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ABSTRACT

This paper describes the "Complex" method of optimisation applied to a simulation model of a compressor. Criteria which may be optimised and constraints to be imposed on design parameters are discussed. The numerical results obtained during a search to optimise valve design for a particular compressor are presented.

INTRODUCTION

In recent years mathematical models which describe compressor behaviour have been developed and the extent of their validity assessed. Such models can now be used to predict the performance of a particular compressor design. However, the problem remains of selecting values of the many parameters involved so that the "best" design is achieved. To this end the designer must decide the criteria to be optimised and the constraints to be applied. The problem, in general terms, is to determine the maximum of a nonlinear, multivariable function (the objective function) $F(x_1, x_2 \ldots, x_N)$, subject to non-linear inequality constraints:

$$G_i \leq x_i \leq H_i, \quad i = 1, 2, \ldots, M$$

where G_i (lower constraints) $= g_i(x_1, x_2, \ldots, x_N)$

H_i (upper constraints) $= h_i(x_1, x_2, \ldots, x_N)$

The implicit variables $x_{N+1}, \ldots, x_M$, which may not be required, are dependent functions of the explicit independent variables $x_1, x_2, \ldots, x_N$.

The choice of optimisation method depends upon the nature of the function $F(x_1, x_2, \ldots, x_N)$, the availability of the partial derivatives $\partial F/\partial x_1$, $\partial F/\partial x_2, \ldots, \partial F/\partial x_N$ and the type of constraints encountered. In the present problem partial derivatives are not available and the function $F(x_1, x_2, \ldots, x_N)$ is so complicated that an exact expression cannot be found for it. This precludes analytical methods and gradient techniques.

There remain the non gradient direct search "hill-climbing" methods. These are widely applicable, are easy to use, and have received much attention during recent years. Perhaps the most promising is the Complex method, which can readily handle inequality constraints.

THE COMPLEX METHOD

The N independent variables are regarded as the coordinate directions in an N-dimensional vector space. If a value is given to each of the variables a point is identified for which the objective function may be calculated. If the objective function is evaluated at several such points, the values may be compared and the point with the worst value moved in a way likely to produce an improvement in the objective function. If this procedure (with some refinements) is repeated a number of times the average value of the objective function can be made to improve at each step until the design converges to the optimum. This procedure is the basis of the Complex method due to Box (1). The method was derived from the Simplex method of Spendley, Hext and Himsworth (2) in which the points are chosen always to form a regular figure called a simplex: all points are equidistant, resulting in N + 1 points in an N variable problem. (Simplexes in 2 and 3 dimensions are a triangle and tetrahedron respectively.) The method is not easily adapted to handling constraints.

In the Complex method, the number of points, K, is increased to approximately 2N points which include a feasible starting point and K-1 additional points generated in a pseudo-random fashion from the constraints on the independent variables (the explicit constraints):

$$X_{ij} = G_i + r_{ij}(H_i - G_i)$$

$$i = 1, 2, \ldots, N$$

$$j = 2, 3, \ldots, K$$

r_{ij} are random numbers between 0 and 1

If a point is found which violates the implicit constraints it is moved halfway towards the centroid of the remaining points (or the points so far calculated if this occurs during the formation of the initial complex):

$$X_{ij}\ (\text{new}) = (X_{ij}\ (\text{old}) + \overline{X}_{ic})/2$$

where the coordinates of the centroid of the remaining points, $\overline{X}_{ic}$, are defined by:

$$\overline{X}_{ic} = \frac{1}{L-1}\left[\sum_{j=1}^{L} X_{ij} - X_{ij}\ (\text{old})\right],\ i = 1, 2, \ldots, N$$

($2 \leq L \leq K$, with $L = K$ when an initial valid complex has been formed.)

If an explicit constraint is violated, the offending point is moved a small distance δ inside the constraint. This cannot occur during the formation of the initial complex.

