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Natural Frequencies and Modes of Gases in Multicylinder Compressor Manifolds and their use in Design

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INTRODUCTION

In the following a case study of a four cylinder compressor is presented where particular attention is paid to the role that natural frequencies and modes play in the gas oscillations inside the compressor manifold.

The first part of the paper discusses the equation of motion of the gas that sloshes around in the manifold and concentrates then on the theoretical prediction of the natural frequencies and modes of the gas in the manifold. Typical results are shown.

The second part of the paper illustrates how knowledge of natural frequencies and modes can be used to predict (or explain) the occurrence of large amplitude oscillations under certain conditions.

EQUATION OF MOTION

The equations of motion are derived using the Helmholtz resonator approach. This approach was presented in references [1,2] and will not be discussed here.

The example case is a four cylinder compressor with an eight degree of freedom discharge system in the vibration sense, not counting the discharge line which is taken as anechoic [3]. Referring to Figure 1, the elements labeled 1L, 2L, ..., etc. are considered the mass elements. Cavities 11, 21, 22, etc. represent the interconnecting springs between "plug" masses. The displacements of the plug masses are represented by the symbols \( \xi \), where the subscript \( J \) denotes that the plug is either to the right or to the left of the discharge cavity directly above the \( J \)th cylinder. The acoustic displacement of a given plug is taken to be positive in the clockwise direction. This sign convention is used when the free body diagram and the equation of motion for a plug are written. The displacement of the gas at the entrance to the discharge pipe is \( \xi_p \). This displacement is not analogous to the displacement of a "plug" mass because of the character of the anechoic termination assumed for the pipe where no sound waves are reflected back into the discharge cavity. It can be visualized as some function of the distance down the pipe from the entrance at any instant of time.

With this as background, we may proceed to derive the equations of motion for the system. They can be represented compactly by the matrix equation

\[
[M] \{\ddot{\xi}\} + [C] \{\dot{\xi}\} + [K] \{\xi\} = \{S\Delta p\}
\]

where

- \([M]\) = mass matrix
- \([C]\) = damping matrix
- \([K]\) = stiffness matrix
- \(\xi\) = plug displacement coordinate
- \(S\Delta p\) = force acting on plug

The damping matrix will be diagonal, including not only the anechoic termination effect, but also factors accounting for the energy loss due to friction. While in general the damping effect is very complex, a function of frequency of oscillation, fluid properties and geometric considerations, it is modeled for this compressor as an appropriate damping factor, after the fashion of single degree-of-freedom vibration problems. Thus, damping terms are of the form

\[
C_{ij} = \eta M_{ij}
\]

The mass matrix is diagonal except for the last row which represents the equation for the discharge pipe. Length terms \( L_{11}, L_{1R}, L_{21}, \ldots \) etc. do not represent physical lengths of the orifices between the cavities but are corrected lengths. Oscillatory fluid motion in the neck region was represented by a mass due to the dominance
of the inertial over the compressible properties. Since the area of relatively high particle velocity extends beyond just the physical confines of the neck, an effective length for these masses must be determined. The effective length for a Helmholtz resonator neck is usually formulated as the geometric length plus an "end correction" to include the co-vibrating mass. Rayleigh [4] gives the end correction, \( \Delta L \) for one end of a cylindrical plug, of length \( L \) and effective length of plug \( l_L \), for example is

\[
L_{1L} = \text{geometric length neck } L + 2(0.82)\frac{S}{L} ~ \Delta L
\]

The force vector has terms of the form \( \delta A p \), the area times the difference in pressure between the cavities located at either side of the plug. Now, the cavities located far from the discharge pipe have higher mean thermodynamic pressures, over a cycle, than those nearer to the outlet so that mass moves toward and out the pipe. Thus, the pressure differential across any neck will not necessarily have an average value of zero over a cycle of the compressor but will be biased in the direction of mean mass flow. Since this bias appears in the equations of motion of the plugs as a constant force, the solution would yield ever increasing mean acoustic displacements. These displacements could give an acoustic pressure with a mean value. Since, the acoustic pressure was defined as a perturbation on mean pressure in the cavities which is described by the thermodynamic relationships, the proper force vector will be \( \delta A p \text{avg} \) where the differential \( \delta p \text{avg} \) has an average value of zero over one compressor crank rotation.

Instead of using the displacements of the plug masses as the generalized coordinate, volume displacement, the cross-sectional area of the plug times its displacement, could be used. The solution of the equations for the volume displacement is more descriptive. It is the cavity volume change due to the plug oscillation which when multiplied by the spring constant for the cavity gives the change in cavity acoustic pressure due to the plug oscillation. Equation (1) could be rewritten as

\[
[M'] \{\xi\} + [C'] \{\delta \xi\} + [K'] \{\delta \xi\} = \{\delta A p\} ~ \text{(5)}
\]

The elements in the matrices of equation (1) are shown in Figures 2, 3 and 4. Figure 5 gives the displacement and force vectors. Equation (1) can be solved along with the thermodynamic and valve dynamic equations by numerical integration.

