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CHARACTERIZATION OF REED TYPE COMPRESSOR VALVES BY THE FINITE ELEMENT METHOD

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INTRODUCTION

A primary motivation for studying dynamic behavior of compressor valves is the need to know the dynamic stress level during operating conditions. Such stresses, if sufficiently large, are likely to cause valve fatigue failure well before the serviceable lives of other compressor components are reached. This paper presents a discussion of how the finite element method (Ref. 1) might be applied to the prediction of compressor valve dynamic stress.

Flexible valves are commonly used in small refrigeration compressors for both suction and discharge functions. The valves are self-actuating in that motion is activated by the pressure loading under certain boundary (or stop) constraints. Thus, the valve motion is governed by the inertial and elastic characteristics of the valve and is not kinematically determinable.

Most flexible valves consist of a thin plate like structure of irregular shape. The stiffness and inertial characteristics of the valve must be chosen carefully if satisfactory performance is to be achieved. A valve must be flexible enough to experience rapid response to pressure imbalances and yet stiff enough to rapidly return to seal the flow port and prevent back flow. To prevent excessive flexure stresses, peak valve motion is usually limited or restrained by the use of valve stops to provide increased stiffness for large displacements. In addition to suitable stiffness and inertial characteristics, a valve must be designed so as to produce minimal flow resistance so as to not degrade overall compressor performance.

Because of these varied design restrictions, flexible valves are usually of an irregular shape, and local stress concentrations pose a real problem. Classical stress analysis methods successful in dealing with regular geometries are of limited value in these instances. The finite element method, however, can treat such irregular geometries relatively easily and in an automated fashion.

VALVE CHARACTERIZATION

In order to determine valve stress under operating conditions, dynamic loading conditions due to the refrigerant pressure must be obtained. Since this pressure is dependent upon valve motion, the most expedient method to obtain a solution is to incorporate an analytical description of valve motion with a computer simulation of the compressor cycle. (Refs. 2,3) The instantaneous valve loading can then be determined from the current valve position and the future valve position predicted. The coupled simulation proceeds in incremental fashion by the use of time integration techniques.

A convenient way to formulate the anlytical description of valve response is to assume that the valve behaves according to Kirchhoff thin plate theory and to make use to the technique of modal expansion. (Ref. 4) In this fashion, the valve motion can be expressed as

$$w(x,y,t) = \sum_{i=1}^{n} a_i(t)\ \phi_i(x,y) \qquad (1)$$

$w(x,y,t)$: transverse valve displacement

$a_i(t)$: dynamic participation factor for the i^{th} mode shape

$\phi_i(x,y)$: i^{th} dynamic mode shape of the valve

x,y : plane coordinate system

n : the number of mode shapes to be used in the solution, typically 2 or 3

Equations of motion now take the form of ordinary, time dependent, differential equations

$$\frac{d^2 a_i(t)}{dt^2} = -\omega_i^{\,2} a_i(t) + P_i(t). \quad i = 1, n \qquad (2)$$

ω_i : circular frequency of the i^{th} mode

$P_i(t)$: dynamic modal pressure corresponding to the i^{th} mode.

Equation (2) is in a form which can be integrated in time incremental fashion during the compressor simulation resulting in numerical solutions of the $a_i(t)$ functions. The dynamic stress values at any time t may now be evaluated by weighting the modal stresses for each of the n modes with the corresponding a_i value and summing the results.

The technique of modal expansion can be modified to include the effect of intermitant "stop" contact by using different sets of the mode shapes $\phi_i(x,y)$ during that portion of the simulation cycle in which stop contact exists. (Refs 2,3) This modification can be achieved by incorporating suitable logic checks in the computer simulation to test for the stop contact conditon. Velocity distributions immediately prior to contact serve as initial conditions to the integration of the new set of equations.

The time integration procedure for valve response is independent of how the valve mode shapes are determined. For an accurate set of mode shapes, the modal particpation factors can be determined to a high degree of accuracy by utilizing sophisticated time integration schemes and small time steps. Thus the accuracy of the dynamic stress results depends on the number of mode shapes used in the integration and on the accuracy with which the "stress modes" of these mode shapes are known. It is easily recognizable that small errors in the mode shapes $\phi_i(x,y)$ tend to become large errors when differentiation is performed in computing the flexure stresses. The accuracy of the dynamic stress results can degrade substantially if the mode shapes are only slightly inaccurate.