The value of the objective function is calculated for each of the points in the complex, and the point with the lowest value is rejected. The coordinates of the centroid of the remaining points are calculated as above and the rejected point is replaced by a new point at a certain distance beyond the centroid on the line joining the rejected point and the centroid:

$$X_{ij}\ (\text{new}) = \epsilon\,(\overline{X}_{ic} - X_{ij}\ (\text{old})) + \overline{X}_{ic}$$
$$i = 1, 2, \ldots, N$$

Box (1) recommended a value of 1.3 for ϵ. This amplification factor tends to hasten convergence and counteracts the tendency of the complex to collapse in constraint corners. The coordinates of the new point are checked against the constraints and if the point proves to be valid, the objective function is calculated. This value is compared with the remaining values for the complex and again the point with the lowest value is moved. This process is repeated until convergence occurs. If a new point continues to have the lowest value for the objective function, it is moved one half the distance towards the centroid and retested. This process is repeated up to τ times after which the centroid is used as the new point. If this still fails to yield an improvement, the solution is said to have converged. This form of convergence is unusual but could occur because the numerical nature of the simulation model might cause the computed objective function surface to be insufficiently smooth in the vicinity of the optimum. Convergence is assumed to have occurred when, for a specified number of consecutive iterations, γ the objective function at new points satisfying all the above conditions lies within a specified distance, β from the best value.

THE COMPRESSOR SIMULATION MODEL

Models of reciprocating compressors have become more sophisticated and are capable now of simulating a complete compressor system. At Strathclyde University two models exist, an early model (3) based on that by Costagliola (4) and a later model (5) capable of simulating the interaction between the compressor and its valves with the pipework system within which the compressor operates. To demonstrate the application of the Complex method the early model was used, in which the effect of pipework is neglected. This probably reduced computer time by a factor of at least 10, an important consideration at the development stage since one optimisation may require solution of the compressor simulation model several hundred times.

The early model is formulated by the following pairs of simultaneous equations: the first and second order differentials present are with respect to θ. The compressor cycle described is illustrated in Figure 1.

For the suction process:

Gas flow

$$\psi' = MG_i\,\alpha\,\psi^{\frac{1}{\gamma}}\sqrt{1 - \psi^{\frac{\gamma-1}{\gamma}}} - N\psi$$

Valve dynamics

$$q_i^2\alpha_i'' = J_i\,(1 - \psi) - \alpha_i - \lambda_i$$

For the discharge process:

Gas flow

$$\phi' = -MG_d\,\alpha\,\phi^{\frac{\gamma-1}{\gamma}}\sqrt{\phi^{\frac{\gamma-1}{\gamma}} - 1} - N\phi$$

Valve dynamics

$$q_d^2\,\alpha_d'' = J_d\,(\phi - 1) - \alpha_d - \lambda_d$$

In addition the re-expansion and compression processes are described mathematically.

VALVE DESIGN

Important limiting factors in valve design are flow passage size and valve plate permitted lift. A large valve flow area is sought to achieve a high gas throughput without incurring excessive gas velocities and throttling losses. (Provision of adequate areas has become increasingly difficult as advances in compressor speed have resulted in smaller compressors for a given throughput of gas.) A higher permitted valve lift may decrease the throttling loss and increase the throughput but may be accompanied by an increase in valve plate impact velocity. The design must conform to acceptable limits for impact velocity since valve life is related to this parameter. Specifying acceptable values for impact velocity remains a problem.

OPTIMISATION OF VALVE DESIGN

The objective function is chosen to suit the particular design problem. The criterion "the greatest gas throughput for the least power input" was adopted as the objective function in the form

$$F(x_1, x_2, \ldots, x_N) = \frac{\text{volumetric efficiency}}{1 + \dfrac{\text{valve losses}}{\text{thermodynamic work input for ideal cycle}}}$$

When this objective function is large, the volumetric efficiency is high and/or the valve losses are low. Then the compressor has a high efficiency so the aim is to maximise this objective function.

