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Accurate Experimental Determination of Frequencies, Mode Shapes, and Dynamic Strains in Plate Valves of Reciprocating Compressors

H. K. Reddy

J. F. Hamilton

ACCURATE EXPERIMENTAL DETERMINATION OF FREQUENCIES, MODE SHAPES AND DYNAMIC STRAINS IN PLATE VALVES OF RECIPROCATING COMPRESSORS

Harsha K. Reddy, Engineer
Advanced Reactors Division
Westinghouse Electric Corporation
Madison, Pennsylvania

James F. Hamilton, Professor
Ray W. Herrick laboratories
Purdue University
Lafayette, Indiana

INTRODUCTION

One of the components of a compressor which fails frequently is the valve, with most valve failures due to metal fatigue. It is important to have some method of predicting valve life under operating conditions. One method of predicting valve life is to determine the valve stress levels by computer simulation of the valve dynamics and relate these stress levels to the fatigue life.

The valve dynamics can be expressed as a linear combination of the normal displacement modes of vibration of the valve. The flexural strains in the valve can then be expressed as a linear combination of the first and second order spatial derivatives of the displacement modes. If the valve modes are derived analytically then the strain in the valve can be determined quite accurately. However, for all but the simplest valve geometries analytical derivation of the modes is extremely difficult, if not impossible, and one must resort to experimental methods to determine the displacement mode shapes. Since the strain values are proportional to the derivatives of the displacement modes, local inaccuracies in the experimental data are highly magnified especially in the regions of high strain gradients. A method to obtain directly modal strain values for the valves (1) is utilized here to overcome this problem. The size of the valves is usually so small that it is difficult to obtain the necessary accuracy in the modal information from experimental tests. A method is utilized in this work that uses scaled up models of the valves to obtain accurate modal data (2).

MATHEMATICAL MODELING TECHNIQUE FOR RING VALVE PLATES

The technique of the computer simulation is described in detail in (1) and (2).and will not be dealt in here. Most computer simulations utilize valve dynamics in terms of the modal expansion technique whereby the dynamic deflection, $W(r,\theta,t)$ of the ring plate valve is given in polar coordinates, for example, by

$$W(r,\theta,t) = \sum_{n=1}^{\infty} \phi_n(r,\theta) \cdot q_n(t) \qquad (1)$$

where $\phi_n(r,\theta)$ are the necessary displacement modes and $q_n(t)$ are the participation factors obtained from a typical modal equation

$$\ddot{q}_n(t)+2\xi\omega_n\dot{q}_n(t)+\omega_n^2 q_n(t)= \frac{F_n(t)}{M_n} \qquad (2)$$

where

ξ = damping coeffient
ω_n = natural frequency of the n^{th} mode
$F_n(t)$ = generalized modal force
M_n = generalized modal mass

The dynamic strains in the tangential and radial directions are generally given as derivatives of the dynamic deflections as

$$\varepsilon_\theta(r,\theta,t) = -c\left[\frac{1}{r^2} \frac{\partial^2 W}{\partial \theta^2} + \frac{1}{r} \frac{\partial W}{\partial r} \right] \qquad (3)$$

$$\varepsilon_r(r,\theta,t) = -c.\frac{\partial^2 W}{\partial r^2} \qquad (4)$$

where

ε_θ = strain in the tangential direction
ε_r = strain in the radial direction
c = plate thickness
r = radial coordinate
W = deflection normal to the plate surface
θ = tangential coordinate

Substitution of Eqn.(1) in Eqn.(3) and Eqn.(4) presents the strains in terms of the derivatives of the deflection modes as

$$\varepsilon_\theta(r,\theta,t) = -c \sum_{n=1}^{\infty} q_n(t) \left[\frac{1}{r^2}\frac{\partial^2\phi_n}{\partial\theta^2} + \frac{1}{r}\frac{\partial\phi_n}{\partial r}\right] \quad (5)$$

$$\varepsilon_r(r,\theta,t) = -c \sum_{n=1}^{\infty} q_n(t) \frac{\partial^2\phi_n(r,\theta)}{\partial r^2} \quad (6)$$

