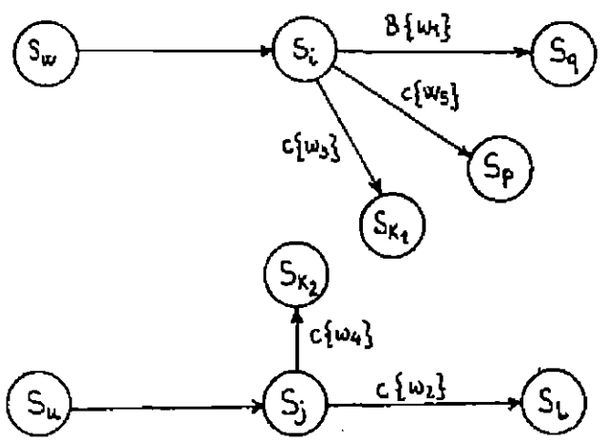
DEF 5.3-A consistent FLR(k) machine is said to be strongly consistent iff
for all i,j and for all m
$s_k \epsilon RP_i(p)$ and $s_l \epsilon RP_j(p)$ at level m
are both shift states or transfer states.
Otherwise it is said to be weakly consistent.■

We now prove that an FLR(k) machine is consistent iff it is generated by an FLR(k) grammar.

THEOREM 5.1-An FLR(k) machine for a grammar G=(VN,VT,P,S) is consistent iff G is an FLR(k) grammar.
Proof:
IF
Suppose G is FLR(k) and the FLR(k) machine is not consistent. Then there will be at least one rule p (whose left part symbol be A) that has at least two different reduce paths $RP_i(p)$ and $RP_j(p)$ such that

$VAL_i(p)=m\neq n=VAL_j(p)$.

Thus there will be at least two states $s_i \epsilon RP_i(p)$ and $s_j \epsilon RP_j(p)$ in the form



where $\{ s_{k1}, s_i \} \epsilon RP_i(p)$
$\{ s_{k2}, s_j \} \epsilon RP_j(p)$
$\{ s_q, s_i \} \epsilon RP(p_1)$
$\{ s_p, s_l \} \epsilon RP(p_2)$

$\{ s_1, s_j \} \in RP(p_3)$,
such that s_i is a shift state, while s_j is a transfer state.
We note that we must have:

$S \Rightarrow^* \delta \alpha A \sigma B w_1$

$S \Rightarrow^* \beta \alpha A \sigma c w_2$

where
$\delta \neq \beta$ (if not $s_u \equiv s_w$, by LR(k) construction)
$|\sigma| \geq k$ (if not $B\{w_1\}$ and $c\{w_3\}$ would be read in different states, by LR(k) construction).
Thus condition 1 doesn't hold.
Condition 2 is not true because there is a rule

$A \rightarrow A\sigma c\lambda_1$

and

$FIRST_1(c\lambda_1) \in VT$.

Since $FIRST_k(cw_3) = FIRST_k(Bw_1)$, condition 3 is not true.
Condition 4 doesn't hold, because we have no nonterminal input in state s_j. Therefore G is not a Strong FLR(k) grammar.
Condition 5 is not true because

$SD(\delta, S) = VAL_i(p)$ and $SD(\beta, S) = VAL_j(p)$

by definition. Therefore G is not an FLR(k) grammar.

ONLY IF
Suppose the FLR(k) machine M for G is consistent, while G is not FLR(k).
Then for all p and for all i,j

$VAL_i(p) = VAL_j(p)$

If none of the conditions ranging from 1 to 4 is true, we have the following situation:

- there are at least two different paths $RP(p_1)$ and $RP(p_3)$ that intersect in a state s_f.
- there are two paths $RP_i(p)$ and $RP_j(p)$ relative to a rule p, in the form $A \rightarrow \sigma\lambda$ such that
  $RP_i(p)$ intersects $RP(p_1)$ starting from state s_i at level m (relative to p)
  $RP_j(p)$ intersects $RP(p_3)$ starting from state s_j at level m (relative to p)
- s_i is a shift state, while s_j is a transfer state.

