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ABSTRACT

Devices to dampen pressure pulsations in positive displacement compressor systems frequently contain a section of perforated pipe. This paper describes an analytical and experimental study of unsteady flow in a compressor system which incorporated such a section. The equations governing one dimensional finite amplitude unsteady flow were modified to account for mass transfer through perforations in a pipe wall. The equations were solved numerically by composite schemes which incorporated either the Lax-Wendroff Method or the Leapfrog Method coupled with the Method of Characteristics at the pipe ends. The analytical results were compared with experimental records from a single stage reciprocating air compressor system in which a perforated section was incorporated in the inlet line. The analytical predictions of the effect of damping on the pressure pulsations at three stations in the inlet line and on the movement of the suction valve were in satisfactory agreement with experimental results provided that the ratio of the area of the perforation(s) to the cross sectional area of the pipe did not exceed about 0.15 (when the perforations were spaced at intervals of 1.5 pipe diameters along the pipe).

INTRODUCTION

The sequence of events during a cycle of operation in a positive displacement compressor generates an intermittent flow resulting in pressure pulsations which cause vibration and noise (1). Dampers to reduce these undesirable effects may be classified according to their principle of operation as resistive, reactive or non-reactive (2). Figure 1 shows schematically dampers classified in this manner. In dampers of the resistive type energy is dissipated by imposing additional resistance to the flow: an attraction of this type is its insensitivity to pulsation frequency, but the pressure loss may be excessive. Reactive dampers can achieve satisfactory attenuation at particular frequencies and the design of these dampers (resonators) is usually based on acoustic theory or accomplished by a trial and error procedure (3). The non-reactive type does not have the resonant characteristics of the reactive type and hence its efficiency is not a function of frequency. In the absorption version of the non-reactive type energy is absorbed in a porous lining fitted to a length of the pipe wall (4). In the snubber version the high static pressure generated by the pulsations drives gas through perforations in the pipe wall: energy is thereby dissipated and the amplitude of pressure pulsations in the pipe is reduced.

PREVIOUS ANALYSES OF FLOW IN A PERFORATED PIPE

The attenuation of pressure pulsations by dampers which include a perforated pipe has usually been estimated using the small wave ("acoustic") theory for the Helmholtz resonator. However when pressure pulsations were large, agreement between analytical and experimental results was poor (5). Trengrouse (5, 6) examined, analytically and experimentally, the attenuation of finite amplitude waves in such dampers. Equations for the motion of the gas upstream and downstream of the holes were integrated using the Method of Characteristics: the single row of holes was treated as a boundary condition for the numerical integration. Woollatt (7) also considered the flow through a hole in the wall of a pipe as a boundary condition. Multiple perforations in the pipe were considered by Rudinger (8) who derived the equations for unsteady flow of gas in a pipe with mass removal. The solution to the equations was obtained iteratively or graphically by the Method of Characteristics. Brablik (9) also studied this type of damper and presented equations for unsteady flow of gas with mass addition or removal: a finite difference scheme was applied to obtain a solution to a linearised ("acoustic") version of the equations of continuity and momentum. Azim (10) used the Method of Characteristics to solve the non-linear version of these equations when simulating the gas exchange processes in the cells of a Comprex type compressor. Szumowski (11) and Wu and Ostrowski (12) studied the attenuation of a shock wave travelling through a pipe which was perforated uniformly. Flow equations were derived which accounted for mass removal and were solved by the Method of Characteristics.

EQUATIONS OF FLOW IN A PERFORATED PIPE

In the present study the flow in a perforated pipe was assumed to be one-dimensional and the fluid was considered to behave as an ideal gas. Heat transfer, gradual changes in cross sectional area and friction were accounted for. When considering the flow through the perforations the following assumptions were made:

(i) the fluid in the enclosure surrounding the perforations was stagnant
(ii) particles leaving the stream carried the average momentum and energy of the stream at the instant of outflow
(iii) particles entering the stream gained the average momentum of the stream at the instant of inflow
(iv) at any instant the flow through the pipe walls obeyed steady state flow relationships.

