A Review of Theories for Sound Transmission through Infinite Double Panels and Identification of Asymptotic Behavior

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A REVIEW OF THEORIES FOR SOUND TRANSMISSION THROUGH INFINITE DOUBLE PANELS AND IDENTIFICATION OF ASYMPTOTIC BEHAVIOR

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Contents

- **Review of Existing Theories**
  Classic models by Beranek and Work, London, Mulholland et al., Heckl, Fahy, and Hamada and Tachibana

- **Asymptotic Behavior**
  Asymptotic behavior of double-panel systems with stiff panels

- **Porous Lining and Average Transmission Loss**
  Effect of limp porous lining and the average transmission loss over a range of incidence angles

- **Conclusion**

Sound transmission through an infinite double-panel systems
Existing Theories

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Different panel impedance leads to,

- Limp panel
- Limp panel + Constant resistance
- Stiff panel + Constant resistance
- Stiff panel + Hysteric resistance
Existing Theories

- Beranek and Work’s equation for a double-limp-panel system without resistance \((Z = j\omega m)^1\),

\[
p_0/p_5 = \left( \cos kd - \frac{\omega m_1}{\rho c} \sin kd \right) + j \left[ \sin kd + \frac{\omega (m_1 + m_2)}{\rho c} \cos kd - \frac{\omega^2 m_1 m_2}{\rho^2 c^2} \sin kd \right]
\]

- Normal incidence
- Different panels
- The total sound pressure ratio can be transferred to transmission coefficient\(^3\).
- This limp panel impedance can be substituted into other models, i.e. London’s model.
Existing Theories

- London’s expression of transmission coefficient for identical panels

\[
T = \frac{p_t}{p_i} = 1\left[ 1 + \frac{Z_w \cos \theta}{\rho c} + \frac{Z_w^2 \cos^2 \theta}{4 \rho^2 c^2} \left( 1 - e^{-j2kd \cos \theta} \right) \right]
\]

- A generalized version of London’s model

\[
|T|^2 = \left| 1 + \frac{Z_1 + Z_2}{2 \rho c} \cos \theta + \frac{Z_1 Z_2 \cos^2 \theta}{4 \rho^2 c^2} \left( 1 - e^{-2jkd \cos \theta} \right) \right|^2
\]

- Oblique incidence
- Different panels
Existing Theories

- Mulholland et al. derived multiple-reflection theory\(^4\),

\[ |T|^2 = \left| \frac{x^2}{1 - (1 - x)^2 e^{-j2kd \cos \theta}} \right|^2 \]

- A generalized version derived from\(^9\),

\[ |T|^2 = \left| \frac{x_1 x_2}{1 - (1 - x_1)(1 - x_2)e^{-j2kd \cos \theta}} \right|^2 \]

- Equivalent to Beranek and Work’s and London’s model\(^4\)
Existing Theories

- Mulholland et al. extended Beranek and Work’s method\textsuperscript{3},

\[
T = \left[ \frac{2 \rho_2 c_2 \cos \theta_1}{(Z_f \cos \theta_1 + \rho_1 c_1) \cos \theta_2} \right] \left[ \frac{\cosh \Phi}{\sinh(\jmath k_2 d \cos \theta_2 + \Phi)} \right] \left[ \frac{\rho_1 c_1}{\rho_1 c_1 + j \omega \rho_1 \rho_2 c_2 \cos \theta_2} \right]
\]

- Oblique incidence
- Different media and panels

\[
\Phi = \text{arccoth}\left[ \frac{(j \omega \rho_2 \rho_1 \rho_1 c_1 \cos \theta_2)}{\rho_2 c_2} \right] \frac{\cos \theta_1}{\cos \theta_1}
\]

\[
Z_f = \frac{\rho_2 c_2 \coth(j k_2 d \cos \theta_2 + \Phi) + j \omega m_1 \cos \theta_2}{\cos \theta_2}
\]
Existing Theories

\[ m_1 = 7 \text{ kg/m}^2 \quad m_2 = 7 \text{ kg/m}^2 \quad d = 0.23 \text{ m} \]

• Governed by mass law – 40 dB/dec
• Minima of transmission loss go to zero
• Resonances shift to higher frequencies at oblique incidence
Existing Theories

- double-limp-panel
  \[ m_1 = 28 \text{ kg/m}^2 \quad m_2 = 7 \text{ kg/m}^2 \quad d = 0.23 \text{ m} \]

- double-limp-panel at \( \theta_i = \pi/4 \)
  \[ m_1 = 28 \text{ kg/m}^2 \quad m_2 = 7 \text{ kg/m}^2 \quad d = 0.23 \text{ m} \]

- Governed by mass law – 40 dB/dec
- Minima of transmission loss do not go to zero
- Resonances shift to higher frequencies at oblique incidence
**Existing Theories**

