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THE SHAW TYPE COMPRESSOR -
A NEW CONCEPT IN COMPRESSOR DESIGN

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SUMMARY

The paper considers the potential of a Shaw-type rotary compressor, in which two drums rotate continuously about inclined axes so that hollow cranked sleeves which connect them can be used as cylinders in which the gas is compressed. It is shown that the machine is in perfect balance, and that the areas available for siting the valves are much larger than in reciprocating compressors. Multi-stage and oil-free delivery versions are shown to be feasible, and there is the possibility of a compressor driven by an internal combustion engine, all contained within one pair of drums, which would provide an exceptionally compact air supply.

INTRODUCTION

For some three years work has been in progress at University College Cardiff on the development of a rotary internal combustion engine of the type described in refs. 1 and 2, which is referred to as the Shaw rotary engine. The present paper describes a compressor which works with essentially the same geometry, and includes a specimen design in some detail, based on experience with the engine development.

A Shaw compressor is a positive displacement machine. Chlumský (ref.3) categorises positive displacement compressors as follows:- piston compressors, membrane compressors and rotary compressors. The piston type of compressor is defined as a compressor in which the gas volume changes due to the action of one or two reciprocating pistons moving axially within a cylinder. This type of compressor is usually used in applications where a relatively high delivery pressure is required (3 bar and above), since leakage is small. The Shaw machine would best fit into this division as its gas compressing volume is bounded by two pistons which have a relative oscillatory motion to their cylinder. It is envisaged that the Shaw machine would be used in applications where multi-cylinder piston type compressors are currently used, as it would have the basic properties of a piston type compressor together with the inherent balance and some of the compactness of the rotary types of compressor. Ref.3 defines a rotary compressor as one that falls into one of the following sub-divisions:- vane type, rotating eccentric type, liquid piston type and double impeller type. These types are usually used in applications where relatively low delivery pressures are required. They are compact, as the lack of reciprocating masses allows them to be run at higher rotational speeds, but their components are subject to higher wear rates. The Shaw compressor shares at least some of these advantages, although it is not a rotary machine in the sense defined by Chlumský.

DESCRIPTION OF THE WORKING PRINCIPLES

A Shaw machine consists of two drums which rotate about axes that are inclined to each other and this is illustrated in fig.1. Each drum has a number of bores that are equi-spaced around a pitch circle diameter: only one set of bores is shown in fig.1. Corresponding bores in each drum are linked with a cranked sleeve, the crank angle of the sleeve being the same as the angle of inclination between the two fixed axes. By fixing pistons to each drum within the bores of the cranked sleeve, an enclosed volume is obtained which varies sinusoidally with angle of drum rotation. Valves are arranged so that the gas would be inducted through the crown of one piston during the expansion part of the cycle and delivered through the other piston crown during the compression part of the cycle.

A set of bevel gears is placed at the centre of the two drums to ensure synchronous drum rotation and to transmit drive through one drum to the other so that each drum transmits an equal driving force to each side of the cranked sleeves. To drive a cranked sleeve through the compression part of the cycle, work must be performed in opposing the net gas pressure imbalance caused by the cranked nature of the sleeve. This gas force has no turning moment about the axes in the maximum and minimum volume positions of the working chamber: this is analogous to the top and bottom dead centre positions on a conventional reciprocating machine where the piston gas forces have no turning moment about the crankshaft axis. For a given gas pressure, the maximum turning moment of the gas imbalance

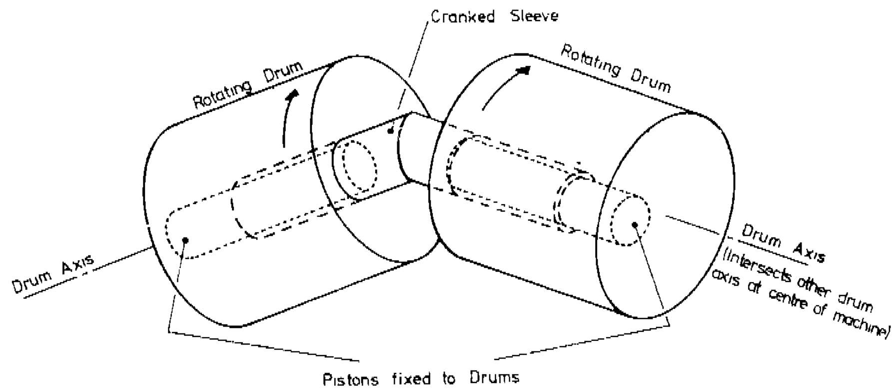

Fig.1 Basic Shaw Machine Components

force on the Shaw machine is 90° away from the maximum and minimum volume positions of the working chamber.

