1976

A Formula for Estimating Dynamic Pressure Changes in Compressor Suction and Discharge Plenums

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INTRODUCTION

In high speed compressors it is somewhat difficult to avoid large pressure changes in front of the suction valve and behind the discharge valve. These pressure changes do not only reduce the mass flow rate (poor volumetric efficiency) but also increase the required power (poor power efficiency).

Previous investigations by the author and his students [1,2,3,4,5,6,7,8] and by others (a partial listing is given in references [9,10] have shown that these pressure changes are less and less dominated by friction effects as the compressor speed increases and more and more by inertial effects. The valve opening times are so short that a large amount of mass is dumped rather suddenly into (or extracted from) a plenum without giving the mass at the entrance of the pipe connecting the plenum to the rest of the system a chance to move. Before the inertial resistance of this mass is overcome, rather severe pressure changes have taken place that impede the flow of the gas through the ports.

In the case of certain gases, these inertia effects may become severe at compressor speeds that would be considered "slow" by, lets say, refrigeration compressor designers. A case study illustrating this on the example of a helium compressor is presented elsewhere in these Proceedings.

The purpose of this paper is to find a way to estimate these pressure changes without resorting to a full fledged computer analysis.

THEORETICAL CONSIDERATIONS

First, we will assume that the mass flow rate into the plenum can approximately be computed knowing bore, stroke and a volumetric efficiency of the compressor based on geometry. The mass delivered per cycle is

$$m = \gamma_s \, \eta_v \, V$$

where

$$\eta_v = 1 - \frac{V_c}{V}\left[\left(\frac{p_d}{p_s}\right)^{\frac{1}{n}} - 1\right]$$

γ = specific mass at suction condition $[\frac{kg}{m^3}]$

V = swept volume $[m^3]$

V_c = clearance volume $[m^3]$

p_d, p_s = nominal discharge and suction pressure $[\frac{N}{m^2}]$

The value of the mass is obviously overestimated, but this will be partially compensated for by the assumption that the mass flow rate is constant during the duration of valve opening. We will estimate suction valves to be open

$$T_s \cong \frac{25}{n} \text{ to } \frac{30}{n}$$

where

n = rotation speed of the compressor $[\frac{rot}{min}]$

T_s = time [sec]

and discharge valves to be open

$$T_d \cong \frac{15}{n} \text{ to } \frac{25}{n}$$

The estimate will of course be dependent on pressure ratio and volumetric efficiency. More exact opening times can be calculated using the ideal p-V diagram.

Let us now consider Figure 1. It shows a discharge valve with discharge plenum and a very long discharge pipe. The displacement rate of gas entering is

$$\dot{\xi}_1 = \frac{m}{\gamma \; S_1 \; T} \; [u(t) - u(t-T)]$$

where

γ = specific mass at discharge condition $[\frac{kg}{m^3}]$

S_1 = effective valve flow area $[m^2]$

$$u(t) = \begin{cases} 0 & \text{if} \quad t < 0 \\ 1 & \text{if} \quad t \geq 0 \end{cases}$$

$$u(t-T) = \begin{cases} 0 & \text{if} \quad t < T \\ 1 & \text{if} \quad t \geq T \end{cases}$$

It will be shown that S_1 will subsequently drop out of the derivation and will thus not have to be known.

In reference [7] it was shown that the pressure increase Δp at the entrance of a long pipe where pressure wave reflections from the other end can be neglected is related to the gas displacement ξ at the pipe entrance by

$$\frac{d\xi}{dt} = \frac{\Delta p}{C\rho}$$

where

C = speed of sound $[\frac{m}{sec}]$

ρ = mean mass density = $\frac{\gamma}{g}$ $[\frac{N \; sec^2}{m^4}]$

g = local gravitational constant $\cong$ 9.81 $[\frac{m}{sec}]$

t = time [sec]

It was also shown that the pressure change Δp in the plenum is

$$\Delta p = \frac{C^2 \rho}{V} (S_1 \; \xi_1 - A \; \xi)$$

where

A = effective flow area of pipe $[m^2]$

V = volume of plenum $[m^3]$

The pressure increase Δp at the pipe entrance must be equal to the pressure increase Δp of the plenum; provided the largest plenum dimension L_{max} is much smaller than the wavelength associated with the running speed frequency of the compressor. This means that

$$\frac{60C}{n} >> L_{max}$$

This restriction is negligible for most practical cases.