With hydrogen between panels and air outside
\[ \rho_2 = 0.08988 \text{ kg/m}^3 \quad c_2 = 1270 \text{ m/s} \]

- double-limp-panel
  \[ m_1 = 15 \text{ kg/m}^2 \quad m_2 = 15 \text{ kg/m}^2 \quad d = 0.23 \text{ m} \]

\[ \text{at } \theta_i = \pi/24 \]

\[ m_1 = 15 \text{ kg/m}^2 \quad m_2 = 15 \text{ kg/m}^2 \quad d = 0.23 \text{ m} \]

- Higher sound speed shifts resonances to higher frequencies
Existing Theories

- **Fahy’s expression of transmission coefficient**\(^6\),

\[
T = -\frac{2j\rho^2c^2\sec^2\theta \sin(kd \cos \theta)}{z'_1 z'_2 \sin^2(kd \cos \theta) + \rho^2c^2\sec^2\theta}
\]

\[
z' = j\omega m + r + \rho c \sec \theta [1 - j \cot(kd \cos \theta)]
\]

  - panel impedance
  - limp panel + constant resistance

- **London’s panel impedance**\(^2\)

\[
Z_w = \frac{2r}{\cos \theta} + j\omega m \left(1 - \frac{f^2}{f_{c}^2} \sin^4 \theta\right)
\]

  - critical frequency
  - stiff panel + constant resistance
**Existing Theories**

With Fahy’s model and panel impedance:

\[
\begin{align*}
\text{double-limp-panel} & \\
 m &= 15 \text{ kg/m}^2 & r &= 1000 \text{ kg/m}^2\text{s} & d &= 0.23 \text{ m} \\
\end{align*}
\]

\[
\begin{align*}
\text{double-limp-panel at } \theta_i &= \pi/4 \\
 m &= 15 \text{ kg/m}^2 & r &= 1000 \text{ kg/m}^2\text{s} & d &= 0.23 \text{ m} \\
\end{align*}
\]

- Resistance in formulations means that minima do not go to zero
Existing Theories

With London’s model and panel impedance

\[ \text{double-stiff-panel} \]
\[ m = 5 \text{ kg/m}^2 \quad r = 500 \text{ kg/m}^2\text{s} \quad d = 0.6 \text{ m} \]
\[ f_c = 1062 \text{ Hz} \]

\[ \text{double-stiff-panel at } \theta_i = \pi/6 \]
\[ m = 5 \text{ kg/m}^2 \quad r = 500 \text{ kg/m}^2\text{s} \quad d = 0.6 \text{ m} \]
\[ f_c = 1062 \text{ Hz} \]

- Minimum at coincidence frequency at oblique incidence
- Mass law no longer applies at frequencies higher than coincidence frequency
Existing Theories

- Heckl’s model with locally-reacting material between panels\(^5\),

\[
|T|^2 = \left| \frac{1}{1 - \omega^2 \frac{m'_1 + m'_2}{2s}} + j\omega \frac{m'_1 + m'_2}{2Z} \left( 1 - \omega^2 \frac{m'_1 m'_2}{s(m'_1 + m'_2)} + \frac{Z^2}{s(m'_1 + m'_2)} \right) \right|^2
\]

\(Z = \rho c / \cos \theta\)

- Stiffness per unit area in between

\(m' = m[1 - (k^4 D / \omega^2 m) \sin^4 \theta]\)

\(D = D'(1 + j\eta)\)

- Stiff panel + hysteretic resistance

- Loss factor

\(\text{Noise-} \text{Con 2019, San Diego, CA}\)
**Existing Theories**

With Heckl’s model

\[ m_1 = 8 \text{ kg/m}^2 \quad D_1 = 20000 \text{ kg} \cdot \text{m}^2/\text{s}^2 \quad \eta_1 = 0.0015 \]
\[ m_2 = 16 \text{ kg/m}^2 \quad D_2 = 10000 \text{ kg} \cdot \text{m}^2/\text{s}^2 \quad \eta_2 = 0.001 \]
\[ d = 0.6 \text{ m} \quad s = 1 \times 10^7 \text{ kg/s}^2\text{m}^2 \]

• No wave propagation between panels, so inter-panel resonances are absent

\[ m_1 = 8 \text{ kg/m}^2 \quad D_1 = 20000 \text{ kg} \cdot \text{m}^2/\text{s}^2 \quad \eta_1 = 0.0015 \]
\[ m_2 = 16 \text{ kg/m}^2 \quad D_2 = 10000 \text{ kg} \cdot \text{m}^2/\text{s}^2 \quad \eta_2 = 0.001 \]
\[ d = 0.6 \text{ m} \quad s = 1 \times 10^7 \text{ kg/s}^2\text{m}^2 \]
Existing Theories