In practice, the two inclined axes on which the drums rotate would be formed by two shafts joined at the centre point of the machine and attached at their outer ends to a rigid frame. This frame could take the form of a structure that encloses all the rotating parts of the machine as is shown in fig.2, which is a demonstration layout of a Shaw type compressor with four cranked sleeves. Further discussion of this layout is given in the section entitled 'Description of a Design for a Shaw Type Compressor'.

MACHINE KINEMATIC EQUATIONS

The motion of the sleeves is considered in detail in the Appendix, where it is shown that a set of any number of sleeves equally spaced round the drums is in perfect balance. Despite the apparent complexity of the motion, there are no out-of-balance components, which is ideal as it means that no balance masses need to be added.

In addition to the inertia force, the following other forces act on a cranked sleeve:- net gas force due to the cranked nature of the sleeve and machine mechanism, friction/drag forces and couples arising from the sleeve to bore bearing reactions,

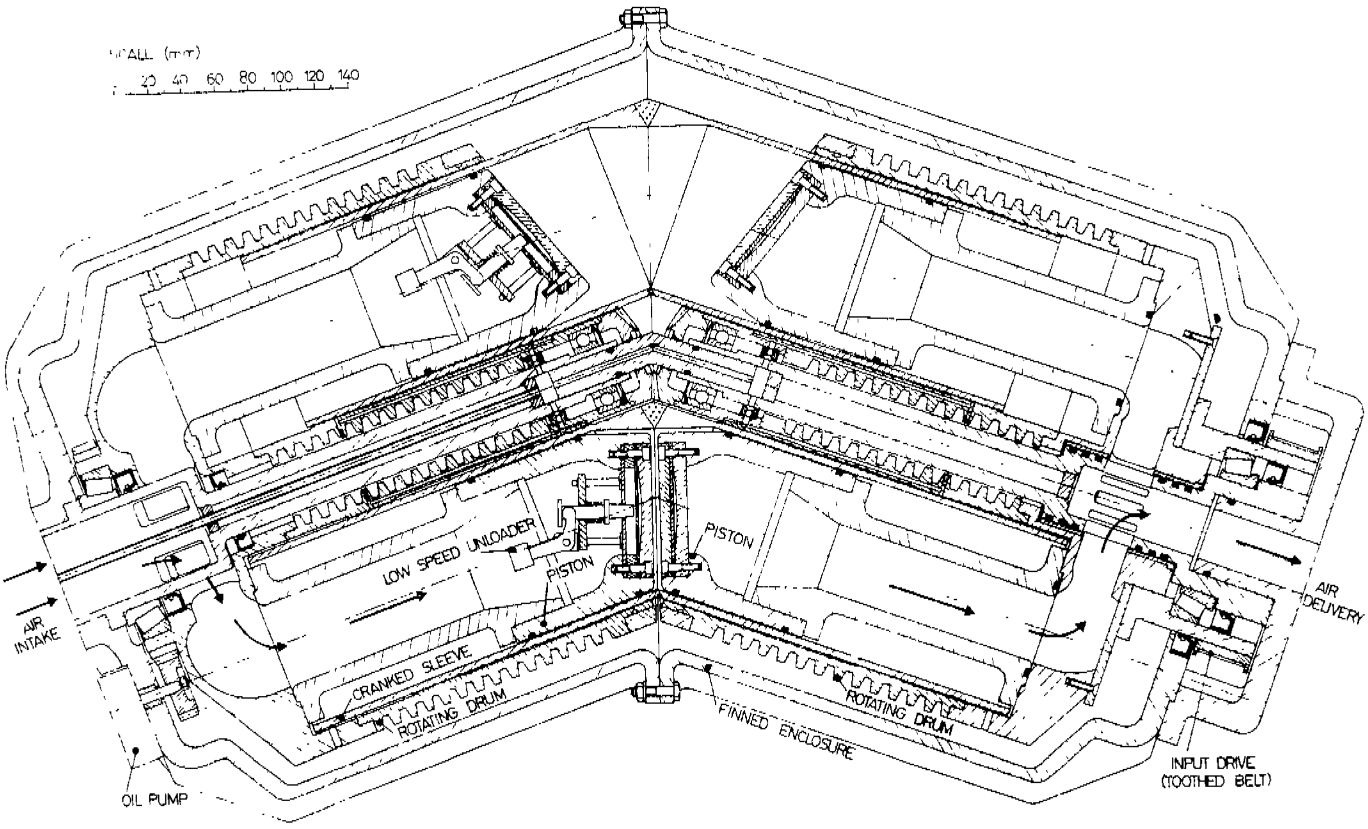

Fig.2 Shaw Type Rotary Compressor - 40° sleeve angle

and the sleeve to bore bearing reactions themselves. In order to balance the bearing friction/drag couples satisfactorily, it is necessary for the bore reactions to have a reasonable moment arm with which to act. This implies that, for a given engine angle and machine design, the length of the sleeve to bore bearing surface must be above a certain value. Not enough is yet understood about the sleeve to bore reactions to make a more specific statement about the minimum value acceptable. The prototype engine under development appears to have a satisfactory engine angle/sleeve length configuration and this configuration is used for the compressor design put forward in fig.2.