STRAIN MODE RATIO

Eqn. (5) can be rewritten as

$$\varepsilon_\theta(r,\theta,t) = c \sum_{n=1}^{\infty} q_n(t)\, \phi''_{n_\theta}(r,\theta) \quad (7)$$

where

$$\phi''_n(r,\theta) = -\left[\frac{1}{r^2}\frac{\partial^2\phi_n}{\partial\theta^2} + \frac{1}{r}\frac{\partial\phi_n}{\partial r}\right]$$

Eqn. (7) shows that strain is a function of the first and second derivatives of the deflection function, $\phi_n(r,\theta)$. For valve plates of such geometry and boundary conditions that analytical solutions to $\phi_n(r,\theta)$ can be obtained, the calculation of strain, ε, from Eqn.(7) poses no problem as far as accuracy is concerned. But if the valve configuration is such that an analytical solution for $\phi_n(r,\theta)$ cannot be obtained, then the strain must be calculated from experimentally measured deflection functios, $\phi_n(r,\theta)$. If the measurement of $\phi_n(r,\theta)$ is slightly in error, then any derivative of these $\phi_n(r,\theta)$ will be in much greater error. In such cases it seems logical to measure strains on the surface of the plate valves experimentally.

Adams (1) defined a nondimensionalized quantity called the Strain Mode Ratio as

$$S_n(r_i,\theta_i) = \frac{E_n(r_i,\theta_i).}{h.W_n(r_j,\theta_j)} \quad (8)$$

where

$S_n(r_i,\theta_i)$ = strain mode ratio which is characteristic of the location (r_i,θ_i) for the n^{th} mode

$E_n(r_i,\theta_i)$ = measured strain at location (r_i,θ_i) on the plate for the n^{th} mode for a corresponding deflection $W(r_j,\theta_j)$ at location (r_j,θ_j) on the valve plate

h = thickness of plate, in.

α = characteristic dimension of the plate

$W_n(r_j,\theta_j)$ = measured deflection at location $(r_j, _j)$ for the nth. mode

The values of E_n and W_n are measured experimentally during the excitation of the valve at a natural frequency.

Knowing the strain mode ratio for the particular valve plate and the modal deflections one can calculate the strain at a point on the valve plate.

The modal deflection at the point (r_j,θ_j) is

$$W_n(r_j,\theta_j,t) = q_n(t).\phi_n(r_j,\theta_j) \quad (9)$$

The modal strain at the point, (r_i,θ_i) at which the strain mode ratio is known is

$$\varepsilon_n(r_i,\theta_i,t) = S_n(r_i,\theta_i).\frac{h}{\alpha^2}.W_n(r_j,\theta_j,t) \quad (10)$$

Substituting Eqn.(9) in Eqn.(10)

$$\varepsilon_n(r_i,\theta_i,t) = \frac{h}{\alpha^2} S_n(r_i,\theta_i)q_n(t)\ {}_n(r_j,\theta_j) \quad (11)$$

The total strain at (r_i,θ_i) due to the contribution of all the modes is

$$\varepsilon(r_i,\theta_i,t) = \sum_{n=1}^{\infty} \frac{h}{\alpha^2} S_n(r_i,\theta_i)\phi_n(r_j,\theta_j)q_n(t) \quad (12)$$

It can be seen that can be computed if $S_n(r_i,\theta_i)$ and $\varepsilon(r_j,\theta_j)$ are known, where ε is either the radial or tangential strain.

STOP FORCE RATIO

In compressors with high flow rates the valve deflections can be excessively high. To limit the maximum deflections valve stops are provided. A simple configuration is shown in Fig. (1) for the case of a reed valve. During the history of its motion the valve can be in two states:

1) valve lifts off the seat and is between seat and stop - state 1.

2) valve hits the stop and remains against the stop - state 2.