One technique to circumvent this problem is to use experimental procedures to determine accurate valve mode and stress mode shapes. (Ref. 2) In this fashion, the mode shapes needed for the time integration are measured at selected points utilizing displacement transducers. Instead of numerical differentiation to determine stress modes, strain gages are employed to make direct measurements at selected positions on the valve. This technique, however, requires prior knowledge or insight as to where large stresses are likely to occur.

USE OF THE FINITE ELEMENT METHOD TO FORMULATE VALVE EQUATIONS OF MOTION

In recent years, the finite element method has become an important solution procedure to attack difficult structural analysis problems. (Ref. 1) While many approximate methods for the solution of structural analysis problems exist, few have received such rapid acceptance in the form of financial commitment and human endeavor as has the finite element method.

One of the primary advantages which the method allows the analyst is the ability to model structures of complex or irregular geometry. This characteristic exists because of the almost arbitrary geometrical descretization permissible in finite element usage. Unlike finite difference methods, the analyst is not constrained to the use of rectangular or polar coordinate systems.

The finite element method is a computer oriented method capable of being highly automated. The structural effects of geometrical modifications can be easily assessed by making minor data changes. In addition, the automated graphical output of stresses, strains, or displacements greatly facilitates the interpretation of results. It seems that this structural analysis method is ideally suited for the purpose of analyzing compressor valve behavior.

A finite element computer program has been written with special emphasis given to the modal analysis of the geometries similar to those of most compressor valves. Particular emphasis has been given to the importance of accurately predicting and plotting dynamic strain and stress modes in addition to the displacement modes.

The program uses a triangular plate bending element with three nodal points each of which possesses six degrees of freedom. (Refs 5,6,&7) In addition, the side of the element adjacent to a curved free edge can be modeled as a series of straight lines or chords rather than the straight side only. Thus simulation of the intricate geometries does not require using more elements than are necessary for accurate strain prediction.

The rate of convergence of modal strain with respect to the number of elements used and the benefits gained over using regular triangular elements are documented in Ref. 7.

Some typical mesh configurations used to analyze suction valves are shown in Figures 1 and 4. Both models were used to analyze the first several symmetrical modes; hence the mesh domain of Figure 1 included only half of the valve geometry while that of Figure 4 included only one quarter of the valve geometry.

At each nodal point, six finite element degrees of freedom exist. These consist of the deflection, the two slopes, and the three second partial derivatives of deflection: w, w_x, w_y, w_{xx}, w_{xy}, w_{yy}. These degrees of freedom are of course associated with all elements common to a given node. Within each element, the valve deflection is assumed to be expressible as a fifth degree polynomial in the in-plane coordinates. By imposing constraints between the polynomial coefficients and the nodal degrees of freedom, deflection at any point internal to an element is now expressible as a unique function of the 18 nodal degrees of freedom associated with that element.

The infinite number of linearly independent configurations of the continuum problem has now been reduced by the descretization process to a finite number -- six times the number of nodes in the mesh. Actually, the number is somewhat less than this after boundary condition constraints are imposed on some of the nodal degrees of freedom.

All possible displacements of the finite element model (at nodal points as well as at internal element points) are now uniquely described by the nodal degrees of freedom. The practical utility of this intricate "description method" becomes apparent with the realization that system energy expressions (kinetic and potential) are also expressible in terms of nodal degree of freedom values. After making use of variational principles such as Lagrange's equations or the Raleigh-Ritz method, the equations of motion for the finite element model become:

$$[K] \{w\} + [M] \{\ddot{w}\} = \{F\ (t)\} \qquad (3)$$

In this expression, [K] represents the stiffness matrix computed from the finite element strain energy expression. The mass matrix [M] is computed from the finite element kinetic energy expression, and the load vector $\{F(t)\}$ is computed from the potential energy expression of the externally applied loading. $\{w\}$ represents the global degree of freedom vector of nodal displacements.