DESCRIPTION OF A DESIGN FOR A SHAW TYPE COMPRESSOR

Fig.2 shows a general assembly drawing in plan view of a single stage Shaw type air compressor with four cranked sleeves and a 40° machine angle (machine angle is defined as the angle of inclination between the two machine axes). Although this drawing probably does not represent the optimum layout for this type of compressor, a reasonably detailed design has been done to show that the basic principles can be translated into a practical machine. The components of this compressor were designed so that they could be produced as simple sand castings while all the machining operations could be performed on standard machine shop equipment. These considerations mean that the compressor design would be somewhat different if it had to be produced on production machines with production methods. This should be borne in mind during the following paragraphs where the machine design is first described and then appraised.

It can be seen that the two rotating drums are contained within an enclosure that is formed by two longitudinally finned casings. The similarity between these two casings is such that they could be machined from the same aluminium castings. The function of these casings would be to contain the oil thrown from the rotating drums. The oil would be collected in a sump which is not visible in the plan view of the machine. A high pressure oil pump is shown and this delivers oil to an oil feed pipe that runs down the centre of one of the fixed shafts. The oil is distributed to each rotating drum and thence to the sleeve to bore bearing surfaces. A high capacity low pressure oil pump would also be required and this is not visible in the drawing. This would take its drive from the same end as the high pressure oil pump and it would feed oil onto the rotating elbows of the cranked sleeves as well as to the ball and roller bearing assemblies. Thus, the compressor would be oil cooled and it is intended that an oil cooler would be included in the low pressure oil pump feed line.

Each rotating drum is made up from six components:- four finned liners that form the bores in which the sleeves run, a centre plate onto which the synchronising gear is mounted, and a back plate which contains the air passages that lead the air to and from the piston stems. The drum on the intake side would be identical with the drum on the exhaust side apart from certain obvious differences on the back plates. The finned liners would best be produced in centrifugally cast iron. All eight pistons would be identical apart from the valve assemblies mounted in their crowns. The shape of the piston crowns takes the form of a cone truncated by an inclined sectioning plane. This shape can fill the working chamber completely, apart from allowances for manufacturing tolerances, thus satisfying the requirement for minimal clearance volume which is an essential requirement for an efficient compressor design. The pistons are mounted on stems which in turn are rigidly mounted to the back plates. The pistons would be designed to run clear of the internal cranked sleeve bores: this can be achieved because the only force that acts to deflect the pistons towards the walls is a centrifugal force. Calculations show that a piston side deflection of the order of only 0.005 mm is caused by the centrifugal force at 1500 rev/min. The piston rings would be conventional in design, although they would have to be pegged because of the relative rotation between the piston and sleeve surfaces. The valves shown in the crowns of the pistons are conventional automatic strip type valves. A device to unload the compressor during starting is shown on the intake valve assemblies. This device is spring loaded to open the intake valves when the compressor is stationary and at the low rotational speeds, but as the compressor speeds up the valves are allowed to operate normally.

The cranked sleeves would be fabricated from two steel tube sections welded at the elbow. The fabrication would be stress relieved after welding and, with the help of a special fixture, the inside and outside diameters would be finish turned on a lathe. This method of construction has proved entirely satisfactory for the cranked sleeves used on the prototype Shaw internal combustion engine. On the compressor version, there is no requirement for an accurate finish on the external part of the sleeve elbow: this part is the most difficult to machine as it requires a specially adapted lathe for turning. The basic sleeve material would be a mild steel for machinability and ease of welding, while a chrome plating could be added to give the necessary wear properties.

Two rotary lip seals are included to seal the transfer of air from the fixed shaft to the rotating drum. These seals prevent the oil mist present within the enclosure from contaminating the intake air. The transfer of high pressure air from the rotating drum to the fixed shaft on the delivery side is sealed by 2 sets of 3 externally sprung rings running against a liner pressed into the drum back plate. Alternatively, a type of stuffing box arrangement could have been used, but this could increase the mechanical losses.

Access to the valves could present a substantial practical problem in the design shown in fig.2. This could be improved by use of large-diameter face seals in place of the lip seals at the outer ends of the shafts, and redesign of the piston mounting so that pistons may be withdrawn, complete with valve assemblies, through access holes in the drum backplates. The corresponding problem of changing sparking plugs in the engine version has, in fact, been solved in this way.