3) valve comes off the stop and is again between seat and stop - state 1 again

In modeling the valve dynamics it is necessary to know at what point in time the boundary conditions change. It is necessary to know when the valve contacts the stop, which can be determined by monitoring the deflection at the stop point of the valve. When the deflection equals the stop depth it indicates that the valve has come in contact with the stop and the boundary conditions should be changed to reflect the new valve dynamics.

To find when the valve departs the stop it is necessary to monitor the shear force in the valve tip. The shear force can be calculated in terms of the third derivative of the deflections modes and thus requires extreme accuracy in the knowledge of the deflection modes. If the deflection mode information is not extremely accurate the third derivative of such information will be highly inaccurate.

In order to overcome this problem the shear force can also be measured experimentally, and similar to the strain mode ratio a nondimensionalized quantity called the "Stop Force Ratio" can be defined as:

$$F_n(r_k,\theta_k) = \frac{f_n(r_k,\theta_k).\alpha^2}{D.W_n(r_j,\theta_j)} \qquad (13)$$

where

$f_n(r_k,\theta_k)$ = measured force at the stop, (r_k,θ_k) for the given mode, n and for a given displacement, $W_n(r_j,\theta_j)$ at the location (r_j,θ_j)

α = characteristic dimension of the valve

D = modulus of rigidity of the valve material

$W_n(r_j,\theta_j)$ = measured modal deflection at the location (r_j,θ_j)

Knowing the stop force ratio, one can calculate the modal contribution to the force at the stop. The deflection at (r_j,θ_j) due to the n th. mode is

$$W_n(r_j,\theta_j,t)=T_n(t).\phi_n(r_j,\theta_j) \qquad (14)$$

It is important to note here that $W_n(r_j,\theta_j,t)$ is the deflection of the valve during the state of the valve against the stop. So are the participation factors $T_n(t)$ and the mode shapes $\psi_n(r_j,\theta_j)$.

The contribution of the n^{th} mode to the total stop force is given by

$$F_n(t) = F_n(r_k,\theta_k).\frac{D}{\alpha^2}.\ W_n(r_j,\theta_j,t) \qquad (15)$$

Substituting Eqn. (14) in Eqn. (15)

$$F_n(t) = F_n.\frac{D}{\alpha^2}.\ \psi_n(r_j,\theta_j)\ T_n(t) \qquad (16)$$

The total stop force is the sum of the contribution of all the modes. So the total stop force is given by

$$F(t) = \frac{D}{\alpha^2}\sum_{n=1}^{\infty} F_n.\psi_n(r_j,\theta_j).T_n(t) \qquad (17)$$

To determine the time at which the valve leaves the stop the force F(t) is monitored. When it becomes zero, the valve is leaving the stop.

USE OF SCALING LAWS

The actual valves are frequently too small on which to make the deflection and strain measurements conveniently. It is difficult to properly simulate the boundary conditions in the experimental set up because of space limitations and the valves are usually of such stiffness that excitation at their natural frequencies is difficult. The deflections which can be achieved are also small such that accuracy is adversely affected. To overcome this problem scaled models of the actual valves can be used. The measurements can be made on the larger scaled models instead of the actual valves. The scaled models of larger size and lower stiffness can be excited easier and the frequencies and mode shapes can be measured with greater accuracy.

Frequency Scaling Law

The relation between the natural frequencies of two geometrically similar plates is given by (3)

$$f_2 = f_1.[\frac{h_2}{h_1}].[\frac{\alpha_1}{\alpha_2}]^2\sqrt{\frac{E_2}{E_1}.\frac{\rho_1}{\rho_2}\frac{(1-\nu_1)}{(1-\nu_2)}} \qquad (18)$$

where

f_1,f_2 = natural frequencies

h_1,h_2 = thickness of plates

α_1,α_2 = characteristic dimension

E_1,E_2 = Young's modulus

ρ_1,ρ_2 = density of the two materialssof the plates

ν_1,ν_2 = Poisson's ratio for the two materials

In the case of ring plate valves the characteristic dimension could be either the inner or outer radius. For two geometrically similar valves it is implied that planar dimensions be similar, therefore

$$\frac{\alpha_1}{\alpha_2} = \frac{R_1}{R_2} = \frac{r_1}{r_2} = k \qquad (19)$$

where

R_i (i=1,2) = outer radii

r_i (i=1,2) = inner radii

k = scaling factor

It is not necessary for the thickness of the two valves to be of the same ratio for the valves to be geometrically similar. The thickness ratio may be chosen arbitrarily.