The eigenvalue formulation for extracting valve mode shapes and frequencies is simply:

$$[K] \{w\} = \omega^2\ [M]\ \{w\} \qquad (4)$$

After solving equation (4) for the eigenvalues and eigenvectors of interest, the displacement modes and modal stresses can be obtained by transforming from the eigenvectors back to the element polynomial coefficients and, in the case of modal stresses, by performing suitable differentiation.

EXAMPLES

Finite element eigenvalue formulations using the technique just discussed have been performed on several production model suction valves. The first valve (see Figure 1) is a cantilevered plate structure. Large bending stresses for the first mode can be expected in the clamped region since this is probably where substantial bending takes place. Stresses should decrease toward the free end of the valve. These trends are observable in the plotted output of Figure 2. The plot for the second mode, Figure 3, clearly shows the stress effects of curvature reversal between the clamped support and the zero deflection node line. Also noticeable in the second mode are some transverse bending effects.

Figures 4, 5 and 6 show plotted output from a formulation of a ring type suction valve. A 90 degree model was used to extract only the symmetrical modes. In the first mode stress plot, it can be clearly seen that circumferential bending stresses are much larger than the radial stress components. This effect tends to decrease for higher modes as can be seen in Figure 6.

CONCLUSIONS

The difficulties of modeling dynamic structural behavior of compressor valves can be alleviated by making use of the finite element method in conjunction with a compressor simulation model. One of the main advantages which the finite element method possesses over other techniques is that intricate valve geometries may be handled in an automated fashion.

Due to the automated nature of computer aided design techniques, it would seem that the investigation of valve fatigue failures could be expedited by using a compressor simulation program which utilizes the finite element method to model valve structural behavior.

By employing computer graphics techniques in a program such as this, the highly automated and economical investigations could be performed on a variety of contemplated valve design configurations. Such a design method would reduce the amount of time and effort normally required for eliminating fatigue problems associated with compressor valves.

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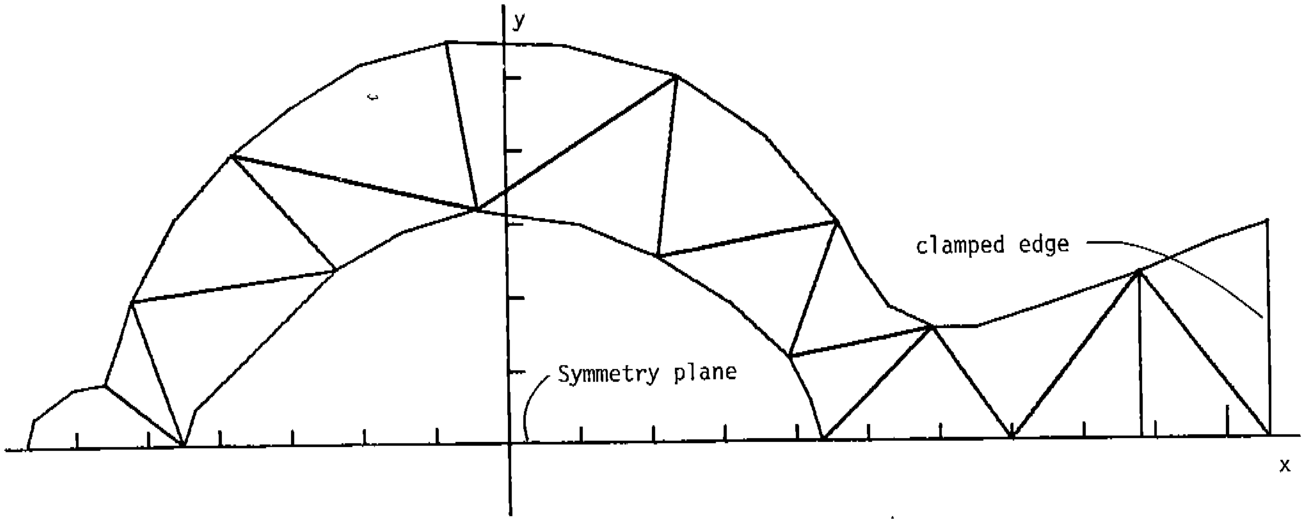