The compressor would have a 360 m^3/hr piston displacement at an operating speed of 1500 rev/min. This operating speed is typical of contemporary medium size piston type positive displacement compressors. The Shaw machine would require approximately 18 kW input shaft power for an air delivery pressure of 3.5 bar.

APPRAISAL OF SHAW TYPE COMPRESSORS

For a new type of compressor to be successful in a given application, it must show some significant advantages over existing types of machinery without introducing any significant disadvantages. With this statement in mind, an appraisal is made of Shaw type compressors with particular reference to the design described in the preceding section.

Piston speed, valve area and inertia forces

These are three fundamental design considerations by which the performance of a piston type positive displacement gas compressing machine can be judged.

In recent years there has been a tendency to reduce the stroke/bore ratio on reciprocating compressor designs from ratios of greater than 1.0 to, typically, 0.85. One of the reasons for this has been to obtain a reduction in the mean piston speed as long service life is a very important requirement for a compressor. Mean piston speed would not have the same significance on a Shaw type machine as, for one reason, there is no piston side thrust. The total stroke/bore ratio of the 4×10^{-3} m^3 Shaw machine is 1.2, but as there are two pistons the individual ratio for each piston is 0.6. This implies a low mean piston speed, although the situation is complicated by the additional rotary motion between the piston and the sleeve surfaces.

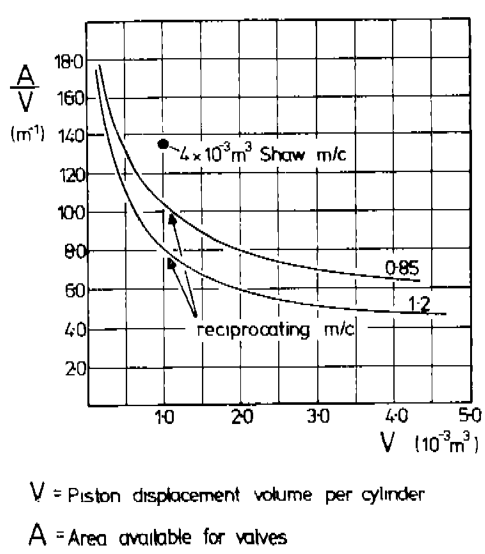

Fig.3 Available valve area - Shaw machine versus conventional machine

Fig.3 shows plots of the ratio of (area available for valves)/(piston displacement volume per cylinder) versus piston displacement volume per cylinder for conventional reciprocating compressors with stroke/bore ratios of 0.85 and 1.2, the area available for valves being optimistically taken as the area of the cylinder bore. The figure of the available valve area for the 4×10^{-3} m^3 Shaw compressor is also plotted - this figure being based on twice the area of the cylinder bore (as the area of both piston crowns is available for valves) less a 15% allowance for piston skirt thickness. There is clearly considerable scope for larger valve areas, and hence smaller pressure losses on the Shaw machine. In practice the area occupied by valves in the conventional machine would rarely exceed 60% of the value assumed in fig.3, so the comparison would in practice be even more favourable to the Shaw machine.

No direct inertia loading comparisons can be made between the Shaw and conventional reciprocating compressors because the mechanisms and components involved are completely different. A measure of the inertia loading on the 4×10^{-3} m^3 Shaw machine can be obtained by substituting values into the kinematic equations derived in the Appendix to this report. This shows that the maximum acceleration of the cranked sleeve would be 2.2×10^3 m/s^2, resulting in an inertia force of 5.56 kN assuming a sleeve mass of 2.5 kg. The maximum force resulting from the gas loading on the sleeve that has to be reacted by the liner surfaces is 1.7 kN for a working chamber gas pressure of 3.5 bar, and this will be ignored in the subsequent reasoning. If it is assumed that the maximum sleeve inertia loading is taken uniformly over the projected liner surface that is common to the sleeve, the resulting maximum bearing pressure is 450 kN/m^2. For comparison, the maximum connecting rod bearing pressure on a current automobile engine is quoted as 29,775 kN/m^2 in ref.4. Admittedly, the sleeve to liner motion on the Shaw machine is complicated by the addition of an oscillatory motion superimposed on to the rotary motion, but the degree of difference between the two bearing pressure values suggests that there should be no difficulty on this score.

Machine balance and speed fluctuation characteristic

The section of this paper on machine kinematics has shown that the Shaw type compressor is in complete dynamic balance providing that it has two or more equi-spaced cranked sleeves. This is in contrast with reciprocating compressors with in-line cylinders where six is the least number of cylinders with which complete dynamic balance can be obtained.