Thus, we get the equation

$$\frac{d\xi}{dt} + \frac{CA}{V} \xi = \frac{CS_1}{V} \xi_1$$

or, differentiating both sides,

$$\frac{d^2\xi}{dt^2} + \frac{CA}{V} \frac{d\xi}{dt} = \frac{Cm}{V\gamma T} [u(t) - u(t-T)]$$

Note that the equation is general. If we look specifically at the discharge plenum, we introduce the subscript d where appropriate and if we analyse the suction plenum, we use the subscript S.

Solving this equation we get

$$\xi = \frac{m}{\gamma TA} [\frac{V}{CA} (e^{-\frac{CA}{V} t} - 1) + t] \; u(t)$$

$$- \frac{m}{\gamma TA} [\frac{V}{CA} (e^{-\frac{CA}{V} (t-T)} - 1) + (t-T)] \; u(t-T)$$

The plenum pressure change becomes, therefore,

$$\boxed{\Delta p = \begin{cases} = \bar{A} \; e^{-\bar{B}t} \; (e^{\bar{B}t} - 1) & 0 \leq t \leq T \\ = \bar{A} \; e^{-\bar{B}t} \; (e^{\bar{B}T} - 1) & T \leq t \end{cases}}$$

where

$$\boxed{\begin{aligned} \bar{A} &= \frac{Cm}{gTA} \\ \bar{B} &= \frac{CA}{V} \end{aligned}}$$

COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS FOR A REFRIGERATION COMPRESSOR

Reference [4] gives experimental results of pressure rise in a refrigeration discharge plenum. In this case the pipe is very long and wave reflection from its termination was probably truly absent. The parameters of the system were

$$C = 169 \; \frac{m}{sec}$$

$$A = 1.27 \; (10^{-4}) \; m^2$$

$$V = 6.19 \; (10^{-5}) \; m^3$$

$$T = 2.33 \; (10^{-3}) \; sec$$

$$m = 7.03 \; (10^{-4}) \; kg$$

$$g = 9.81 \; \frac{m}{sec^2}$$

Results are plotted in Figure 2. As one can see, the overall pressure rise is predicted excellently. What is not predicted is a resonance at about 1750 Hz that occurs in the plenum.

CONCLUSIONS

There is reason to believe that the derived formula can be useful to compressor designers. It was used in the diagnosis of a unsuitable plenum in a helium diving compressor and also in the redesign of this compressor with good success. This is presented as a case study elsewhere in these proceedings.

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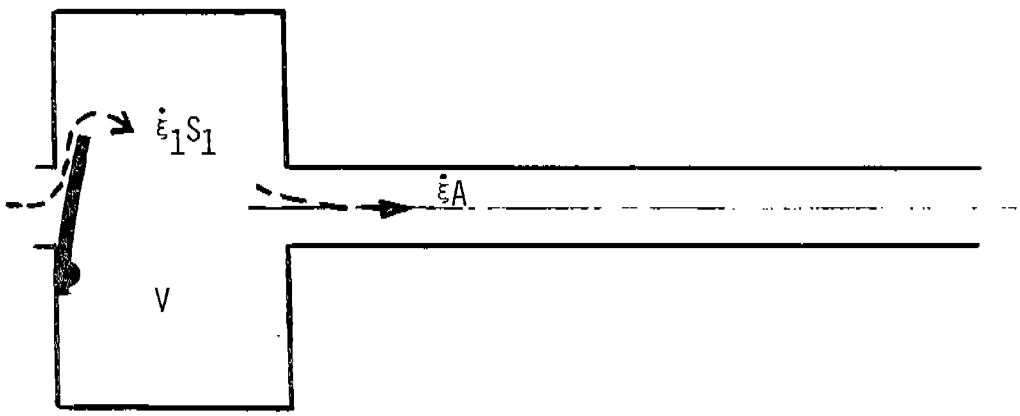

