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Near-Field Acoustical Holography Incorporating Compressive Sensing

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Acoustic Beamforming and Holography

Near-Field Acoustical Holography Incorporating Compressive Sensing

Tongyang Shi
Weimin Thor
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Outline

• Introduction

• Near-Field Acoustical Holography Methods

• Experiments

• Summary
Introduction and Motivation

• Near-Field Acoustical Holography (NAH)

Choose the type of basis functions

Estimate coefficients from measurements

Predict the sound field at any location

Transfer path

Reconstruction

Unknown sound source

Measurement data

• Limitation of NAH

  • Generally a large number of measurements is required

  • Expensive and hard to conduct experiment in industrial setting
Sparse Equivalent Source Method (S-ESM)

- Monopole based Equivalent Source Method
  - Sound pressure generated by a monopole with source strength $q$
    \[
    P_{S0} \left( \vec{X} | \vec{X}_0, \omega \right) = q \cdot g_0 \left( \vec{X} | \vec{X}_0, \omega \right) = q \frac{e^{-jk\|\vec{X} - \vec{X}_0\|}}{4\pi \|\vec{X} - \vec{X}_0\|},
    \]
    \(\vec{X}:\) Field point position
    \(\vec{X}_0:\) Monopole position
  - Matrix formation with multiple monopoles and measurements

\[
\begin{bmatrix}
P_1(\zeta_1, \omega) \\
P_2(\zeta_2, \omega) \\
\vdots \\
P_M(\zeta_M, \omega)
\end{bmatrix}
= \begin{bmatrix}
g_1(\zeta_1 | \vec{X}_1, \omega) & \cdots & g_W(\zeta_1 | \vec{X}_W, \omega) \\
g_1(\zeta_2 | \vec{X}_1, \omega) & \ddots & \vdots \\
\vdots & \ddots & g_W(\zeta_2 | \vec{X}_W, \omega) \\
g_1(\zeta_M | \vec{X}_1, \omega) & \cdots & g_W(\zeta_M | \vec{X}_W, \omega)
\end{bmatrix}
\begin{bmatrix}
q_1(\omega) \\
q_2(\omega) \\
\vdots \\
q_W(\omega)
\end{bmatrix}
\]

\(P_1, P_2, \ldots, P_M:\) Monopoles
\(g_1, g_2, \ldots, g_W:\) Transfer functions
\(q_1, q_2, \ldots, q_W:\) Source strengths
\(\zeta_1, \zeta_2, \ldots, \zeta_M, \zeta_W:\) Source positions
\(\xi_1, \xi_2, \ldots, \xi_m, \xi_W:\) Field point positions
\(x_1, x_2, x_3, x_4, x_5, \ldots, x_W:\) Microphone positions

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Sparse Equivalent Source Method (S-ESM)

- **Wideband Acoustical Holography**


### Diagram

- **Source strength vector** $q_k$
- Calculate residual vector
- **Residual vector:** $r(\tilde{q}) \equiv \|\tilde{P}_m - A\tilde{q}\|^2$
- Calculate the step $\Delta q$ to minimize the residual vector
- **Steepest gradient method**

### Algorithm

1. **Check the threshold condition**
2. **Turn off the sources below threshold**
3. **Calculate residual vector**
4. **Calculate the step $\Delta q$** to minimize the residual vector
5. **Final Result**
6. **Get the next solution candidate**

- **Error between reconstruction pressure and measurement pressure is less than $e$**
- $D_k$ reach the maximum value

$$q_{k+1} = q_k + \alpha \Delta q$$
Sparse Equivalent Source Method (S-ESM)

- \( l_1 \)-Norm Minimization
  - Objective function
    \[
    \text{minimize } ||\hat{q}||_1 + \lambda \left\| A\hat{q} - \hat{p}_m \right\|_2
    \]
  - Weighting parameter
  - Solution sparsity
  - Solution accuracy
  - Careful choosing the weighting parameter
  - \( l_1 \)-norm for source strength and \( l_2 \)-norm for residual

- \( l_1 \)-norm and \( l_2 \)-norm
  - \( l_1 \)-norm: \( ||\hat{q}||_1 = \sum_{i=1}^{m} |q_i| \)
  - \( l_2 \)-norm: \( ||\hat{q}||_2 = \sqrt{q_1^2 + q_1^2 + \cdots + q_m^2} \)

\[
p = \alpha_1 s_1 + \alpha_2 s_2
\]
Sparse Equivalent Source Method (S-ESM)

- **The Hybrid Method**

1. Begin the search with $l_1$-norm minimization and a relatively large $\lambda$: ghost sources created
2. Take solution of $l_1$-norm minimization as initial solution for WBH: the ghost source can be eliminated by WBH
Loudspeaker Experiment

- Sound field reconstruction
  - Sound source: loudspeaker (Infinity Primus P163)
  - Input signal: white noise (bandwidth 25.6 kHz) generated by PULSE software
  - Intensity probe scan measurement at near and far-field

- Intensity probe
  - Bruel and Kjaer type 3654 intensity probe
  - Two microphone cross-spectral approach
    - Sound intensity at the middle of two microphones
    - \( I_z = \frac{1}{2} \frac{1}{\omega \rho_0 \Delta} \text{Im}(P_1 P_2^*) \)
  - Patched measurement
    - \( p = H_{rp}^T r \)
    - An equivalent simultaneous microphone array measurement can be calculated
Loudspeaker Experiment

Measurement plane
- \(-0.05 \, m < x < 0.22 \, m, -0.05 \, m < y < 0.4 \, m\)
- Measurement taken at 0.055 \, m from loudspeaker front face
- Measurement spacing: 0.03 \, m in x, and y-directions
- 160 near-field measurements in total
- Equivalent of a regular rectangular microphone array