Reciprocating machines usually require a flywheel to minimise speed fluctuations due to differences in driving torque requirements during a machine revolution. A flywheel would be quite superfluous on a Shaw machine as approximately 75% of the engine mass is rotating.

Overall machine dimensions

The Shaw machine detailed in fig.2 has overall casing dimensions of 800 x 475 x 425 mm. The unit overall dimensions would rise to 880 x 625 x 425 mm with the addition of provision for oil cooling and air filtering. For comparison, a commercially available single stage reciprocating compressor with a similar piston displacement and a 3.5 bar air delivery pressure has overall dimensions of 625 x 865 x 1095 mm (length x width x height) without motor. Thus, the 4 x 10^{-3} m^3 design of Shaw machine shows some advantage in overall volume occupied although it would depend on the installation as to whether this could be used to advantage. The noise and vibration produced in existing types of large, rather slow, reciprocating compressors are a particular nuisance when they are installed in buildings below offices, hospital wards and the like. The cost of suitable foundations and acoustic treatment can become substantial. The Shaw compressor would require only slight foundation and should be intrinsically much quieter.

As has been previously stated, the 4 x 10^{-3} m^3 Shaw machine design represents a demonstration layout where simplicity of manufacture has been an important consideration. A gain of 25% in piston displacement volume could be obtained by incorporating five instead of four sleeves together with making certain modifications to the air intake and delivery passages. Further increases in piston displacement for a machine of given overall dimensions could be obtained from careful optimisation of the following factors:- engine angle, pitch circle diameter of the bores, bore size, sleeve length and number of sleeves. A case could be made for obtaining still greater output by adopting a relatively high machine rotational speed. The case would be based on the inherent balance and the large area available for valves on this type of machine.

Mechanical efficiency and sealing losses

Several promising ideas for new types of compressor have failed because either it has been impossible to design a satisfactory sealing arrangement or the machine's internal losses have been excessive. One of the advantages of the Shaw type machine is that conventional piston rings are required to seal the working chamber and conventional rotating shaft seals are used to seal the transfer of gas from the rotating drums, so that no new seal technology need be evolved. The leakages from the working chamber of the Shaw machine would be greater because it is sealed by two pistons instead of one. However, consideration could be given to partially pressurizing the regions behind the pistons as the percentage change in volume of these regions during a machine revolution is relatively low. Leakage losses would also occur in the transfer of gases from the rotating drum to the fixed shaft.

The mechanical losses in a Shaw machine would be made up from the following components:- piston and piston ring losses, losses between the liner to sleeve surfaces, bearing and seal friction losses between the rotating drums and the fixed shafts, pumping losses transferring gas to and from the working chamber, windage losses and ancillary losses such as oil pump, cooling fan, etc. These loss components are compared with equivalent reciprocating machine losses in the following table:-

Loss Component	Shaw machine versus equivalent reciprocating machine
Piston and piston rings	There would be no piston losses on a Shaw machine. This contrasts with the piston side thrust losses present on the connecting rod/crankshaft mechanism of a reciprocating machine. Ref.5 shows that the piston side thrust losses are usually the greatest single factor in a loss table. The piston ring losses would be theoretically greater on a Shaw machine because of the extra distance travelled by the rings relative to the sleeves due to the extra rotary motion.
Losses between the liner to sleeve surfaces	The magnitude of these losses depends on the lubrication regime that exists between these two surfaces. If full film lubrication can be established, this component of the losses would be relatively small. If boundary lubrication exists, this loss would be significant.
Rotating drum bearing and seal friction losses	These losses should be about equal to the crankshaft bearing and seal losses on an equivalent reciprocating machine.
Pumping losses	Little difference between the two types of machine.
Ancillary losses	Little difference between the two types of machine.
Windage losses	Greater on the Shaw machine, but not a significant loss at 1500 rev/min rotational speed.

This table shows that the Shaw machine gains because it has no piston side thrust losses, but loses because of extra piston ring losses and sleeve bearing losses. It would be too speculative to estimate figures for these losses, as they can only really come from actual machine measurements.

Manufacturing Cost

The Shaw machine can only start to show its advantages of balance and compactness in versions using three or more cranked sleeves. As the most important single factor in the manufacturing cost of a conventional compressor is the number of cylinders, it can be deduced that Shaw machines would most likely be competitive in applications where conventional compressors with three and more cylinders are currently used. This would tend to be in the medium (200 m^3/hr piston displacement and above) and large compressor field. A Shaw machine could be manufactured from less raw materials than an equivalent reciprocating machine because of its compactness and moderately stressed components. An earlier section of this paper has demonstrated that no sophisticated machining techniques are necessary to produce this type of machine. These considerations lead to a claim for a low manufacturing cost for the Shaw machine.