FIGURE 1 DISCHARGE SYSTEM

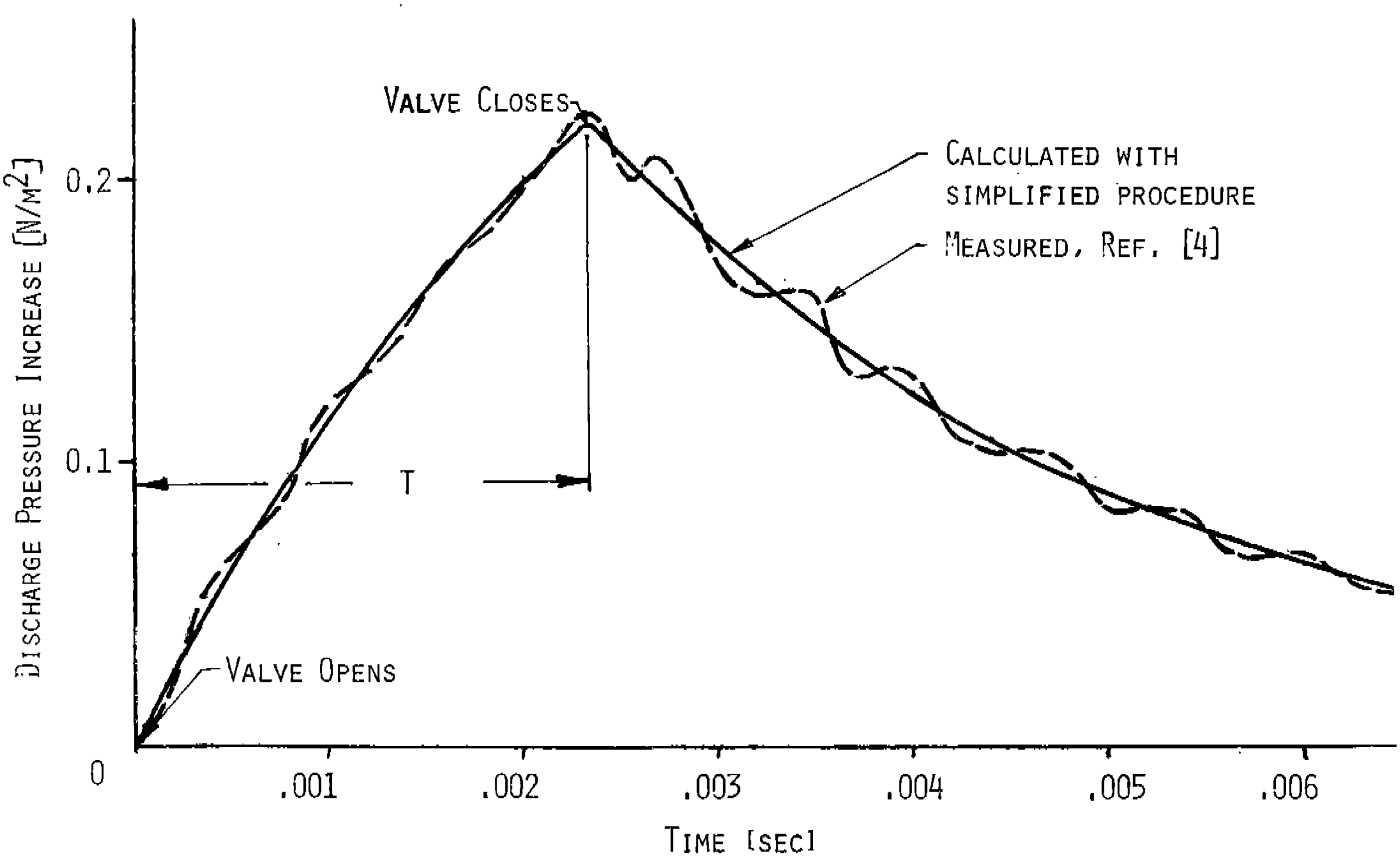

FIGURE 2 COMPARISON OF CALCULATED PLENUM PRESSURE WITH MEASURED VALUES