If condition 5 doesn't hold, then $VAL_i(p) \neq VAL_j(p)$, against the hypothesis. ■

LEMMA 5.1–An FLR(k) machine is strongly consistent iff it is generated by a Strong FLR(k) grammar. ■

EXAMPLE 5.1
Referring to Example 2.1, we note that G_1 is a Weak FLR(0) grammar because only condition 5 of def 5.1 holds. $M(G_1)$ is, as expected, a weakly consistent FLR(0) machine.
Let's note that G_1 is a Strong FLR(1) grammar, because condition 3 of def 5.1 will hold.

EXAMPLE 5.2
Referring to example 2.2, we note that G_2 is a Strong FLR(0) grammar, since condition 4 of def 5.1 is true. $M(G_2)$ is a strongly consistent

FLR(0) machine.

EXAMPLE 5.3

Define grammar $G_3=(VN,VT,P,S)$ as

S → a B c E |
    b B c d
B → B c f |
    b
E → e

G_3 is a Strong FLR(1) grammar, but it is not FLR(0) even if it is LR(0).

6-EQUIVALENCE OF LR(k) AND FLR(k) RELATED AUTOMATA

In this section we shall prove that an FLR(k) machine doesn't stack those and only those states which are useless for further analysis.

DEF 6.1-Two parsing machines M_1 and M_2 are related if
- they are generated by the same grammar and
- they are consistent (conflict-free)

They are equivalent if
- they are related and
- if given the same input string w, either they both accept w or they both reject w at the same symbol in w.

They are strongly equivalent if
- they are equivalent and
- while parsing w, they produce an identical sequence of control transferred states. ■

THEOREM 6.1-Two related machines M and M', where M is FLR(k) and M' is LR(k), generated by the FLR(k) grammar G, are strongly equivalent.

Proof:
Suppose condition 2 doesn't hold.
Then there will be two sequences of control transferred states, while parsing w:

$s_0 \ldots s_k s_i \ldots$ for M'
$s_0 \ldots s_k s_j \ldots$ for M

and $s_i \neq s_j$.
Two possibilities arise:
-transfer from s_k to s_i in M' is achieved by direct transfer (i.e. there is a transition from s_k to s_i, involving the input of a symbol). Then, by construction transfer from s_k to s_i in M is achieved by direct transfer, and $s_i=s_j$, against the hypothesis.
- transfer from s_k to s_i in M' is achieved by indirect transfer (i.e. s_k is a reduce state for a rule p and the sequence is

$s_0 \ldots s_i \ldots s_k s_i \ldots$

and $|s_i \ldots s_k|$ is equal to the length of the right part of p plus 1). By construction, transfer from s_k to s_j in M is achieved by indirect transfer, giving the sequence $s_0 \ldots s_i \ldots s_k s_j \ldots$
Since $s_i \neq s_j$ this means that we unstack more or less than VAL(p) items from the stack. But this is, by construction, impossible.
It is straightforward to see that condition 1 must hold too. ■

Therefore FLR(k) parsers have the same properties of LR(k) parsers:
-they are deterministic
-they parse a string w in time αn, $n=|w|$.
Let's note, however, that the proportionality constant α is smaller for FLR(k) parsers.
-they recognize an error before shifting the symbol which is not consistent with the analysis.

7-TENTATIVE FLR(k) MACHINES.

Since the FLR(k) condition is too restrictive we should develop feasible ways to solve FLR(k) conflicts.

DEF 7.1-A tentative FLR(k) (TFLR(k)) machine is a machine which
- doesn't have any LR(k) conflicts
- does have at least one FLR(k) conflict
- can be made consistent by local modification.■

By local modification we mean modifying the reduce paths for each of the troublesome rules (those, we recall, that don't have a unique value for the reduce path) in order to eliminate the inconsistencies of the machine.
We'll have Strong or Weak TFLR(k) machines, whether local modifications in point 3 induce strong or weak local consistency.

There are two major ways to locally modify an inconsistent FLR(k) machine:
- state splitting
- replacement of transfer states by shift states.

The first method derives from the following theorem, which can be proved considering condition 3 of the FLR(k) condition (def 5.1):

THEOREM 7.1-If a grammar G is LR(k), $k=k_1$, then G is FLR(k), $k=k_2$ and $k_2 \geq k_1$.■

Thus, if we have two states s_i and s_j where
$s_i \epsilon RP_i(p)$ is a shift state
$s_j \epsilon RP_j(p)$ is a transfer state
and VAL(p) is not unique, then we can split state s_i in at least two states s_{i1} and s_{i2}, such that
$s_{i1} \epsilon RP_i(p)$ is a shift state
$s_{i2} \epsilon RP_i(p)$ is a transfer state.
Such a state splitting is achieved by locally increasing the lookahead length.