The choice of the independent variables, as with the objective function, depends upon the particular design problem. Any of the parameters in the compressor simulation model which requires an input value could be used as an independent variable. This would infer a situation which involved some 30 variables, which would be unmanageably large, certainly for an initial study. It would also give the situation greater dimensionality than it actually has as most of the variables appear in the model only within lumped parameters which are the coefficients in the pairs of simultaneous equations: it is a change in these coefficients rather than in the individual variables which affect the solutions yielded by the model.

In the present study the following assumptions were made:

1) the mean operating conditions (ω, p_i, T_i, p_d) are fixed
2) the basic compressor design (s, r, l, Ap, c) is fixed
3) the empirical coefficients in the model (Cd, CD, e) are fixed

These assumptions leave twelve variables (A_V, A_i, L, k, h_o, W_V for suction and discharge valve). The problem has been reduced to that of dimensioning the valves to maximise the objective function for a particular compressor with specified operating conditions. The particular single stage air compressor investigated has a cylinder 6 in bore x $4\frac{1}{2}$ in stroke with suction and discharge valves, each of which has a single ring plate backed by three coil springs. The speed range was 400 - 600 rev/min and the discharge pressure was up to 100 $\mathrm{lbf/in^2}$.

At the first attempt to obtain a solution the explicit constraints on the independent variables were set arbitrarily at some distance on either side of the values in the existing design. Only one implicit constraint was applied: namely that the valve port areas could not be larger than the respective valve plate areas. Convergence to an optimum occurred in 110 iterations. The valve lift, port area and valve plate area were placed on the upper constraint boundaries and the spring preload, spring stiffness and valve plate weight were placed on the lower constraint boundaries. An increase in volumetric efficiency and a decrease in the throttling losses were obtained but the computed values for the impact velocity became very large.

The solution was modified by applying an arbitrary maximum permissible impact velocity for the plate in both the suction and discharge valves. If a computed value for impact velocity fell outside the limit, the value was subtracted from the objective function, thus introducing a penalty for designs with impact velocities outside the specified limits. This proved effective and resulted in designs in which the independent variables were not erroneously impeded by the arbitrarily chosen explicit constraints.

Three sets of results A, B and C are listed in Table 1. The convergence to the optimum of the objective function and of some of the independent variables is shown in Figures 2 to 5. The convergence criteria were $\gamma = 40$, $\delta = .0001$, $\beta = .005$ and $\tau = 5$. A plot of the points at which new best values were generated is given in Figure 2: the points plotted in Figures 3 to 5 are of mean values of some of the valve variables.

Comparisons between cases A and B illustrate how reduction of the upper limits on the acceptable values for impact velocity reduced the maximised value of the objective function. The values of the suction valve plate weight, W_{VS}, in these two designs were appreciably different. Since a heavier (thicker) valve plate may tolerate a greater impact velocity, a better method of incorporating the impact velocity limits might be to make the limits a function of the valve weight. Case C incorporates this suggestion: the limit was specified as:

$$\text{Limit} = \frac{(\text{impact velocity } V \text{ ft/s}) \times W_V}{\text{acceptable } W_V \text{ for an impact velocity } V \text{ ft/s}}$$

This limit could be easily altered to a functional relationship based on the latest theoretical and experimental studies on valve impact stressing.

CONCLUDING COMMENTS

The Complex method can be used in conjunction with a simulation model to optimise a compressor or valve design. To assess the validity of the procedure the valves in the compressor examined should be modified to the optimised dimensions and the improvement in compressor efficiency measured experimentally for comparison with that predicted by the analysis. The initial study should be extended in a number of directions. The sensitivity of the maximised objective function to changes in each independent variable could be examined. This would assist in identifying suitable manufacturing tolerance for valve components.

