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**THE DAMPING OF PANELS BY
THICK LAYERS OF
ELASTIC POROUS MEDIA**

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PROBLEM CONSIDERED

Transverse Displacement of and Sound Power Radiated by Line-Excited Panels Loaded by "Acoustical" Media (normal stress), "Poro-Elastic" Media (normal and shear stresses) or Both

i. Semi-Infinite Fluid:

ii. Layered Structure:

LITERATURE REVIEW

• FLUID-LOADED PLATES

— Morse and Ingard (1968), Nayak (1970), Keltie and Peng (1985), Feit and Liu (1985), Crighton (1972,1977,1979,1983,1988)

— Procedure:

- i. Solve problem in the wavenumber-frequency domain

$$\tilde{V}_f(\gamma, \omega) = 1/(Z_a + Z_p)$$

- ii. Inverse Transform to obtain spatial response

$$\tilde{V}_f(x, \omega) = (1/2\pi) \int_{-\infty}^{\infty} \tilde{V}_f(\gamma, \omega) e^{i\gamma x} d\gamma$$

- iii. Evaluate transform by: contour integration, numerical integration, or combination of the two

PRESENT APPROACH

- **Arbitrarily complicated layered structures:**
 - not amenable to analytical approach
- **Evaluate inversion integral numerically:**
 - along steepest descent path, branch cuts, or real axis?
- **Evaluate integral directly along real axis:**
 - add damping to avoid singularities
 - perform numerical integration by using *Fast Fourier Transform* algorithm to evaluate the *Inverse Discrete Fourier Transform*
- **Solve boundary value problem in wavenumber domain**
 - to determine panel velocity in presence of loading
- **Calculate acoustical response**
- **Result:**
 - efficient method for calculating response of damped structures
 - useful for design exercises, especially treatments involving porous layers

GOVERNING EQUATIONS - ACOUSTIC LOADING

- **Euler-Bernoulli Panel:**

$$D \frac{\partial^4 w(x,t)}{\partial x^4} + m_s \frac{\partial^2 w(x,t)}{\partial t^2} = -p(x,0,t) + f(t)\delta(x)$$

- **Spatial and Temporal Transform:**

$$(D\gamma^4 - \omega^2 m_s) \tilde{W}(\gamma, \omega) = -\tilde{P}(\gamma, 0, \omega) + F$$

or equivalently:

$$\tilde{V}_f = \frac{1}{Z_a + Z_p}$$

$\tilde{V}(\gamma, \omega)$: transformed panel velocity ($-i\omega \tilde{W}(\gamma, \omega)$)

$\tilde{V}_f(\gamma, \omega)$: normalized panel velocity ($\tilde{V}(\gamma, \omega)/F$)

Z_a : input impedance of acoustic space ($\tilde{P}(\gamma, 0, \omega)/\tilde{V}(\gamma, \omega)$)

Z_p : impedance of panel ($i[(D/\omega)\gamma^4 - \omega m_s]$)

DISSIPATIVE MECHANISMS

- **Plate Losses:** Complex flexural stiffness

$$D = D_m(1 - i\eta_p)$$

where: D_m and η are real and $\eta_f = \text{loss factor}$

- **Fluid Losses:** Complex wavenumber

$$k = \frac{\omega}{c_o}(1 + i\eta_a)$$

where: $c_o = \text{speed of sound}$, and $\eta_a = \text{attenuation factor}$

- **Porous Material:** Complex density and speed of sound
 - Use Delany and Bazley formulae based on flow resistivity
 - Current results limited to fiber glass-like materials

GOVERNING EQUATIONS - PORO-ELASTIC LOAD

- **Euler-Bernoulli Panel:**

$$D \frac{\partial^4 w(x,t)}{\partial x^4} + m_s \frac{\partial^2 w(x,t)}{\partial t^2} = -q(x,0,t) + \frac{h_p}{2} \frac{\partial \tau_y}{\partial x} + f(t)\delta(x)$$

- **Spatial and Temporal Transform:**

$$(D\gamma^4 - \omega^2 m_s) \tilde{W}(\gamma, \omega) = -\tilde{Q}(\gamma, 0, \omega) + i \frac{\gamma h_p}{2} \tilde{\tau}_y + F$$

- **Solve Boundary Value Problem**

- Use Biot model for elastic porous material
- Solve for plate velocity or other quantities as required