FIGURE 1. Finite element mesh used for the beam valve example

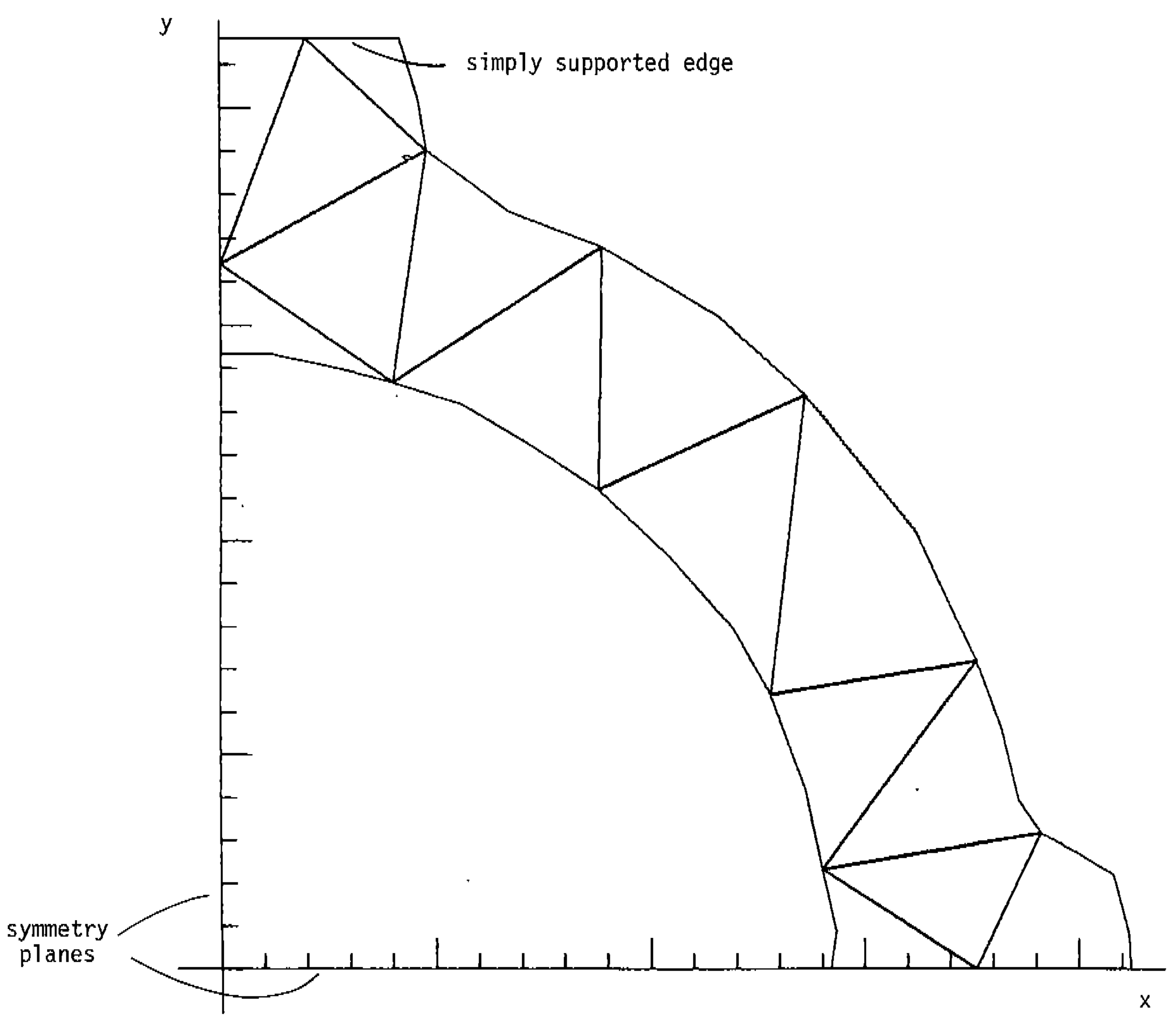

FIGURE 4. Finite element mesh used for the ring valve example