Maintenance

Providing that the initial design is properly done, very little maintenance other than oil changes should be necessary. The large available valve area should allow room for valve components with low stress and long life. If frequent valve servicing is anticipated, there is the possibility of design for the removal of the valve by withdrawal through the hollow piston stem and access holes in the casings, so that complete disassembly would not be required, as indicated in the section describing a design for a Shaw type compressor. Piston ring changes as well as bearing and seal maintenance would provide no particular difficulties to a maintenance engineer.

Oil-free delivery, multi-stage and self contained unit versions

An oil-free delivery version of the Shaw compressor can be obtained relatively easily as there is no side thrust between the piston and the cranked sleeve. Oil exclusion seals would be fitted between the piston stems and the internal bores of the cranked sleeves: this is shown schematically in fig.4. These seals would exclude oil from the central part of the cranked sleeves and hence the working chambers. PTFE-coated or carbon piston rings would be required.

Multi-stage versions of the Shaw machine would be feasible, the second stage cranked sleeves naturally having smaller bores than the first stage cranked sleeves. Versions of the machine with four sleeves would have each first stage sleeve diametrically opposite a second stage sleeve and would require the following conditions to be met for dynamic balance:- the machine drums to be in balance, the sleeves diametrically opposite each other to have the same mass, same centre of gravity position and be the same distance from the drum axis. Two-stage versions with six cranked sleeves would allow differences in mass and centre of gravity position between first and second stage sleeves as they could be considered to be made of two sets of three sleeves: providing each set of sleeves was equi-spaced on the same pitch circle diameter, dynamic balance would be achieved. Six cranked sleeve versions would be preferable as these would give symmetrical intake and delivery characteristics, whereas these would not be possible with four sleeve versions.

Mixed compressor and internal combustion engine versions are also worth consideration as these could be designed to provide very compact compressed air supplies. They would best take the form of four sleeve machines with one set of diametrically opposite sleeves acting as a compressor, while the other set would provide the driving power. The engine could be designed to operate on either a spark ignition or a compression ignition two-stroke cycle. The inlet mixture and exhaust gas flows would be timed by the relative motion between ports positioned in the sleeves and liners, so no extra mechanisms would be necessary.