TABLE 1

Test		A	B	C
Velocity limit at stop (suction) (ft/s)		6.0	8.0	$(W_{vs}/.046) \times 6.0$
Velocity limit at stop (discharge) (ft/s)		10.0	14.0	$(W_{vd}/.0552) \times 10.0$
Objective function (maximum)		.86318	.88087	.86949
Independent variables (optimum)				
A_{vs}	(in^2)	5.318	3.074	1.558
A_{is}	(in^2)	5.310	3.074	1.558
L_s	(lb_f)	0.631	.0001	.0001
k_s	(lb_f/in)	0.0322	1.703	1.422
h_{os}	(in)	0.167	0.190	.155
W_{vs}	(lb)	0.293	.0803	.0503
A_{vd}	(in^2)	1.360	2.758	5.153
A_{id}	(in^2)	1.331	2.758	4.161
L_d	(lb_f)	0.699	0.487	0.425
k_d	(lb_f/in)	2.606	6.065	1.211
h_{od}	(in)	0.123	0.137	0.101
W_{vd}	(lb)	0.0655	0.0655	0.162
V_s	(ft/s)	4.635	6.344	5.724
V_d	(ft/s)	9.751	12.638	16.4
Iterations to convergence		212	212	173
Computer time	(seconds)	3885	4582	4000

NOMENCLATURE (Compressor Simulation Model)

- a Acoustic velocity
- A Area of flow in partly open valve
- A_o Area of flow in fully open valve
- A_p Area of piston face
- A_v Area of valve face
- c Clearance at inner dead centre
- Cd Coefficient of discharge
- CD Coefficient of pressure drag
- d (suffix) Discharge
- e Coefficient of Restitution
- G Lumped dimensionless parameter = $C_d A_o a/\omega A_p s$
- h Valve lift in partly open valve
- h_o Maximum permitted valve lift
- i (suffix) Inlet
- J Lumped dimensionless parameter = $CD\, A_v p/kh_o$
- k Spring constant
- l Length of connecting rod
- L Spring load on closed valve
- M Parameter = $(s\gamma/z)\sqrt{2/\gamma - 1}$
- N Parameter = $(\gamma/z)\, dz/d\theta$
- P Pressure
- Pd Discharge pressure
- P_i Suction pressure
- p (suffix) Piston
- q Dimensionless speed ratio = ω/ω_n
- r Radius of crank
- s Piston stroke
- s (suffix) Suction
- T Temperature
- v (suffix) Valve
- V Velocity
- W_v Weight of valve
- z Piston position from cylinder cover = $c + r\,(1 - \cos\theta + (\frac{r}{2l})\sin 2\theta)$
- α Valve lift ratio = $A/A_o = h/h_o$
- γ Gas isentropic index
- θ Crankangle
- λ Spring parameter = L/kh_o
- ϕ Valve pressure ratio (discharge) = P/Pd
- ψ Valve pressure ratio (suction) = P/P_i
- ω Angular speed of crank
- ω_n Natural frequency of valve

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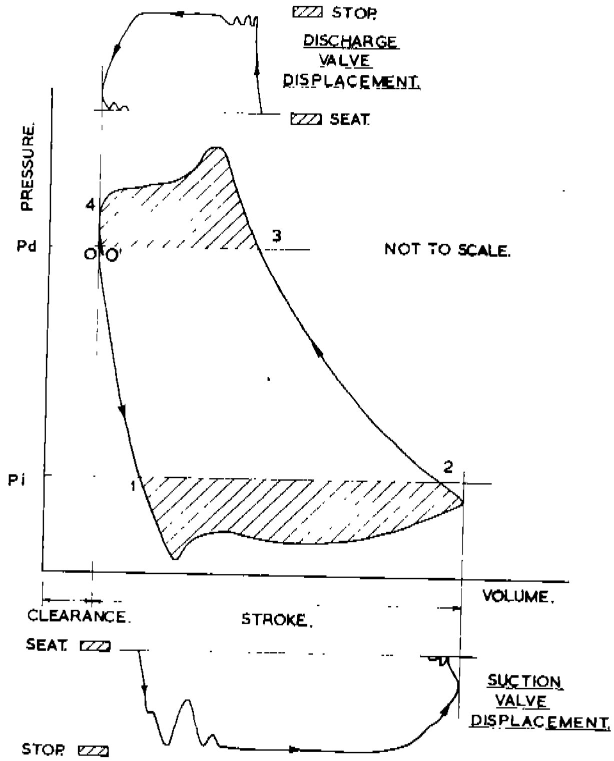

FIG.1. PRESSURE - VOLUME AND VALVE DISPLACEMENT DIAGRAMS.