The other method doesn't increase the lookahead string, but suitably replaces transfer states by shift states until the machine is consistent. This will eventually change the definition of the FLR(k) machine.

DEF 7.2-In a TFLR(k) machine, a shift state s_i is a state all transitions to are labelled S and a transfer state s_j is a state all transitions to are labelled T.■

All the proofs on equivalence with LR(k) parsers we already demonstrated will still hold, if we change the definition of VAL(p) accordingly.

THEOREM 7.2- An LR(k) machine is a conflict-free TFLR(k) machine.

Proof: Obvious, by construction.■

We can now introduce an algorithm that can be applied to any LR(k) grammar. Informally, the proposed algorithm considers each of the troublesome rules and traces all its reduce paths. Then it modifies them in order to have equal values for all of them.

This is obtained by tracing in parallel through all the reduce paths and changing a transfer state s_i in a $RP_i(p)$ to a shift state if there is a $RP_j(p)$ in which state s_j is a shift state and s_j has the same sequence number in the reduce path as s_i.

Obviously in this way we can actually cause other rules to become troublesome, and we'll have to repeatedly scan the rule set in order to determine whether the consistent machine has been constructed or more iterations are required. It should be understood that, while theorem 7.2 proves that the process will halt, it also shows that an LR(k) machine is a possible output (namely the worst case output).

ALGOR 7.1-
Input: an LR(k) grammar G, with P rules and S symbols
Output: a conflict-free TFLR(k) machine for G, with K states and S symbols.
Algorithm:

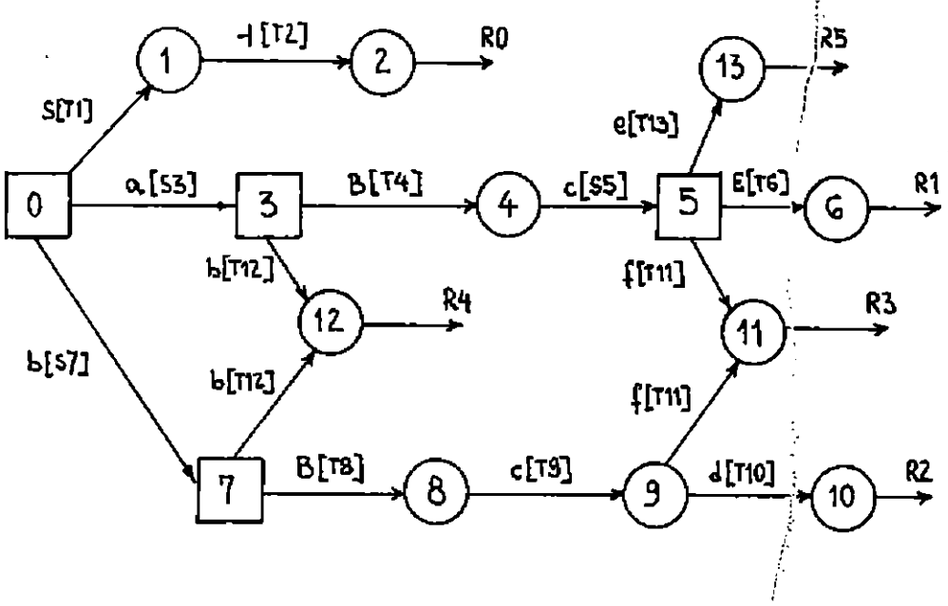
```
BEGIN
     apply algorithm 2.1 ; /* construction */
    WHILE   p, for which VAL(p) is not unique DO
    BEGIN
        FOR p:=1 TO P DO
           IF VAL(p) is not unique THEN
            BEGIN
/* solve conflicts for rule p */
                FOR j:=length of right part of rule p DOWNTO 1 DO
                   FOR i:=2 TO number of different RP(p) DO
/* array element RP[i,j] holds the state number for the i-th
reduce path for rule p, at level j */
                     IF RP[1,j] is a shift state THEN
                       IF RP[i,j] is a transfer state THEN
                         RP[i,j] becomes a shift state
                       ELSE /* is already a shift state: no action
required */
                     ELSE
                       IF RP[i,j] is a shift state THEN
                         RP[1,J] becomes a shift state ;
/* now VAL(i) is unique, but the local modification may have caused
some other VAL(p) to become multivalued */
               END
               FOR p:=1 TO P DO
                   update VAL(p) ;
    END ;
/* now the TFLR(k) machine is conflict-free */
END.  ■
```

The resulting machine will be a Strong TFLR(k), and Strong TFLR(k) machines are a subset of Weak TFLR(k) machines. However, the problem of constructing Weak TFLR(k) machines has not yet been solved and it appears that the amount of computation needed for construction will possibly be so large to be unpractical.