The equations of continuity, momentum and energy were written using the above assumptions (13) leading to a set of equations in conservation-law form:

$$\frac{\partial}{\partial t}\begin{bmatrix} \rho \\ \rho u \\ \frac{\rho u^2}{2} + \frac{p}{k-1} \end{bmatrix} + \frac{\partial}{\partial x}\begin{bmatrix} \rho u \\ \rho u^2 + p \\ \frac{\rho u^3}{2} + \frac{k p u}{k-1} \end{bmatrix} = \begin{bmatrix} -\frac{\rho u}{F}\frac{dF}{dx} \mp \frac{\dot M}{F} \\ -\frac{\rho u^2}{F}\frac{dF}{dx} - \rho\phi \mp \frac{\dot M u}{F} \\ -\frac{1}{F}\frac{dF}{dx}\left(\frac{\rho u^3}{2} + \frac{k p u}{k-1}\right) + \rho q \mp \frac{\alpha}{F} \end{bmatrix} \qquad (1)$$

When two algebraic signs appear together, the upper refers to flow from the pipe to the surroundings, the lower refers to flow in the opposite direction.

The rate of mass transfer through the perforations per unit length of pipe is given by

$$\dot M = C_D \frac{k g p_u}{a_u} H \left[\frac{2}{k-1}\left(\frac{p_d}{p_u}\right)^{2/k}\left(1.0 - \frac{p_d}{p_u}\right)^{k-1/k}\right]^{\frac{1}{2}} \qquad (2)$$

C_D is an empirical flow coefficient which may be determined from steady flow experiments or approximated by analytical methods (14). When mass flows through a perforation to the surroundings it is assumed that any particle leaving the pipe carries the mean energy of the flow in the pipe, i.e.

$$\alpha_{ps} = \dot M \left[\frac{u^2}{2} + \frac{a^2}{(k-1)}\right] \qquad (3)$$

In the case of flow from the surroundings through a perforation into the pipe any particle entering the main stream has the energy level of the surroundings, i.e.

$$\alpha_{sp} = \dot M \left[\frac{a_s^{\,2}}{k-1}\right] \qquad (4)$$

Equation (1) may also be expressed in normal form

$$\frac{\partial}{\partial t}\begin{bmatrix} \rho \\ u \\ p \end{bmatrix} + \begin{bmatrix} u & \rho & 0 \\ 0 & u & 1/\rho \\ 0 & a^2\rho & u \end{bmatrix}\frac{\partial}{\partial x}\begin{bmatrix} \rho \\ u \\ p \end{bmatrix} = \begin{bmatrix} -\frac{\rho u}{F}\frac{dF}{dx} \mp \frac{\dot M}{F} \\ -\phi \\ (k-1)\quad(u\phi+q) - \frac{a^2\rho u}{F}\frac{dF}{dx} \pm \frac{(k-1)\dot M u^2}{2F} \mp \frac{(k-1)\alpha}{F} \end{bmatrix} \qquad (5)$$

This system of equations (5) may be reduced to a system of ordinary differential equations thus giving the characteristic form of the equations. In characteristic form the following equations apply:

Along the Mach line characteristic, $\frac{dx}{dt} = u + a$

$$\frac{da}{dt} + \frac{k-1}{2}\frac{du}{dt} = -\frac{k-1}{2}\frac{au}{F}\frac{dF}{dx} + a\frac{ds}{2c_p} - \frac{k-1}{2}\phi\left[1 - (k-1)\frac{u}{a}\right]$$
$$+ \frac{(k-1)^2}{2}\frac{q}{a} \pm \frac{(k-1)^2}{4}\frac{\dot M u^2}{\rho a F} \mp \frac{(k-1)^2}{2}\frac{\alpha}{\rho a F} \qquad (6a)$$

Along the Mach line characteristic, $\frac{dx}{dt} = u - a$

$$\frac{da}{dt} - \frac{k-1}{2}\frac{du}{dt} = -\frac{k-1}{2}\frac{au}{F}\frac{dF}{dx} + \frac{a}{2c_p}\frac{ds}{} + \frac{k-1}{2}\phi\left[1 + (k-1)\frac{u}{a}\right]$$

$$+ \frac{(k-1)^2}{2}\frac{q}{a} \pm \frac{(k-1)^2}{4}\frac{\dot{M}u^2}{\rho aF} \mp \frac{(k-1)^2}{2}\frac{\alpha}{\rho aF} \qquad (6b)$$