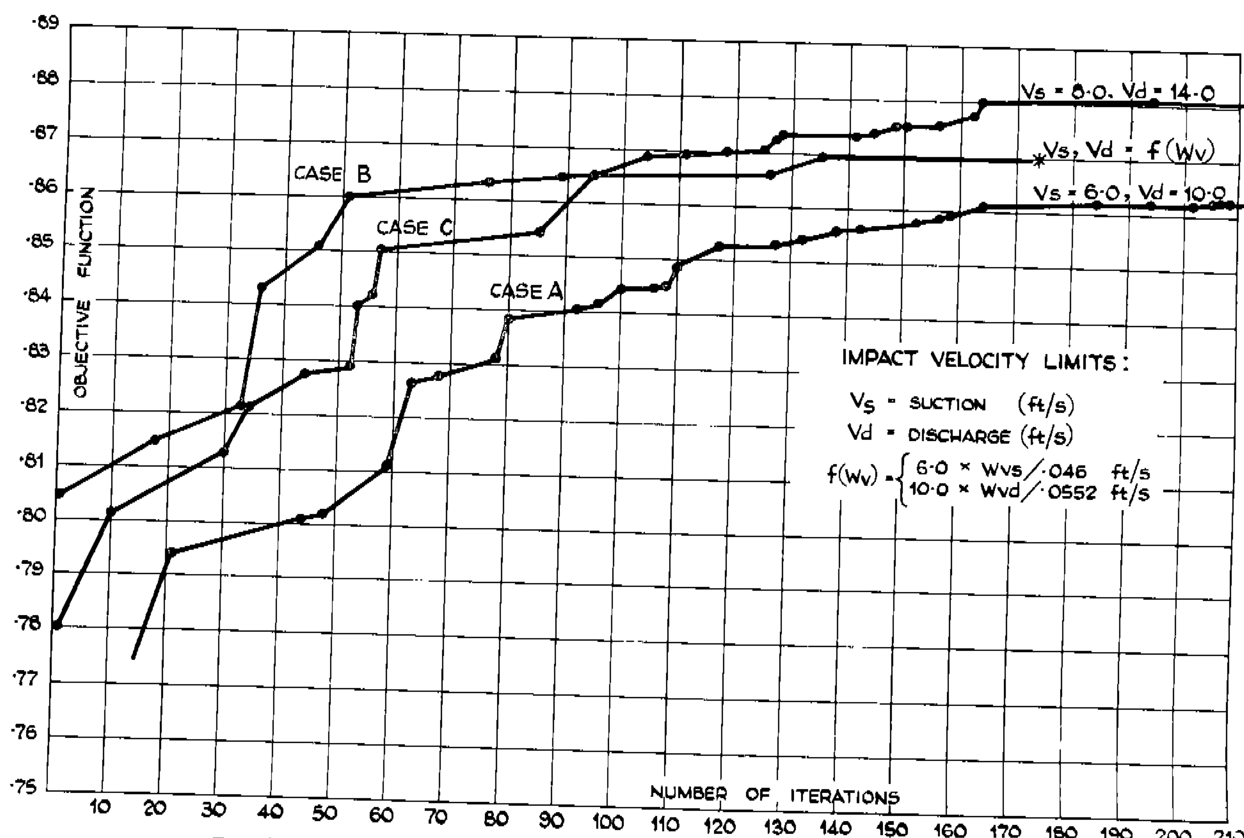

FIG. 2. OPTIMIZATION OF OBJECTIVE FUNCTION

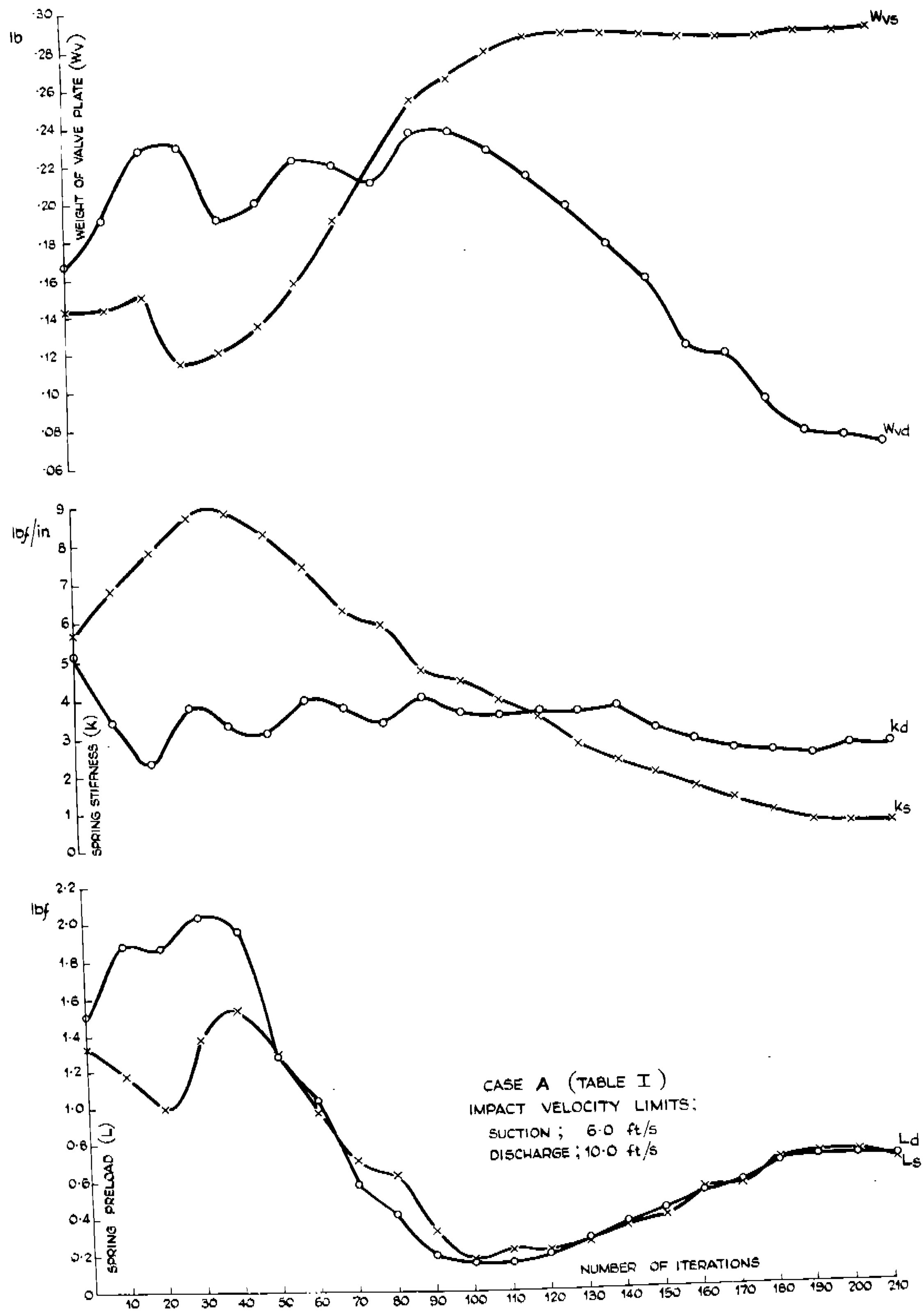

FIG. 3 OPTIMIZATION OF VARIABLES-WEIGHT OF VALVE PLATE, SPRING STIFFNESS, SPRING PRELOAD.