If the material of the two valves is the same

$$E_1 = E_2$$

$$\rho_1 = \rho_2$$

$$\alpha_1 = \alpha_2$$

then the frequency law reduces to

$$f_2 = f_1 \left[\frac{h_2}{h_1}\right] \left[\frac{\alpha_1}{\alpha_2}\right]^2 \qquad (20)$$

It is worth noting that Eqns. (18) and (20) apply to plates in general. The only requirement is that the two plates which are scaled be geometrically similar.

The strain mode ratio and the stop force ratio are both nondimensional quantities and are therefore invarient for corresponding locations on geometrically similar valves. Scaled models of the actual valve can be used and the strain mode ratio and the stop force ratio can be measured much more accurately.

COMPRESSOR VALVE DYNAMICS TEST DATA

Experimental tests were performed on the particular compressor test valves in a scaled up version. Fig. (2) presents a schematic of the experimental test system for measuring the desired values of valve deflection, valve strain and valve stop force at a resonant frequency of the valve. Only the resonant frequency needed to be scaled back to the true size valve.

The results of the frequency tests are shown in Table 1 where the experimental values are the frequencies measured on the true size valves and the predicted values are scaled from the frequency measurements made on the scaled up valve.The higher values obtained on the true valve were probably due to the difficulty of maintaining the proper boundary conditions at the support edges.

The results of the strain mode ratio measurements are shown in Fig. (3), (4) and (5). One can see that more detailed data was obtained by using larger scaled models.

APPLICATION TO COMPUTER SIMULATION

The modal data obtained from the scaled model was utilized to predict the valve strain using a computer model of the compressor. Fig. (6) shows the experimentally measured strain and the strain predicted by using the computer model. The maximum level of strain could be predicted accurately, though it was impossible to predict strain point for point. The important parameter is the maximum strain to predict the valve life. The effect of fluid damping on the maximum valve strain was being studied in this analysis.

SUMMARY AND CONCLUSIONS

A method was presented for measuring accurate modal data of compressor valves utilizing scaled models.

The modal data was used in a computer simulation to sucessfully predict valve strain.

REFERENCES

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2) Reddy,K.H.,"Computer simulation of a reciprocating compressor with special emphasis on the prediction of dynamic strains in ring type valves", Ph.D. Thesis, Purdue University, August, 1974.

3) Soedel, W.,"Similitude approximations for vibrating thin shells", Journal of the Acoustic Society of America, Volume 49, No. 5(part 2), May 1971.

Table 1 Comparison of Experimental and Predicted Values of Frequencies of the Actual Valve

	Two Point Support			Four Point Support		
	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
Experimental	465-485 Hz	2700 Hz	5900 Hz	1900 Hz	4300 Hz	Not Measured
Predicted	403 Hz	1820 Hz	5365 Hz	1141 Hz	4184 Hz	5487 Hz