Along the path line characteristic, $\frac{dx}{dt} = u$

$$\left(\frac{dp}{dt}\right)_{path} = \frac{2k}{k-1}\frac{p}{a}\left(\frac{da}{dt}\right)_{path} - \rho\,(u\phi + q)$$

$$\mp \frac{\dot{M}}{F}\left[\frac{u^2}{2} + \frac{a^2}{k-1}\right] \pm \frac{\alpha}{F} \qquad (6c)$$

SOLUTION OF THE EQUATIONS

The set of non-linear partial differential equations (1) was solved using two finite difference schemes (15). One employed the Two-Step Lax-Wendroff Method, the other used the Leapfrog Method. In both schemes the Method of Characteristics was applied at the pipe boundaries. The equations in the computer program for a simulation model of compressor systems (16) were modified to account for the mass transfer at the perforations in a pipe wall.

EXPERIMENTAL TESTS

Tests were conducted using a single cylinder air compressor, 6.0 inch bore x 4.5 inch stroke (152 x 114 mm), within a system which included an inlet pipe, the compressor, discharge pipe and receiver. Speeds were in the range 400 - 600 rev/min. Tests were conducted with a number of perforated pipes in the inlet line, as illustrated by Figure 2. The discharge coefficient for the perforations, C_D in equation (2), was determined by steady flow tests. The pressure variations at three locations in the system and the movement of the compressor inlet valve were recorded.

RESULTS

The validity of the mathematical model was assessed by comparing analytical predictions with experimental records. For the arrangements illustrated in Figure 2(a) and 2(b), the left hand side and right hand side respectively of Figure 3 shows records of pressures at stations 1, 2 and 3 (suction valve plenum chamber) and in the cylinder during suction, together with suction valve displacement: the compressor speed was 400 rev/min. The inlet line in Figure 2 (b) had 30 perforations (each 0.36 inch diameter) distributed uniformly along the first 5 feet of the 1.375 inch internal diameter pipe. The marked changes in the pressure variations at the three stations and in the movement of the suction valve, caused by the introduction of the perforations, were considered to have been predicted satisfactorily.

Figure 4 shows results when the compressor speed was 600 rev/min, using the pipeline arrangements shown in Figure 2 (c) and 2 (d). In this case the rotational frequency, 10 Hz, corresponded approximately to the natural frequency of vibration of the air in the inlet pipe of the arrangement shown in Figure 2 (c). This resonant condition is illustrated in the left-hand portions of Figure 4. The introduction of the perforated portion of pipe (right-hand portion of Figure 4) greatly reduced the amplitude of the pressure pulsations. Again agreement between predicted and measured results was considered to be satisfactory.

The sound pressure/frequency records at stations 1 and 3 for the conditions pertaining in Figure 4 were obtained from the analytical pressure/time histories. The characteristic of the non-reactive type of damper (snubber) - that it is not sensitive to frequency - may be observed from Figure 5: besides reduction of the sound pressure level at the fundamental frequency by about 20 db in the suction plenum chamber, there was significant damping at the higher frequencies. Corresponding tests (not shown) using dampers of the reactive type (Quincke resonator and Quarter-Wave resonator, Figure 1) in place of the snubber gave even greater reduction at the fundamental frequency but did not appreciably damp the higher frequency pulsations.

The extension of tests over a wide range of conditions revealed that there was a limit beyond which the model was less adequate. It was judged that when the perforations were pitched at 1.5 pipe diameter intervals along the pipe, this limit was reached when the ratio of the area of the perforations to the cross sectional area of the pipe exceeded 0.15. Beyond the limit one dimensional flow theory cannot describe adequately a situation where two and three dimensional effects have become significant. Multi-dimensional effects were particularly significant when flow was from the surroundings into the pipe.