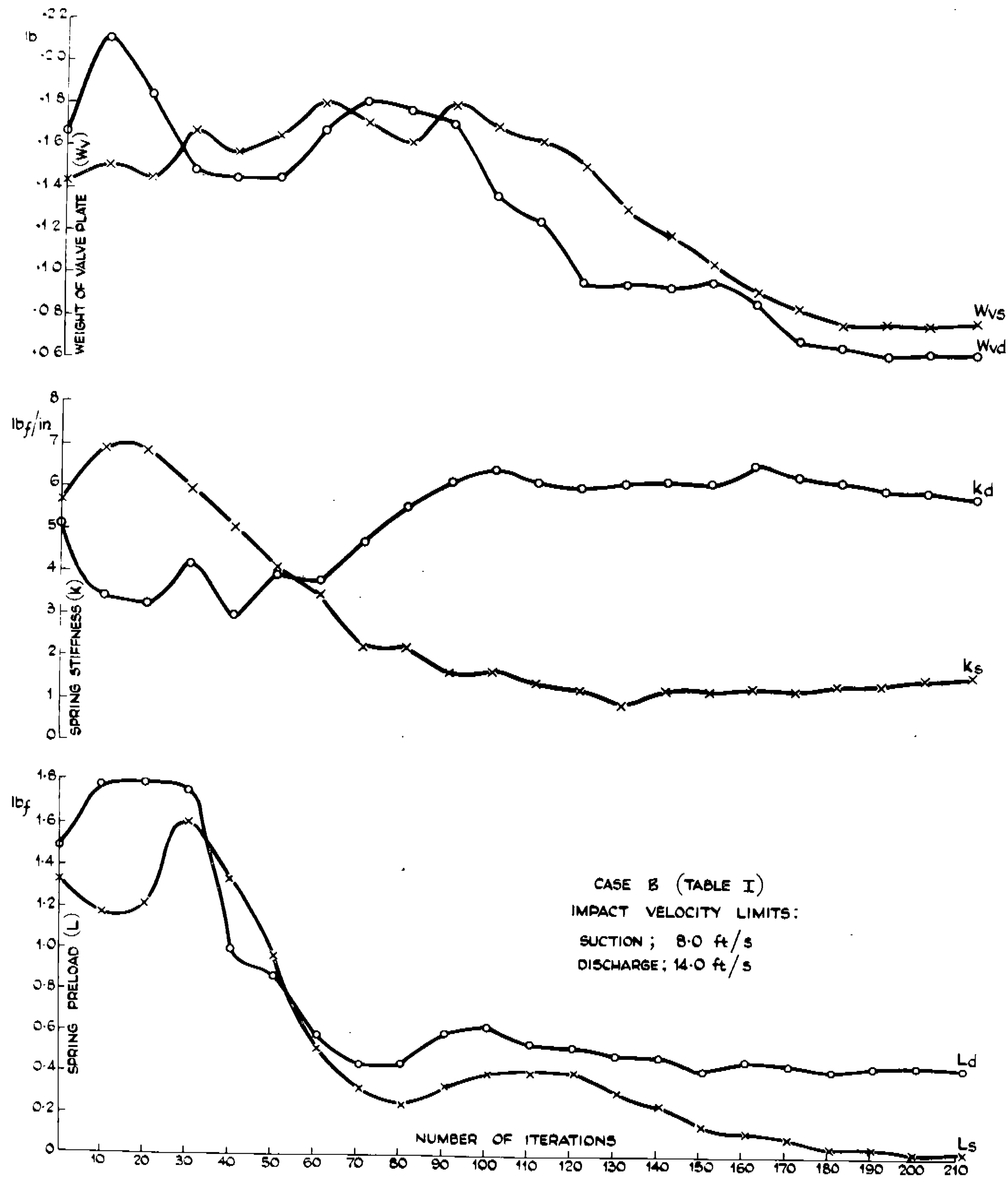

FIG. 4 OPTIMIZATION OF VARIABLES - WEIGHT OF VALVE PLATE, SPRING STIFFNESS, SPRING PRELOAD