- Hamada and Tachibana’s transfer matrix method\textsuperscript{7},

\[
\mathbf{F}_\theta = \begin{bmatrix}
A_\theta & B_\theta \\
C_\theta & D_\theta
\end{bmatrix} = \mathbf{F}_{1\theta} \mathbf{F}_{A\theta} \mathbf{F}_{2\theta}
\]

The matrices for panels,

\[
\mathbf{F}_{i\theta} = \begin{bmatrix}
1 & Z_{i\theta}
\end{bmatrix} \text{ panel impedance}
\]

The matrix for the air gap,

\[
\mathbf{F}_{A\theta} = \begin{bmatrix}
\cos(kd \cos \theta) & \frac{j \rho c}{\cos \theta} \sin(kd \cos \theta) \\
\frac{j \cos \theta}{\rho c} \sin(kd \cos \theta) & \cos(kd \cos \theta)
\end{bmatrix}
\]
Asymptotic Behavior

- The stiffness + hysteretic damping impedance introduced by Cremer\textsuperscript{10},

\[ Z_w = \frac{\eta D}{\omega} k^4 \sin^4 \theta + j(m\omega - \frac{D}{\omega} k^4 \sin^4 \theta) \]

Substitute into generalized London’s model,

\[
\left| \frac{1}{T} \right|^2 = (1 + \alpha (\Re Z_1 + \Re Z_2) + \alpha^2 [(1 - \cos 2\beta)(\Re Z_1 \Re Z_2 - \Im Z_1 \Im Z_2) - \sin 2\beta (\Re Z_1 \Im Z_2 + \Re Z_2 \Im Z_1)])^2 + \{\alpha (\Im Z_1 + \Im Z_2) + \alpha^2 [(1 - \cos 2\beta)(\Re Z_1 \Im Z_2 + \Re Z_2 \Im Z_1) + \sin 2\beta (\Re Z_1 \Re Z_2 - \Im Z_1 \Im Z_2)]\}^2
\]

with \( \alpha = \cos \theta / 2 \rho c, \beta = kd \cos \theta \)
Asymptotic Behavior

- when $\cos 2\beta = -1$ (maxima of transmission loss)

$$\left| \frac{p_i}{p_t} \right|^2 \approx 4\alpha^4 \left[ \left( \frac{\eta_1 D_1 k_x^4}{\omega} \right)^2 + \left( \omega m_1 - \frac{D_1 k_x^4}{\omega} \right)^2 \right] \left[ \left( \frac{\eta_2 D_2 k_x^4}{\omega} \right)^2 + \left( \omega m_2 - \frac{D_2 k_x^4}{\omega} \right)^2 \right] = O(\omega^{12})$$

$k_x = k \sin \theta$

corresponding to $d/\lambda = 1/4, 3/4, 5/4, \text{etc. at normal incidence}$

- when $\cos 2\beta = 1$ (minima of transmission loss)

$$\left| \frac{p_i}{p_t} \right|^2 = \left[ 1 + \frac{\alpha k_x^4}{\omega} \left( \eta_1 D_1 + \eta_2 D_2 \right) \right]^2 + \alpha^2 \left[ \omega (m_1 + m_2) - \frac{k_x^4}{\omega} (D_1 + D_2) \right]^2 = O(\omega^6)$$

$60 \text{ dB/dec}$

corresponding to $d/\lambda = 1/2, 1, 3/2, \text{etc. at normal incidence}$
Asymptotic Behavior

\[ m_1 = 20 \text{ kg/m}^2 \quad D_1 = 10000 \text{ kg} \cdot \text{m}^2/\text{s}^2 \quad \eta_1 = 0.002 \]
\[ m_2 = 20 \text{ kg/m}^2 \quad D_2 = 10000 \text{ kg} \cdot \text{m}^2/\text{s}^2 \quad \eta_2 = 0.002 \]
\[ d = 0.01 \text{ m} \quad f_{c1} = f_{c2} = 823 \text{ Hz} \]

\[ m_1 = 15 \text{ kg/m}^2 \quad D_1 = 9000 \text{ kg} \cdot \text{m}^2/\text{s}^2 \quad \eta_1 = 0.003 \]
\[ m_2 = 30 \text{ kg/m}^2 \quad D_2 = 12000 \text{ kg} \cdot \text{m}^2/\text{s}^2 \quad \eta_2 = 0.001 \]
\[ d = 0.01 \text{ m} \quad f_{c1} = 751 \text{ Hz} \quad f_{c2} = 920 \text{ Hz} \]
Asymptotic Behavior

\[ \text{at } \theta_i = \pi/6 \]