CONCLUSIONS

The equations for one dimensional finite amplitude unsteady flow with heat transfer can be modified to account for mass transfer through perforations in a pipe. Consequently a mathematical model can be constructed to simulate the snubber version of the non-reactive type of pulsation damper. Comparison between predictions by such a model and experimental records from a reciprocating air compressor system indicated that the model was valid over the range of conditions likely to be encountered in such a system. However, a limit was found beyond which analysis using one dimensional flow theory become increasingly inadequate.

NOTATION

a	Speed of sound
C_D	Flow coefficient
c_p	Specific heat at constant pressure
D	Pipe diameter
F	Pipe area
f	Friction factor ($= \tau w/\frac{1}{2} \rho u^2$)
g	Gravitational acceleration
H	Area of perforations per unit length of pipe
k	Ratio of specific heats
$\dot{M}$	Rate of mass transfer per unit length of pipe through the perforations
p	Pressure
q	Heat transfer rate per unit mass of gas
t	Time
u	Particle velocity
x	Distance
α	Total energy
ϕ	Friction term ($= \frac{4f}{D} \frac{u}{2} \lvert u \rvert$)
ρ	Density
τ_w	Shear stress at wall

SUBSCRIPTS

d	Downstream
u	Upstream
s	Surrounding
ps	Pipe to surrounding
sp	Surrounding to pipe
path	Denotes motion along the path of a particle

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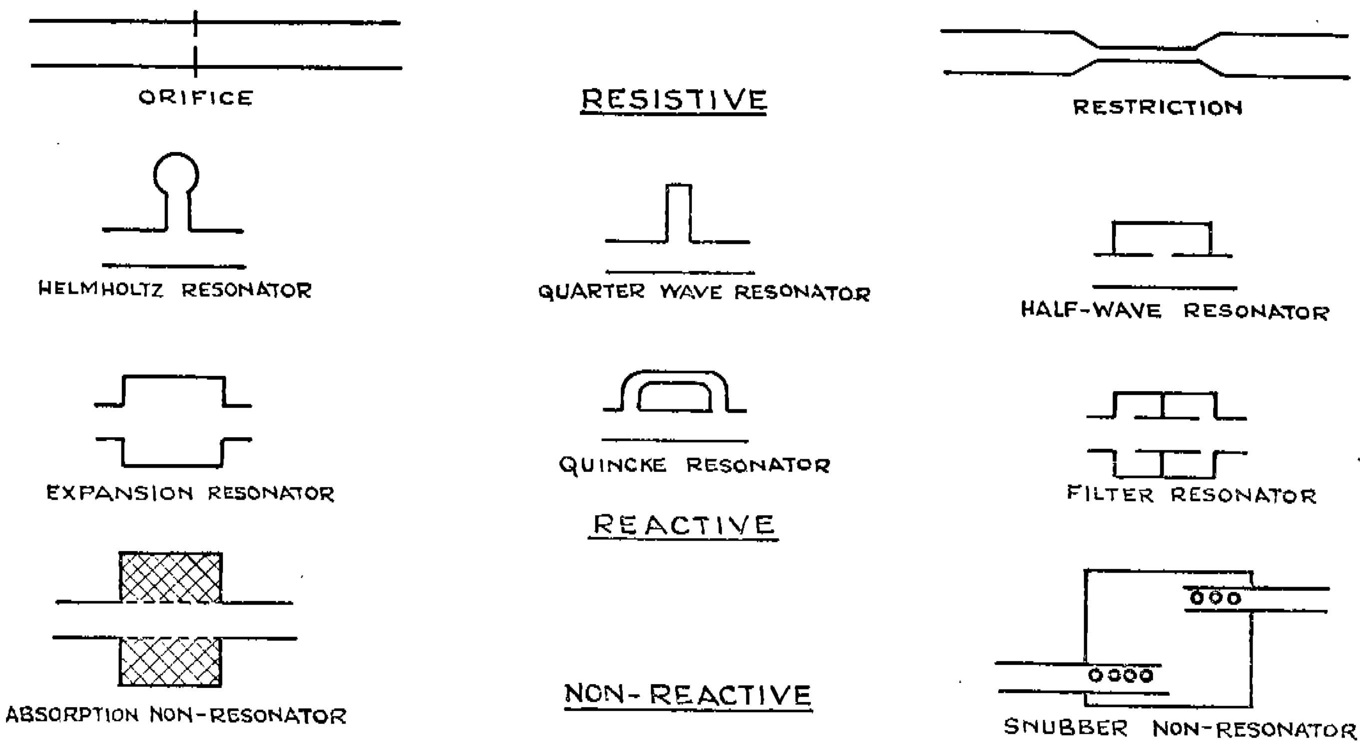