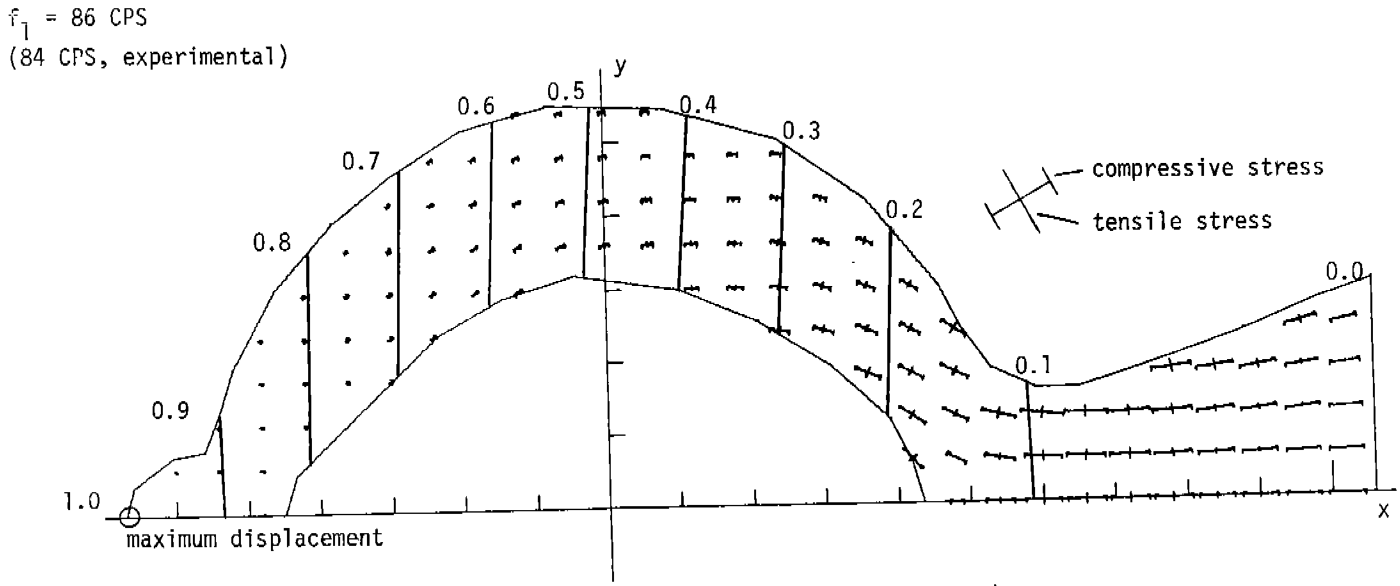

FIGURE 2. Finite element results for the beam valve example -- first symmetrical mode