EXAMPLE 7.1

Refer to grammar G_3 (Example 5.3). As we pointed out G_3 is not FLR(0), although it is LR(0). We apply algorithm 7.1 to construct a TFLR(0) machine for G_3.

After step 1, we have the following FLR(0) machine for G_3:



This machine is inconsistent, because rule 3 has

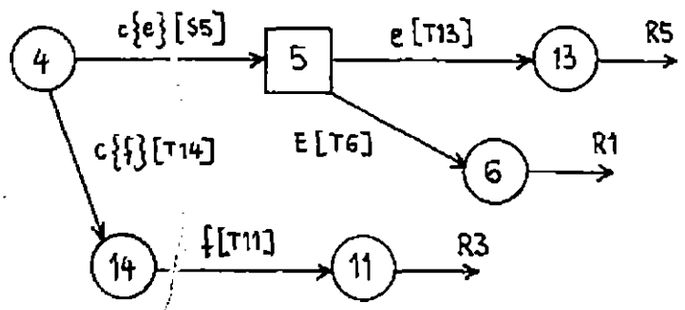
$VAL_1(3)=0$, $RP_1(3)=\{11,9,8,7\}$

$VAL_2(3)=1$, $RP_2(3)=\{11,5,4,3\}$

Applying the remaining steps of the algorithm, state 9 becomes a shift state and the machine is now conflict'free, because every VAL(p) is unique.

If a state splitting technique were used, we would split state 5 into two states, increasing the lookahead for all transitions to state 5. Since G_3 is a Strong FLR(1) grammar, the required lookahead is 1.

The machine would then be locally modified as follows:



Note that, while algorithm 7.1 doesn't increase the number of states of the resulting machine, and therefore appears more appealing, it may, however, produce LR(k) machines as a result. In certain particularly unfavourable cases for algorithm 7.1, a state splitting algorithm may be applied with better results. The decision whether to apply a state splitting algorithm should take into account the cost of lookahead vs. its benefits. Obviously, the rules that produce, after algorithm 7.1, other inconsistencies in the machine, are the natural candidates for state splitting. If, however, these rules happen to be the most used ones, it might be questionable whether the simplification of the machine balances the cost of lookahead in a critical section . The reader should be aware that state splitting and algorithm 7.1 can be used at the same time, for different parts of the machine, and therefore a selection of the troublesome rules is always possible.

8-FLR(k) PARSING FOR REGULAR GRAMMARS

We have seen so far that an FLR(k) machine is an LR(k) machine in which irrelevant stack traffic is avoided. The notion of irrelevant stack traffic obviously depends on the grammar that generates the machine and varies from grammar to grammar according to its complexity. We define informally the complexity of an FLR(k) grammar as the ratio of shift states in its FLR(k) machine minus 1 over the number of states in its LR(k) machine.

In this section we determine the class of grammars that have the minimum complexity and which form their FLR(k) machines have. We will find that left regular grammars have 0-complexity and the FLR(k) machines generated by them are deterministic finite state automata.

THEOREM 8.1-A left regular grammar in the form

$$A \rightarrow B\ x \mid y$$

where x,y are strings on VT, is an FLR(k) grammar.
Proof: An LRG is surely LR(k) and condition 1 of def 5.1 is true.■

We now define a new type of automaton, the Deterministic Pseudo Stack Automaton (DPSA), which may be seen as a degeneration of a DPDA. A DPSA is a DPDA in which, after initialization, the stack is only inspected, but neither popped or pushed: therefore only the stack top (which is assumed to contain the initial state s_0) is meaningful.
It is obvious that a DPSA is equivalent to a DFSA, in which indirect transitions performed to s_0 are made direct.
We'll now prove that LRG's are parsed by FLR(k) DPSA's.