Data processing in PULSE
- 10 second measurement at each point
- Sampling frequency: 25.6 kHz
- Hann window: 0.25 second 50% overlap
- 100 averages
Loudspeaker Experiment

➢ Equivalent source (monopole) plane
  - $-0.05 \, m < x < 0.22 \, m$, $-0.05 \, m < y < 0.4 \, m$
  - 0.02 m behind the loudspeaker front face
  - 0.01 m spacing in both x, and y-directions
  - 1288 monopoles

➢ Reconstruction plane
  - Reconstruction plane is 0.055 m from loudspeaker front face
  - Reconstruction plane is also where the intensity measured
    - $-0.05 \, m < x < 0.22 \, m$, $-0.05 \, m < y < 0.4 \, m$
    - Measurement spacing: 0.03 m in x, and y-directions
    - Reconstruction will be directly compared with measurement
Sound Intensity Reconstruction at 800 Hz

**SONAH**

- Measurement: 47.9 dB
- SONAH: 40.8 dB
- WBH: 27.9 dB
- L1-norm minimization: 34.1 dB
- Hybrid method: 34.4 dB
### Sound Intensity Reconstruction at 2400 Hz

**Measurement** | **46.3 dB**
--- | ---
SONAH | 47.5 dB
WBH | 39.9 dB
L1-norm minimization | 46.5 dB
Hybrid method | 46.1 dB
Sound Intensity Reconstruction at 4000 Hz

<table>
<thead>
<tr>
<th>Method</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>SONAH</td>
<td>46.8 dB</td>
</tr>
<tr>
<td>WBH</td>
<td>43.2 dB</td>
</tr>
<tr>
<td>L1-norm minimization</td>
<td>43.8 dB</td>
</tr>
<tr>
<td>Hybrid method</td>
<td>45.0 dB</td>
</tr>
</tbody>
</table>
Loudspeaker Experiment

- **Measurement plane**
  - \(-0.05 \, \text{m} < x < 0.22 \, \text{m}, -0.05 \, \text{m} < y < 0.4 \, \text{m}\)
  - Measurement taken at 0.055 m from loudspeaker front face
  - Measurement spacing: 0.06 m in \(x\), and \(y\)-directions
  - 40 near-field measurements in total
  - Equivalent of a regular rectangular microphone array

- **Data processing in PULSE**
  - 10 second measurement at each point
  - Sampling frequency: 25.6 kHz
  - Hann window: 0.25 second 50% overlap
  - 100 averages
Sound Intensity Reconstruction at 800 Hz

**SONAH**

- $l_1$-norm minimization

**WBH**

- Hybrid method

**Measurement**

- **SONAH**: 40.7 dB
- **WBH**: 27.9 dB
- **L1-norm minimization**: 33.0 dB
- **Hybrid method**: 33.7 dB
Sound Intensity Reconstruction at 2400 Hz

Measurement

<table>
<thead>
<tr>
<th>Method</th>
<th>Sound Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>SONAH</td>
<td>47.4 dB</td>
</tr>
<tr>
<td>WBH</td>
<td>39.9 dB</td>
</tr>
<tr>
<td>L1-norm minimization</td>
<td>43.8 dB</td>
</tr>
<tr>
<td>Hybrid method</td>
<td>44.7 dB</td>
</tr>
</tbody>
</table>
Sound Intensity Reconstruction at 4000 Hz

**SONAH**

- Measurement: 49.5 dB
- SONAH: 42.3 dB
- WBH: 43.2 dB
- L1-norm minimization: 43.3 dB
- Hybrid method: 43.3 dB
Loudspeaker Experiment

- **Measurement plane**
  - $-0.05 \, \text{m} < x < 0.22 \, \text{m}$, $-0.05 \, \text{m} < y < 0.4 \, \text{m}$
  - Measurement taken at 0.055 m from loudspeaker front face
  - Measurement spacing: 0.12 m in x, and y-directions
  - 12 near-field measurements in total
  - Equivalent of a regular rectangular microphone array

- **Data processing in PULSE**
  - 10 second measurement at each point
  - Sampling frequency: 25.6 kHz
  - Hann window: 0.25 second 50% overlap
  - 100 averages
Sound Intensity Reconstruction at 800 Hz

**SONAH**

**WBH**

**Measurement**

<table>
<thead>
<tr>
<th>Method</th>
<th>Measurement (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SONAH</td>
<td>40.8 dB</td>
</tr>
<tr>
<td>WBH</td>
<td>27.9 dB</td>
</tr>
<tr>
<td>L1-norm minimization</td>
<td>37.9 dB</td>
</tr>
<tr>
<td>Hybrid method</td>
<td>37.8 dB</td>
</tr>
</tbody>
</table>
Sound Intensity Reconstruction at 2400 Hz

<table>
<thead>
<tr>
<th>Method</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>SONAH</td>
<td>43.2 dB</td>
</tr>
<tr>
<td>WBH</td>
<td>38.2 dB</td>
</tr>
<tr>
<td>L1-norm minimization</td>
<td>45.1 dB</td>
</tr>
<tr>
<td>Hybrid method</td>
<td>45.6 dB</td>
</tr>
</tbody>
</table>
Sound Intensity Reconstruction at 4000 Hz

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SONAH</td>
<td>36.8 dB</td>
</tr>
<tr>
<td>WBH</td>
<td>38.1 dB</td>
</tr>
<tr>
<td>L1-norm minimization</td>
<td>42.7 dB</td>
</tr>
<tr>
<td>Hybrid method</td>
<td>43.6 dB</td>
</tr>
</tbody>
</table>
Sparse ESM algorithms are able to identify major sound sources with relatively small number of measurements without spatial aliasing.

SONAH reconstructed most accurate sound field when the number of microphones is large enough.