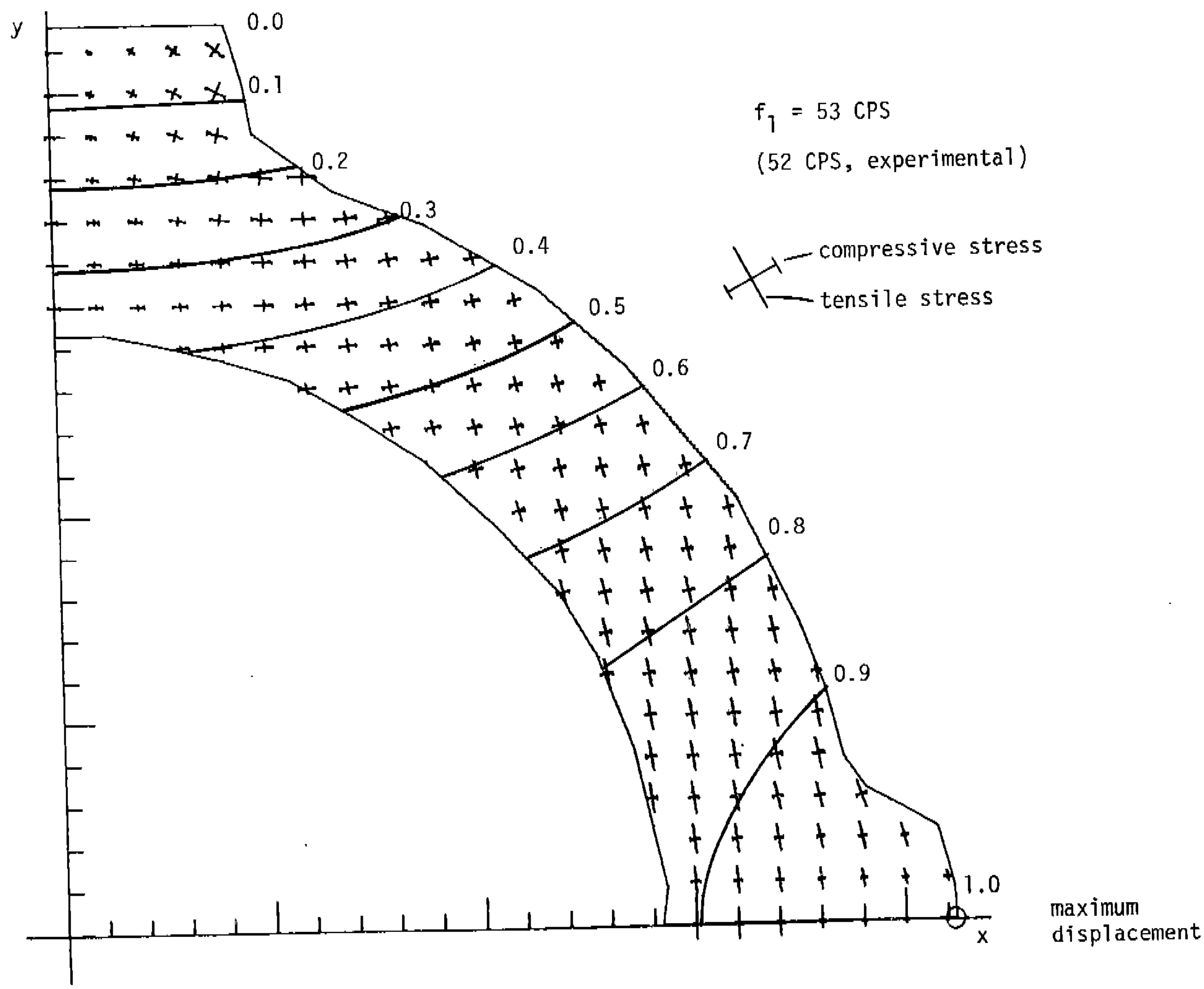

FIGURE 5. Finite element results for the ring valve example -- first symmetrical mode

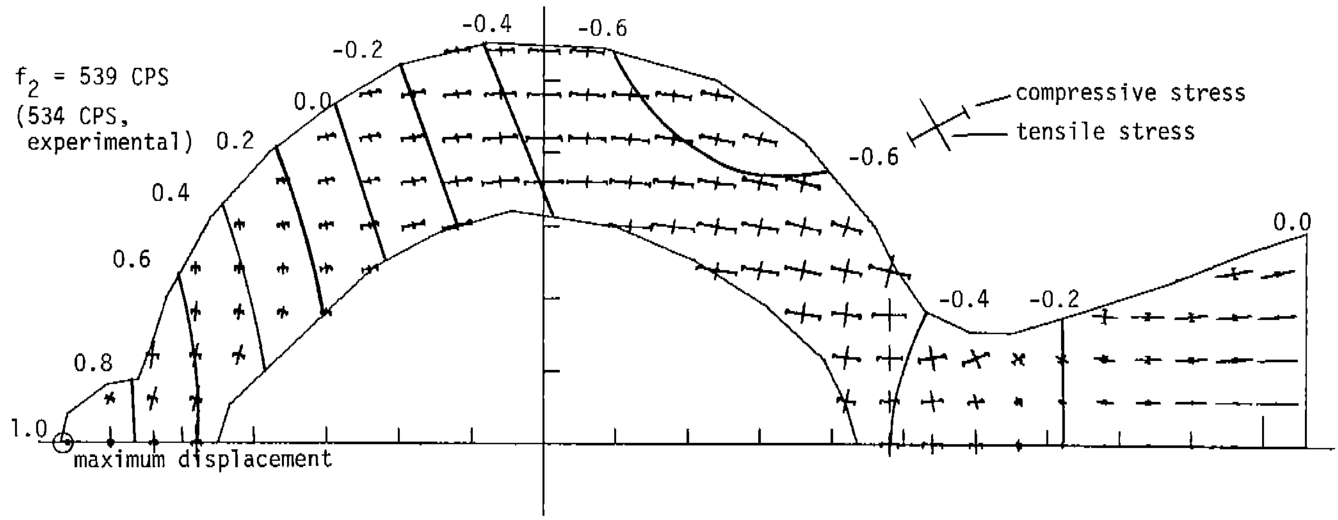

FIGURE 3. Finite element results for the beam valve example -- second symmetrical mode.