FIGURE 1. TYPES OF DAMPER

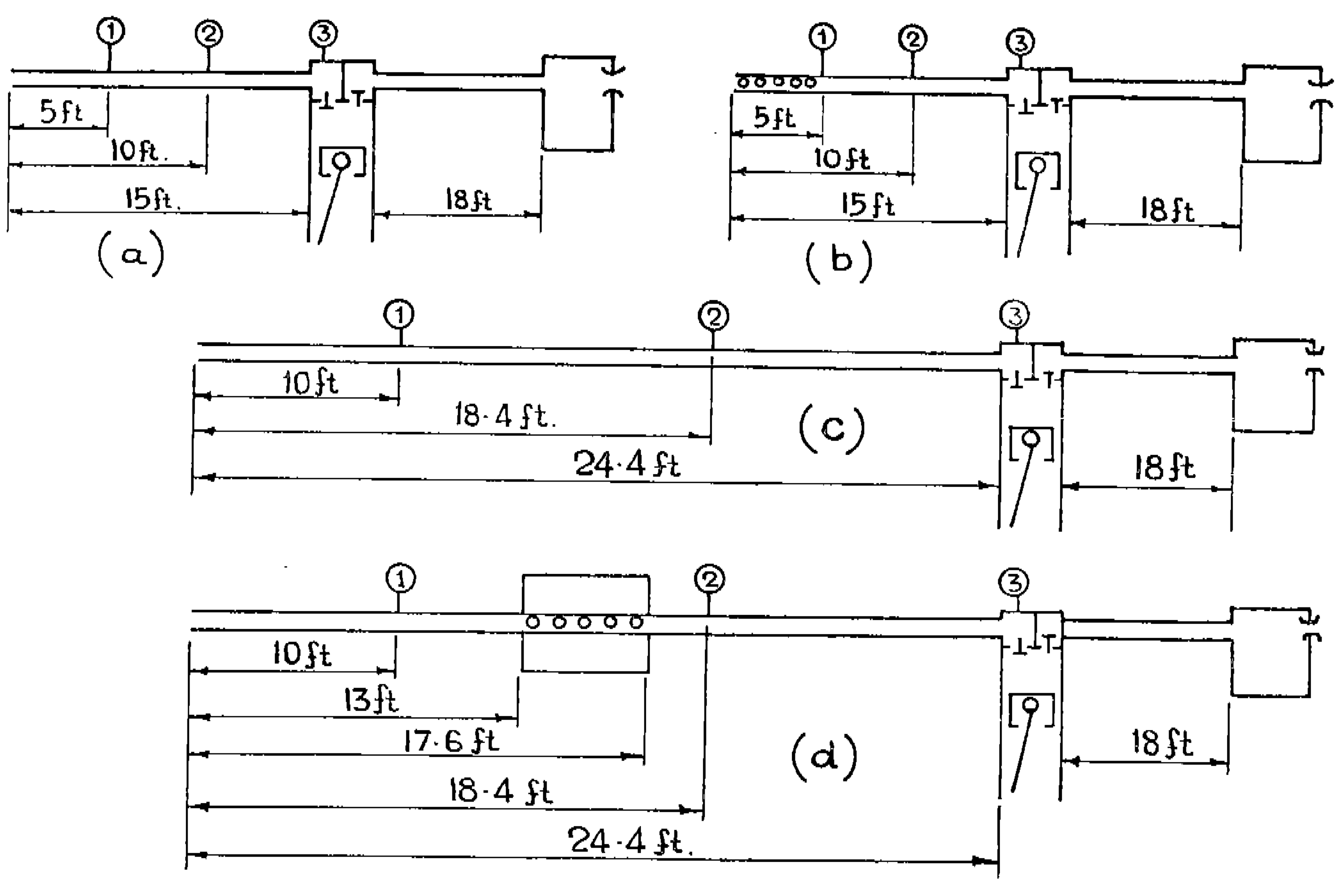

FIGURE 2. INLET LINE CONFIGURATIONS.