SOUND RADIATION

- **Given:** $\tilde{V}_{f_2}(k_x, l, \omega)$
- **Calculate:** $\bar{P}(k_x, l, \omega) = Z_{cn2} \tilde{V}_{f_2}(k_x, l, \omega)$
- **Surface Normal Intensity:**

$$I_n(x, \omega) = \frac{1}{2} \text{Re} \left\{ P(x, l, \omega) \tilde{V}_{f_2}^*(x, l, \omega) \right\}$$

- **Sound Power per Unit Width:**

$$\bar{P}(\omega) = \frac{1}{4\pi} \int_{-k}^k Z_{cn2} |\tilde{V}_{f_2}(k_x, \omega)|^2 dk_x$$

- **Also: Line Input Impedance of Panel 1:**

$$Z_{1i}(\omega) = \frac{1}{V_{f_1}}(0, \omega)$$

DISCRETIZATION

- **CONTINUOUS**

$$V_f(x, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{V}_f(\gamma, \omega) e^{i\gamma x} d\gamma$$

- **DISCRETE**

$$\begin{aligned} V_f(m\Delta x, \omega) &= \frac{1}{L\Delta x} \sum_{l=0}^{L-1} \tilde{V}_f(l\Delta\gamma, \omega) e^{i2\pi lm/L} \\ &= \frac{1}{\Delta x} \text{IDFT} \left[\tilde{V}_f(l\Delta\gamma, \omega) \right] \end{aligned}$$

where: $\Delta x = 2\pi/\gamma_s$, γ_s = spatial sampling frequency,
 $\Delta\gamma = \gamma_s/L$ = spatial frequency resolution,
 L = transform length

Note: IDFT can be evaluated using **FAST FOURIER TRANSFORM** algorithm

GUIDELINES FOR NUMERICAL IMPLEMENTATION

i. Spatial Truncation:

spatial response must decrease to
small values

within spatial record length to ensure that
wavenumber spectrum is
sampled sufficiently quickly

ii. Spectral Truncation:

wavenumber spectrum must decrease to
small values

within spectral record length to ensure that
spatial response is
sampled sufficiently quickly

FLUID-LOADED PLATE

- Fiet and Liu (*JASA* 78, 763-766)

- Present Approach

EXAMPLE PARAMETERS

EXAMPLE 1: *two steel panels, panel 1 - 12.7 mm thick, panel 2 - 6.35 mm thick, medium 1 is air or limp porous material, medium 2 is water,*

$$\eta_a = 0.01, L = 4096, \gamma_s = 60 \text{ rad/m}$$

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INPUT IMPEDANCE

• Real Input Impedance

• Imaginary Input Impedance

TRANSVERSE VELOCITY - BASE PLATE

- **Untreated Plate**

- **Air In Channel**

TRANSVERSE VELOCITY - BASE PLATE

- Porous Material In Channel ($\sigma = 2.5 \times 10^3$ MKS Rayls/m)

TRANSVERSE VELOCITY - OUTER PLATE

• Untreated Plate

• Air In Channel

TRANSVERSE VELOCITY - OUTER PLATE

- Porous Material In Channel ($\sigma = 2.5 \times 10^3$ MKS Rayls/m)

SURFACE NORMAL INTENSITY

- **Untreated Plate**

- **Air In Channel**

SURFACE NORMAL INTENSITY

- Porous Material In Channel ($\sigma = 2.5 \times 10^3$ MKS Rayls/m)

- Total Sound Power

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EXAMPLE PARAMETERS

EXAMPLE 2: *steel panel 12.7 mm thick, 6.35 mm thick layer of poro-elastic medium, external medium is water*

$$\eta_a = 0.01, L = 2048, \gamma_s = 300 \text{ rad/m}$$

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TRANSVERSE VELOCITY - BASE PLATE

- Untreated Plate

- Poro-Elastic Material

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SURFACE NORMAL INTENSITY

- Untreated Plate

- Poro-Elastic Material

CONCLUSIONS

- 1. Spatial response of damped panels may be calculated efficiently by using FFT techniques.**
- 2. Any type of layer-wise homogeneous panel loading may be accounted for (including heavy fluids, limp porous materials and elastic porous materials).**
- 3. The technique allows calculation of total power delivered to structure and radiated from the structure as sound.**
- 4. Present approach is useful when conducting design studies to identify effective noise and vibration control treatments.**