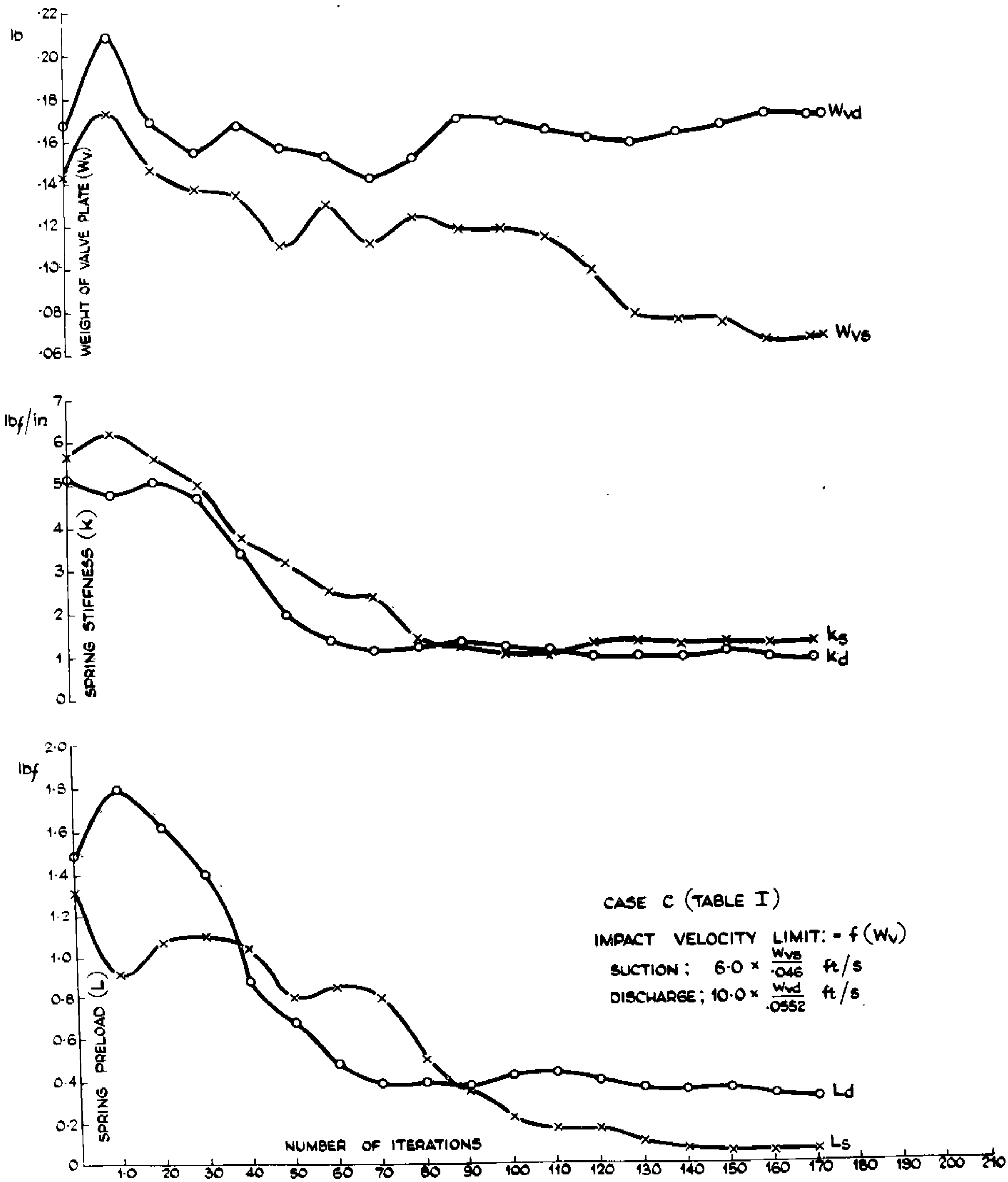

FIG. 5. OPTIMIZATION OF VARIABLES – WEIGHT OF VALVE PLATE, SPRING STIFFNESS, SPRING PRELOAD.