\[ m_1 = 20 \text{ kg/m}^2 \quad D_1 = 20000 \text{ kg} \cdot \text{m}^2/\text{s}^2 \quad \eta_1 = 0.1 \]

\[ m_2 = 20 \text{ kg/m}^2 \quad D_2 = 20000 \text{ kg} \cdot \text{m}^2/\text{s}^2 \quad \eta_2 = 0.1 \]

\[ d = 0.01 \text{ m} \]
Asymptotic Behavior

\[ \theta_i = \pi / 6 \]

\[ m_1 = 20 \text{ kg/m}^2 \quad D_1 = 20000 \text{ kg} \cdot \text{m}^2/\text{s}^2 \quad \eta_1 = 0.01 \]
\[ m_2 = 20 \text{ kg/m}^2 \quad D_2 = 20000 \text{ kg} \cdot \text{m}^2/\text{s}^2 \quad \eta_2 = 0.01 \]
\[ d = 0.01 \text{ m} \]
Asymptotic Behavior

\[ \text{at } \theta_i = \pi/6 \]

\[ m_1 = 20 \text{ kg/m}^2 \quad D_1 = 20000 \text{ kg} \cdot \text{m}^2/\text{s}^2 \quad \eta_1 = 0.001 \]

\[ m_2 = 20 \text{ kg/m}^2 \quad D_2 = 20000 \text{ kg} \cdot \text{m}^2/\text{s}^2 \quad \eta_2 = 0.001 \]

\[ d = 0.01 \text{ m} \]
Asymptotic Behavior

\[ | \text{at } \theta_i = \pi/6 \]

\[ m_1 = 20 \text{ kg/m}^2 \quad D_1 = 20000 \text{ kg} \cdot \text{m}^2/\text{s}^2 \quad \eta_1 = 1 \times 10^{-4} \]
\[ m_2 = 20 \text{ kg/m}^2 \quad D_2 = 20000 \text{ kg} \cdot \text{m}^2/\text{s}^2 \quad \eta_2 = 1 \times 10^{-4} \]
\[ d = 0.01 \text{ m} \]

The 60 dB/dec line shifts toward high frequencies as loss factor decreases.
Porous Lining

- The resistance in system will suppress the dips in transmission loss.

\[ | | \text{at } \theta_i = \pi/6 \]

\[ m = 20 \text{ kg/m}^2 \quad D = 10000 \text{ kg} \cdot \text{m}^2/\text{s}^2 \quad \eta_1 = 0 \]

\[ d = 0.01 \text{ m} \]

Resistance brought by imaginary part of wavenumber.

So we are primarily interested in the maximum transmission loss behavior.
Porous Lining

- A layer of porous material described with,

<table>
<thead>
<tr>
<th>Flow Resistivity</th>
<th>Porosity</th>
<th>Tortuosity</th>
<th>VCL</th>
<th>TCL</th>
<th>Solid Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.5 \times 10^5$ rayls/m</td>
<td>0.97</td>
<td>1.5</td>
<td>20 μm</td>
<td>40 μm</td>
<td>2000 kg/m³</td>
</tr>
</tbody>
</table>

- Effective density and wavenumber calculated with JCA-Limp model$^{11,12}$

\[
\sqrt{120 \text{ dB/dec}}
\]

\[
m = 20 \text{ kg/m}^2 \quad D = 10000 \text{ kg} \cdot \text{m}^2/\text{s}^2 \quad \eta_1 = 0
\]

\[
d = 0.01 \text{ m}
\]

- The increase rate is now greater than 120 dB/dec
Kang et al. proposed an approach of calculating average transmission loss\(^{12}\)

\[
\tau(\omega) = \frac{\int_0^{\pi/2} G(\theta) |T(\omega, \theta)|^2 \sin \theta \cos \theta \, d\theta}{\int_0^{\pi/2} G(\theta) \sin \theta \cos \theta \, d\theta}
\]

A distribution function for incident energy versus incidence angle is applied

\[
G(\theta) = e^{-\zeta \theta^2}
\]

The average transmission loss

\[
TL(\omega) = 10 \log_{10} \left[ \frac{1}{\tau(\omega)} \right]
\]
Average Transmission

- With $\zeta = 1.5$, the average transmission loss of the double panel system with porous material inside in the previous case was calculated,

- A drop of transmission loss occurs at critical frequency $f_c = 823$ Hz
Conclusions

- Classic models were reviewed

- Asymptotic behavior of double-stiff-panel systems at oblique incidence were studied
  - The peaks of the transmission loss increases at 120 dB/dec
  - The minima of the transmission loss increases at 60 dB/dec
  - The minima shift to higher frequencies as hysteretic damping decreases

- Porous lining between panels will suppress the resonance pattern of double panels and change the transmission loss increase rate

- Average transmission loss was obtained with Gaussian distribution applied
Reference