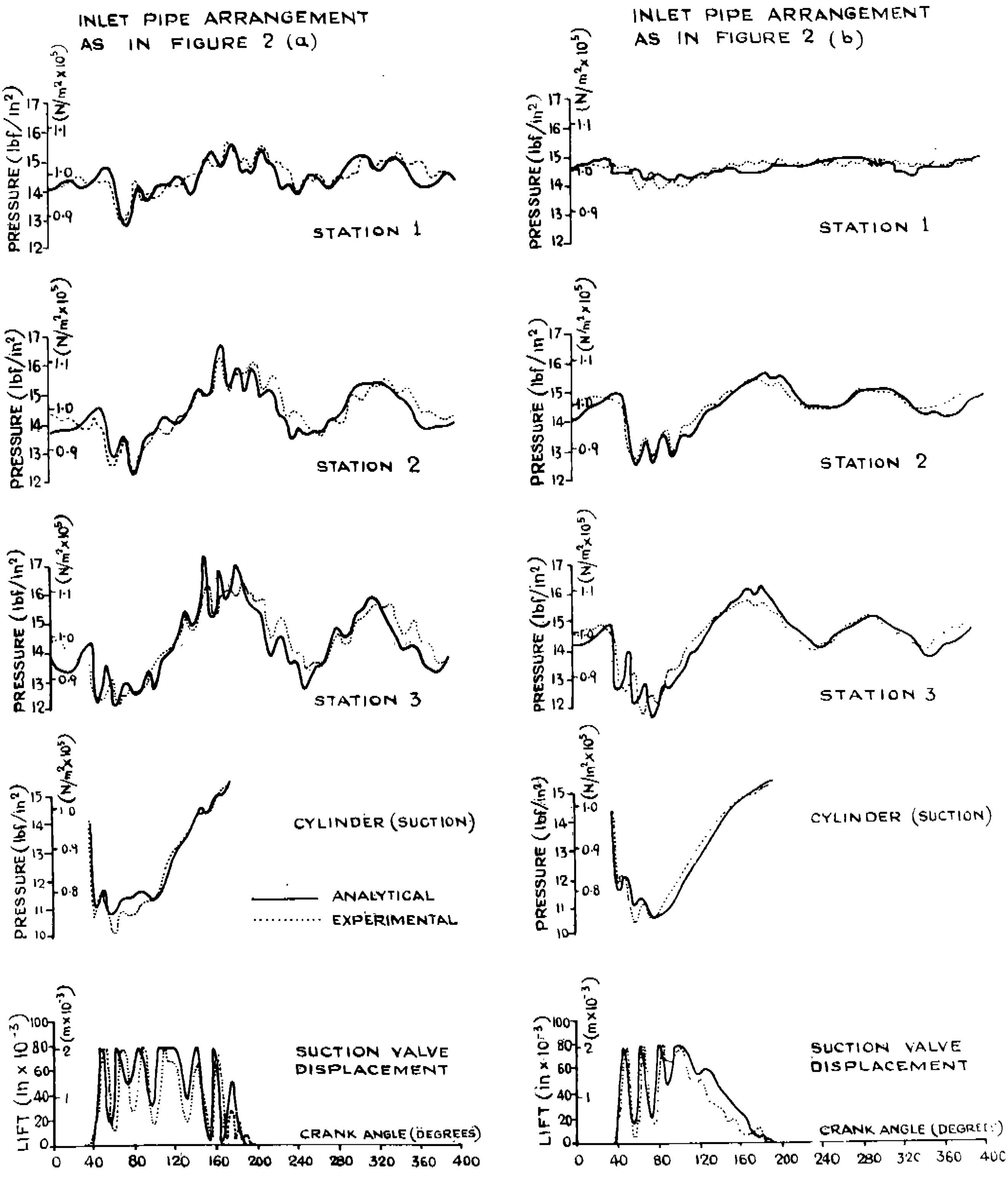

FIGURE 3 COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS

(COMPRESSOR SPEED 400 rev/min)