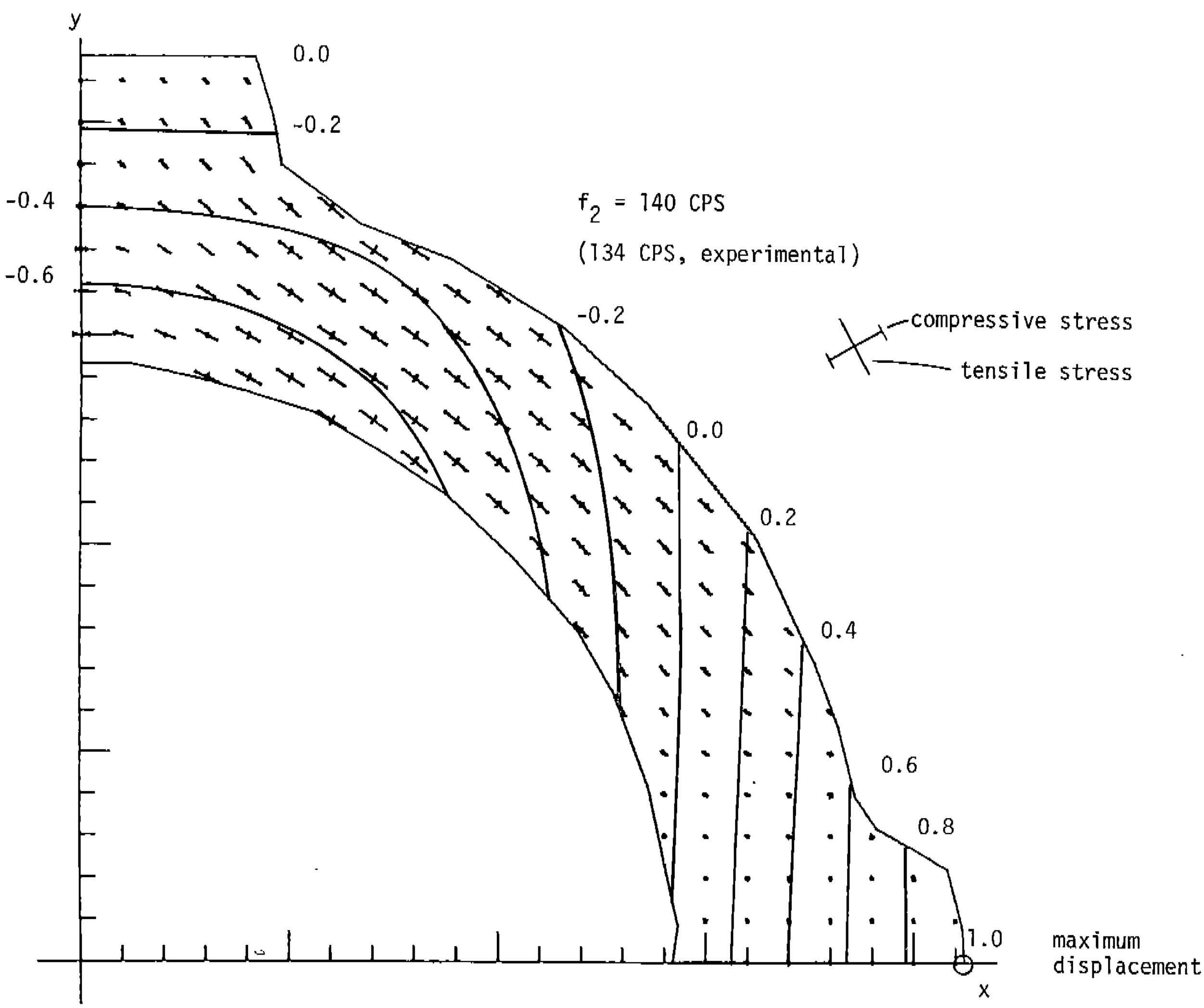

FIGURE 6. Finite element results for the ring valve example -- second symmetrical mode