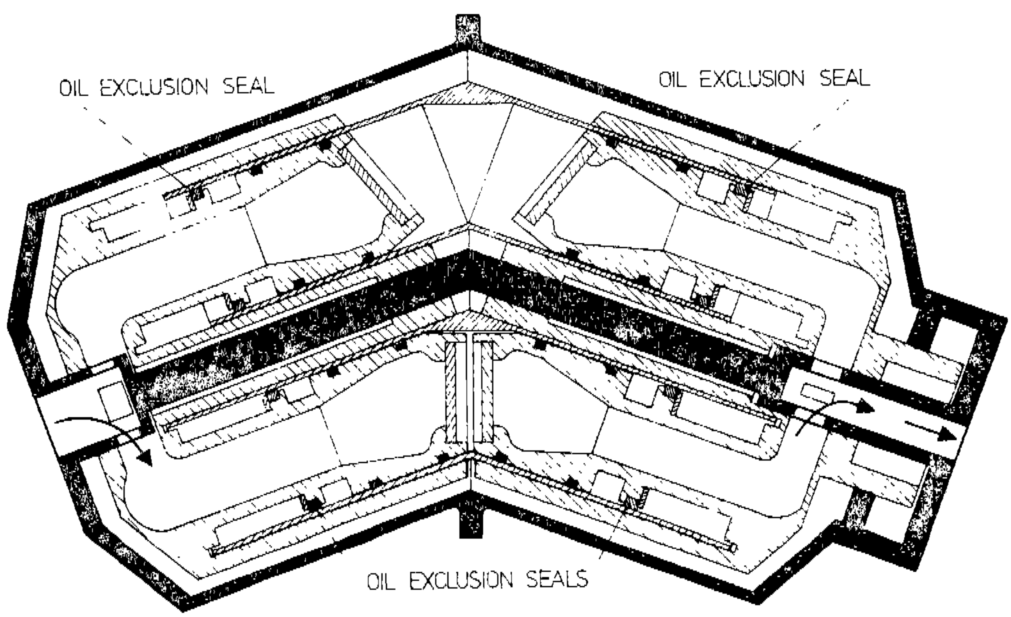

Fig.4 Oil free delivery version of the Shaw machine

CONCLUSIONS

A new design of compressor can be put forward based on the Shaw machine principle. This design of compressor would satisfy the fundamental requirement of having a minimal clearance volume in the working chamber minimum volume position. The working chambers would be sealed by conventional piston rings, and the machine could be manufactured using conventional machine shop equipment. Versions of this machine with two or more "cylinders" would be in complete dynamic balance. A Shaw machine of a given piston displacement could be designed to be considerably more compact than an equivalent reciprocating machine. The Shaw machine compares favourably with existing machines when such factors as mean piston speed and available valve area are considered. The leakage losses could be greater on a Shaw machine and the frictional losses appear comparable with reciprocating machines. The Shaw machine would be competitive in applications where multi-cylinder piston type positive displacement machines are currently used. Multi-stage and oil free delivery versions of the Shaw machine would be quite feasible. A mixed compressor/i.c. engine version could be designed and this would be an exceptionally compact compressed air supply.

ACKNOWLEDGEMENT

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APPENDIX

Nomenclature

R = Pitch circle radius of machine drum (i.e. distance between drum and sleeve axes)

m = mass of sleeve

L = distance between sleeve centre of gravity and sleeve geometric centre

α = half engine angle

θ = angle that sleeve has turned through from its minimum volume position

N = number of sleeves

ω = angular drum velocity

$\dot{\omega}$ = angular drum acceleration

x,y = coordinates that describe position of sleeve centre of gravity

$\dot{x},\dot{y}$ = velocity components of sleeve centre of gravity

$\ddot{x},\ddot{y}$ = acceleration components of sleeve centre of gravity

X,Y = defined in figure

T = time

In this appendix we investigate the kinematics and dynamics of the sleeves. The first step is to establish equations for the position (x,y) and velocity $(\dot{x},\dot{y})$ of the centre of gravity of a sleeve referred to rectangular axes in the plane of symmetry of the machine.

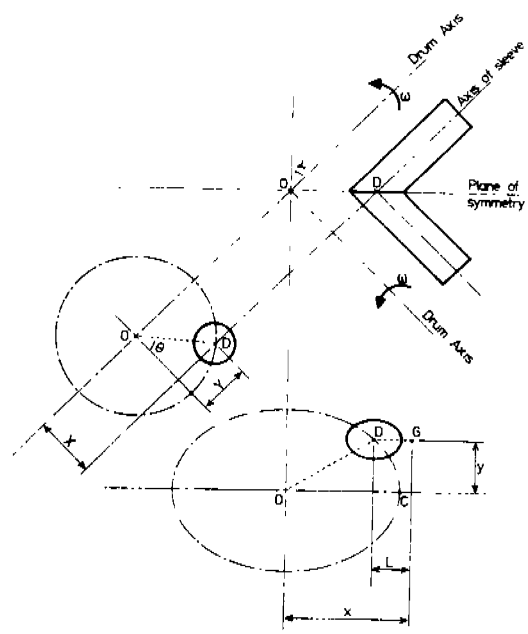

Fig. A.1

In fig.A.1 the essential features of the geometry of the path of the sleeve are indicated as it rotates at constant angular velocity ω about the axis of the drum, its own axis remaining parallel to that of the drum. In the plane normal to the drum axes, the coordinates (X,Y) of the central point D of the sleeve are

$$X = R\cos\theta$$
$$Y = R\sin\theta$$

referred to a centre at the drum axis. In the plane of symmetry, the centre of gravity G is seen to be at the point

$$x = X\sec\alpha + L$$
$$y = Y$$

and substitution for X and Y gives

$$\left.\begin{aligned} x &= R\cos\theta\sec\alpha + L \\ y &= R\sin\theta \end{aligned}\right\} \quad \ldots\ldots (1)$$

The velocity components $(\dot{x},\dot{y})$ follow immediately by differentiation as

$$\left.\begin{aligned} \dot{x} &= -R\omega\sin\theta\sec\alpha \\ \dot{y} &= R\omega\cos\theta \end{aligned}\right\} \quad \ldots\ldots (2)$$

and the acceleration components $(\ddot{x},\ddot{y})$ as

$$\left.\begin{aligned} \ddot{x} &= -R\omega^2\cos\theta\sec\alpha - R\dot{\omega}\sin\theta\sec\alpha \\ \ddot{y} &= -R\omega^2\sin\theta + R\dot{\omega}\cos\theta \end{aligned}\right\} \quad \ldots\ldots (3)$$

The moment of momentum may now be calculated for N sleeves in rotation about O.