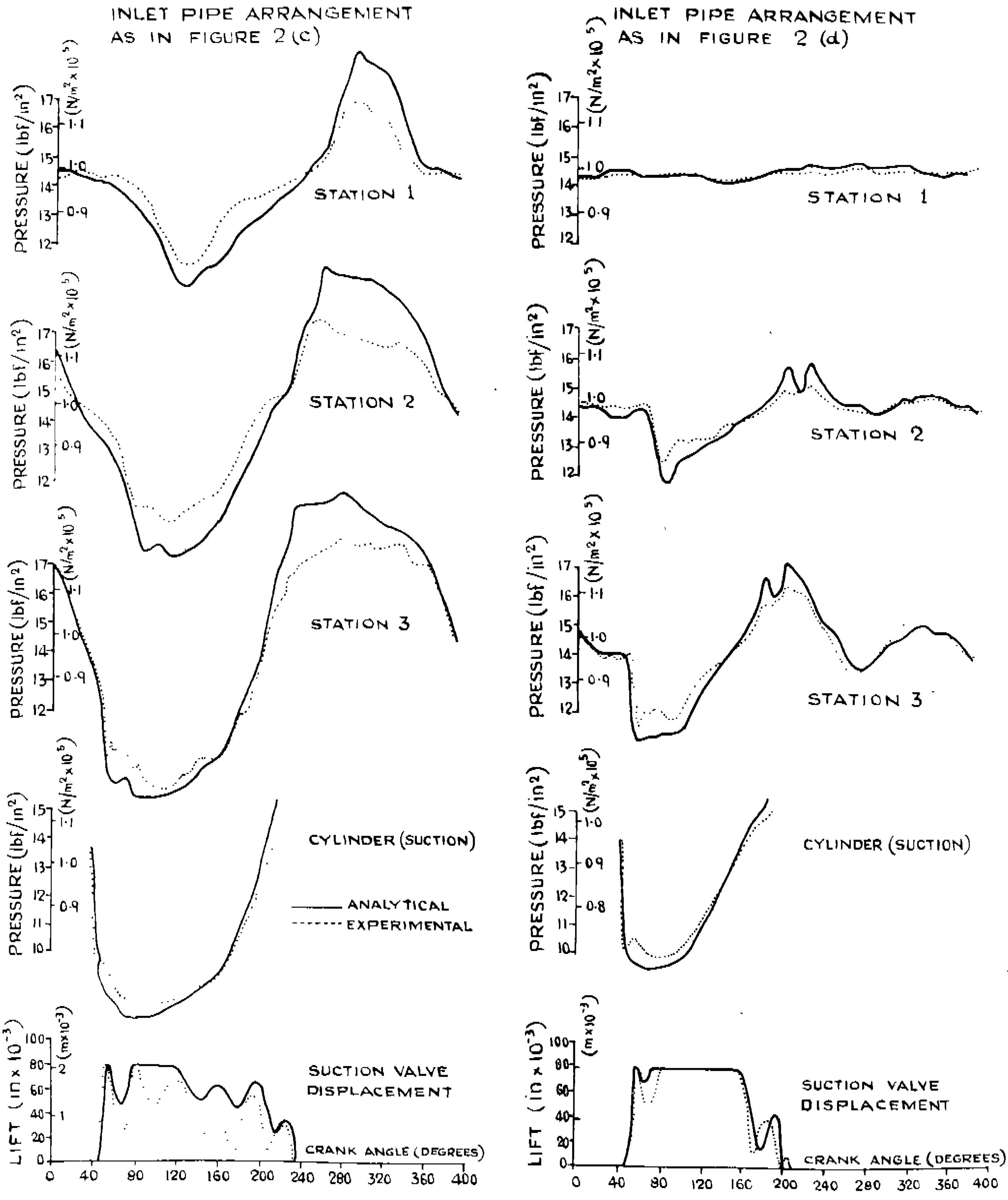

FIGURE 4 COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS (COMPRESSOR SPEED 600 rev./min)