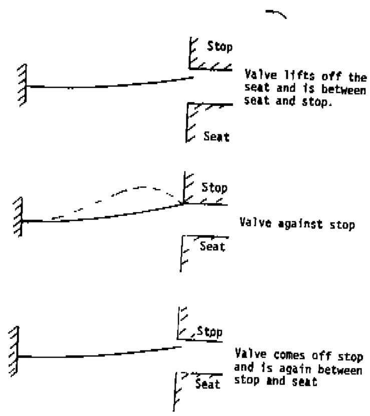

Figure 1 Valve in Three States of Motion

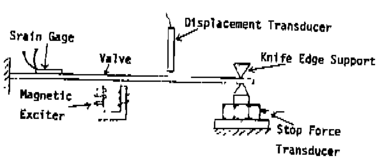

Figure 2 Instrumentation for Force Measurement

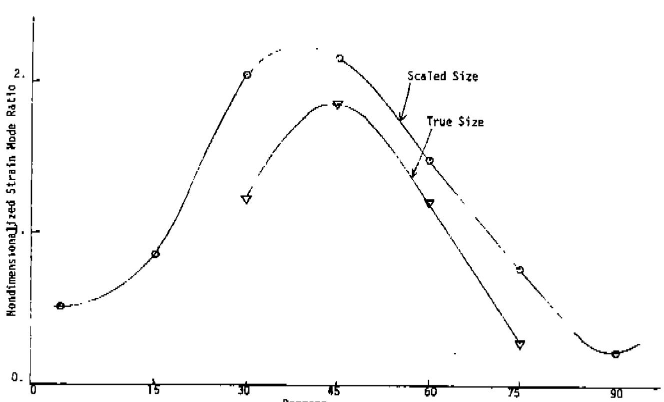

Figure 3 Comparison of Strain Data Obtained from Actual Valve and Model Valve

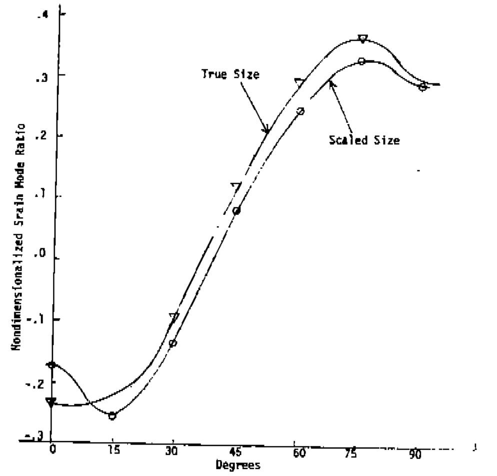

Figure 4 Comparison of Strain Data Obtained from Actual Valve and Scaled Valve

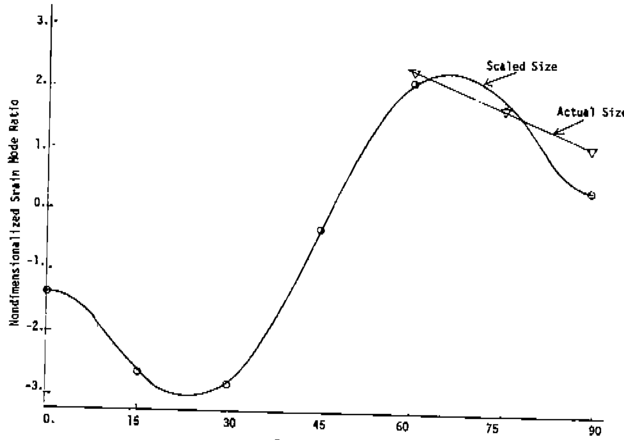

Figure 5 Comparison of Srain Data Obtained from Actual Valve and Model Valve

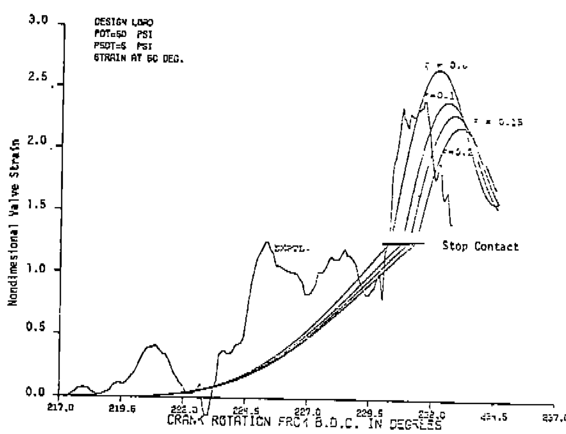

Figure 6 Predicted Total Strain at 60° for Different Values of Damping