THEOREM 8.2-An FLR(k) machine M for a grammar G is a DPSA iff G is a LRG.
Proof:
IF
By construction, s_0 is the only shift state of M.
ONLY IF
Suppose G is not a LRG and M is a DPSA.
Then G must have at least one rule in one of the following forms:

$$A \rightarrow B\ x\ C\ \gamma$$
$$A \rightarrow y\ C\ \gamma$$

where $\gamma \in V^*$, x,y are strings on VT.
But then there will be at least one state $s_i \neq s_0$ in which a nonterminal (namely C) input is allowed. Then s_i is a shift state and M is a DPSA.□

If we define a left regular component of an LR(k) grammar G=(VN,VT,P,S) as the left regular grammar G_1=(VN',VT',P',A), where

$$A \in VN,\ VN' \subseteq VN,\ VT' \subseteq VT,\ P' \subseteq P$$

and P' is in the form $A \rightarrow B\ x \mid y$, $x,y \in VT^*$,
then there exists a k for which the part of the FLR(k) machine for G, that parses G_1 is a finite automaton.
We will give an intuition of the correctness of this assertion. If $G-G_1$ is empty, then G is regular and, as we proved, the FLR(k) machine is a finite automaton. Otherwise, if the part of the machine that

parses G_1 is not a finite automaton, then there is at least a control path which is shared with the part of the machine parsing $G-G_1$. Therefore, increasing the lookahead will cause the elimination of this sharing. Note that, due to the necessary conditions for this sharing (def 5.1 condition 1), lexical rules in the description of artificial languages are not likely to present any trouble.

From a theoretical point of view, the FLR(k) method can be used as a powerful test on left regular components of Context-free grammars.

We have now proved our claim that FLR(k) machines self adjust to the complexity of the grammar they have to parse. We noted that a DPSA is equivalent to a DFSA: therefore we can use the same machine to practically parse both lexical and syntactic rules of the language.

9-OPTIMIZING TECHNIQUES FOR TFLR(k) PARSERS

We'll refer to the methods discussed in [8]. These are of two classes:

A-automaton structure modification
1-SLR(k) methods [2]
2-LALR(k) methods [6]
3-unit reduction elimination [7]
4-final state elimination [7]

B-automaton representation modification
1-code compaction
2-matrix compaction [9]
3-extended matrix compaction [9]

SLR(k) and LALR(k) are the only parsing methods in he LR family used for practical purposes [3]: for artificial languages, k=1 generally proves sufficient. As we claimed before, our method is applicable to them without any modification. We are not going to formally prove this assertion, but rather give an intuition of its correctness.

Both methods use an increased lookahead length to solve RR and RS conflicts in a given machine, generally an LR(0) one. The machine then becomes an hybrid LR machine, partly LR(k_1) and partly LR(k), where k_1<k. Stack operations however don't vary, the only difference between the machines being the lookahead length. Our method is concerned only with stack operations and it is not affected in any way by the lookahead length: as long as stack operations are performed consistently with the LR(k) definition, algorithm 7.1 will work correctly, as well as state splitting techniques. The same informal proof is easily demonstrated for A.3 and A.4. Obviously we'll have to perform the TFLR(k) construction <u>after</u> the application of any method in class A.

The following inclusion rules hold:

$SFLR(k) \subset LAFLR(k) \subseteq FLR(k)$

$STFLR(k) \subset LATFLR(k) \subset TFLR(k)$.

One remark is to be done on final state elimination. Final state elimination causes the reduction to be performed in a final state to be anticipated, by means of a SR (shift-reduce) action. Therefore, states in which for every nonterminal input A, we have a SR action associated to all $a \in VT$ such that $a=FIRST_1(A)$, can be considered transfer states. Note that, applying this considerations to the machine for G_3, we will produce a consistent FLR(k) machine.

Methods in class B simply modify the resulting automaton representation (i.e. the coding of the transition function) and are therefore to be use <u>after</u> the TFLR(k) construction.

The TFLR(k) method can then be practically used for constructing optimized parsers and, if algorithm 7.1 is used, the number of states for a TFLR(k) and an LR(k) machine generated by the same grammar will be the same and the TFLR(k) parsing speed will be, at worst, as fast

. . . .

as the LR(k) one.

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