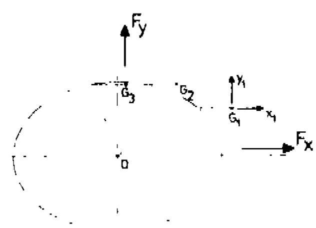

Fig. A.2 Moment of Momentum

$$\text{Moment of Momentum about O} = m(\dot{x}_1y_1 + \dot{x}_2y_2 + \cdots\dot{x}_Ny_N) - m(\dot{y}_1x_1 + \dot{y}_2x_2 + \cdots\dot{y}_Nx_N)$$

$$= -m\omega R^2\sec\alpha\sin^2\theta - m\omega R^2\sec\alpha\sin^2(\theta+\tfrac{2\pi}{N}) - m\omega R^2\sec\alpha\sin^2(\theta+\tfrac{4\pi}{N}) \cdots \text{(N terms)}$$

$$- mR\omega\cos\theta(R\cos\theta\sec\alpha + L) - mR\omega\cos(\theta+\tfrac{2\pi}{N})(R\cos(\theta+\tfrac{2\pi}{N})\sec\alpha + L)$$

$$- mR\omega\cos(\theta+\tfrac{4\pi}{N})(R\cos(\theta+\tfrac{4\pi}{N})\sec\alpha + L) + \cdots\cdots \text{(N terms)} \quad \ldots\ldots (4)$$

This result may be simplified by use of the following standard expressions

$$\sin^2 A + \sin^2(A+B) + \sin^2(A+2B) + \ldots N\text{ terms} = \frac{N}{2} - \frac{\cos(2A + (N-1)B)\sin BN}{2\sin B}$$

$$\cos^2 A + \cos^2(A+B) + \cos^2(A+2B) + \ldots N\text{ terms} = \frac{N}{2} + \frac{\cos(2A + (N-1)B)\sin BN}{2\sin B}$$

$$\cos A + \cos(A+B) + \cos(A+2B) + \ldots N\text{ terms} = \frac{\cos(A + \frac{1}{2}(N-1)B)\sin\frac{1}{2}BN}{\sin\frac{1}{2}B}$$

and the resulting expression is, after some reduction,

$$\text{Moment of momentum} = -m\omega R^2 N\sec\alpha \quad \ldots\ldots (5)$$

apart from when N = 1 or 2, as the trigonometric formulae break down in these cases,

When $N = 1$ Moment of momentum $= - m R \omega (L \cos \theta + R \sec \alpha)$

When $N = 2$ Moment of momentum $= - 2 m \omega R^2 \sec \alpha$

We see, therefore, that except for N = 1 the moment of momentum is independent of θ. There are thus no inertia forces acting to accelerate or decelerate the mechanism which will rotate at constant speed in the absence of friction or other applied moments.

We now calculate the inertia forces at O due to an assembly of N equi-spaced sleeves rotating at constant angular velocity. Referring to Fig.A.2, let F_x and F_y denote force components through O,

$$F_x = m(\ddot{x}_1 + \ddot{x}_2 + \ldots + \ddot{x}_N)$$

Noting in equation (3) that $\dot{\omega} = 0$, we find

$$F_x = m(-R\omega^2 \sec \alpha \cos \theta - R\omega^2 \sec \alpha \cos (\theta+\tfrac{2\pi}{N}) - R\omega^2 \sec \alpha \cos (\theta+\tfrac{4\pi}{N}) \ldots\ldots) \quad \text{(N terms)}$$

$$= -m R\omega^2 \sec \alpha \left[\cos \theta + \cos (\theta+\tfrac{2\pi}{N}) + \cos (\theta+\tfrac{4\pi}{N}) + \ldots\ldots \right]$$

Using one of the previously quoted trigonometrical results, it follows that

$$F_x = 0 \quad \text{apart from when } N = 1 \qquad \ldots\ldots (6)$$

Also

$$F_y = m(\ddot{y}_1 + \ddot{y}_2 + \ldots\ldots \ddot{y}_N)$$

$$= m(-R\omega^2 \sin \theta - R\omega^2 \sin (\theta + \tfrac{2\pi}{N}) - R\omega^2 \sin (\theta+\tfrac{4\pi}{N}) \ldots\ldots) \quad \text{(N terms)}$$

$$= -Rm\omega^2 (\sin \theta + \sin (\theta+\tfrac{2\pi}{N}) + \sin (\theta+\tfrac{4\pi}{N}) + \ldots\ldots) \quad \text{(N terms)}$$

Using the following trigonometrical formula:

$$\underset{\text{(N terms)}}{\sin A + \sin (A+B) + \sin (A+2B) + \ldots} = \frac{\sin (A + \frac{1}{2}(N-1)B) \sin \frac{1}{2} BN}{\sin \frac{1}{2} B}$$

it follows also that

$$F_y = 0 \quad \text{apart from when } N = 1 \qquad \ldots\ldots (7)$$

Thus, apart from N = 1, both F_x and F_y are zero and the machine is in perfect balance.