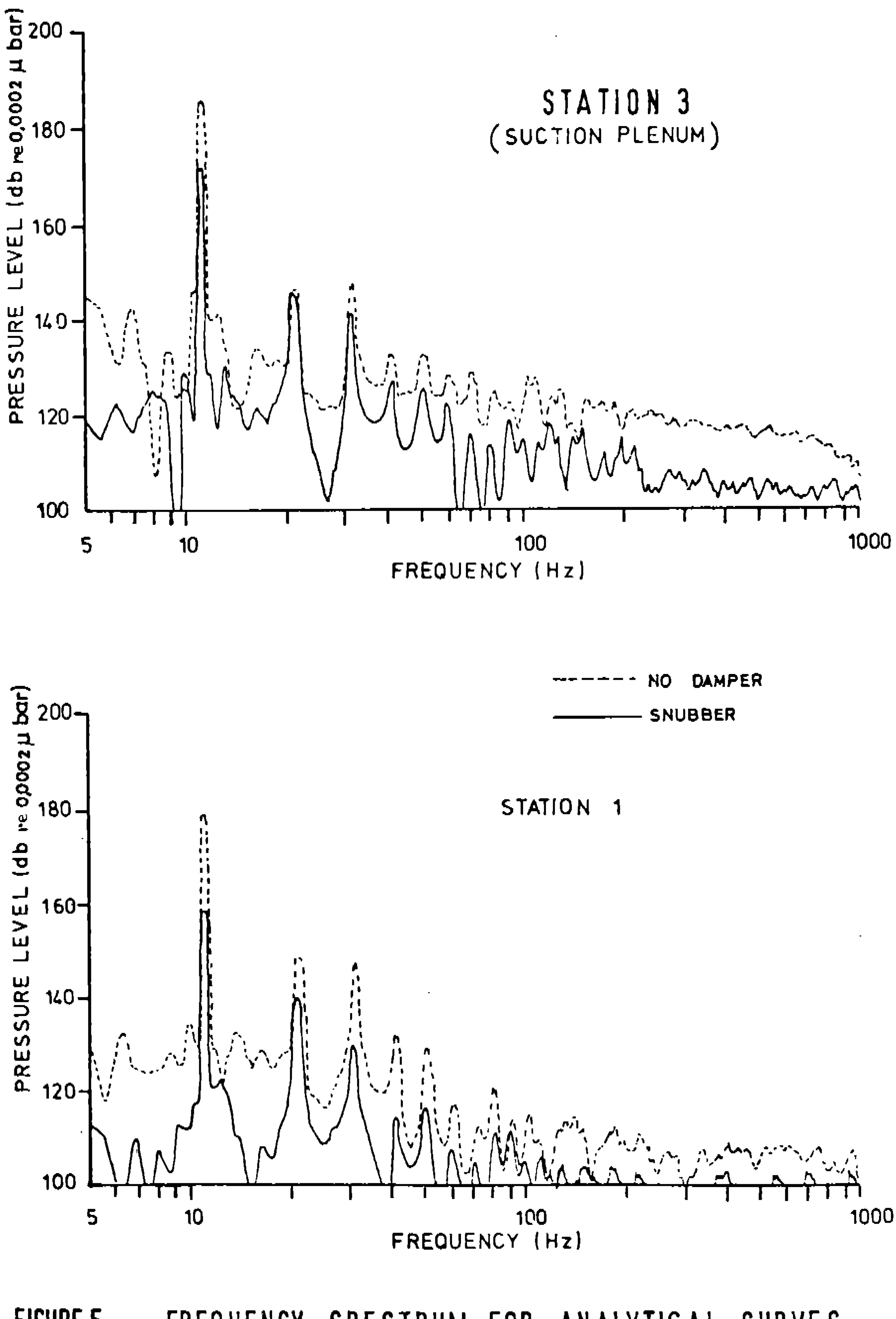

FIGURE 5. FREQUENCY SPECTRUM FOR ANALYTICAL CURVES IN FIGURE 4.