NATURAL FREQUENCIES AND MODE SHAPES

The acoustic behavior of the compressor discharge network has been described by using analogous mechanical system elements of springs, masses and dampers. Thus, it is possible to obtain, just as with a mechanical system, a set of natural frequencies and modes. When excited by a harmonic force at one of these frequencies, the response, in the absence of damping, will become infinite. If however the response is initiated in some way at a natural frequency and forcing is immediately removed, the response will continue at that frequency with a particular pattern. This pattern, or mode shape, is unique to that frequency. In general, an 8-degree-of-freedom system will have 8 natural frequencies and associated mode shapes which may be found by reworking equation (1).

For undamped free vibration, equation (1) can be simplified to

\[
[M'] \{\xi\} + [K'] \{\xi\} = 0 ~ \text{(6)}
\]

where

\[
8x8 ~ 9x9
\]

\[
[M'] = [M] ~ \text{Row 9 deleted}
\]

\[
8x8 ~ 9x9
\]

\[
[K'] = [K] ~ \text{Row 9 deleted}
\]

The deletions of row 9 and column 9 come about because elements in this row and this column describe the influence of the anechoic exit pipe. The influence of the anechoic discharge pipe is that of a damping element and is not considered in the eigenvalue analysis of the undamped system. However note, that if the pipe would not be anechoic, it could not any longer be deleted from the eigenvalue determination.

At a natural frequency, the response of the system will be harmonic

\[
\{\xi\} = \{x\} e^{j\omega t} ~ \text{(9)}
\]

Substituting this into equation (6) yields

\[
[[K'] - \omega^2 [M']] \{x\} = 0 ~ \text{(10)}
\]

This equation is satisfied if

\[
\{x\} = 0 ~ \text{(11)}
\]

However, this solution is a case of no
interest. Another solution is found by letting the determinant of the matrix

\[ [A] = ([K]) - \omega^2 [M] \]  

be zero:

\[ |A| = 0 \]  

This is called the characteristic equation and represents the classical eigenvalue problem. It is solved for the natural frequencies, or eigenvalues. By substituting these eigenvalues into equation (10) one at a time, each eigenvector, or mode shape, may be found. The matrix \([A]\) is given in Figure 6.

A program was written to generate the elements of the matrix, \([A]\), and a library subroutine to solve the generalized eigenvalue problem, given by equation (13), was used. The following natural frequencies were obtained for the example case:

- \(f_0 = 0 \) Hz
- \(f_1 = 267 \) Hz
- \(f_2 = 290 \) Hz
- \(f_3 = 433 \) Hz
- \(f_4 = 764 \) Hz
- \(f_5 = 820 \) Hz
- \(f_6 = 840 \) Hz
- \(f_7 = 897 \) Hz

The numbers themselves are meaningless in the confines of this paper, since compressor dimensions are not defined. What is of interest is, that since none of the lumped elements is "fixed" to ground, a zero natural frequency exists. It corresponds to a rotational mode in which the masses move in a circle around the compressor axis so that they remain the same volume distance from each other with no "springs" stretched.

The first nonzero natural frequency has an acoustic displacement mode shape of:

\[
\{x\}_1 = \begin{bmatrix}
1.00 \\
1.80 \\
1.81 \\
1.47 \\
-1.09 \\
-1.87 \\
-1.76 \\
-1.37 
\end{bmatrix} \tag{14}
\]

with a corresponding volume displacement mode shape of:

\[
\{sx\}_1 = \begin{bmatrix}
1.00 \\
1.33 \\
1.81 \\
1.09 \\
-1.09 \\
-1.38 \\
-1.76 \\
-1.01 
\end{bmatrix} \tag{15}
\]

Again, the absolute numbers have no meaning attached. Only the character of the modes is of interest here.

Equation (14) gives the plug displacement at each of the mass locations, relative to \(X_1 = 1\), for the response at 267 Hz. This can be visualized in Figure 7. As suggested previously when formulating equation (5), volume displacement is often a more interesting quantity since the change in cavity volume is proportional to the acoustic pressure in the cavity. So, the volume displacement mode shape for 267 Hz is shown in Figure 8.

Since \([S_{1R} \{x\}_{1R}]\) is greater than \([S_{1L} \{x\}_{1L}]\), there is an increase in the volume of cavity 11. Thus the "acoustic" pressure in the cavity is negative, denoted by a (-) in Figure 8. The relative magnitude of the pressure is given by

\[
[p^a]_1 = -\frac{c^2 \rho d}{V_1} \begin{bmatrix}
S_{1L} \{x\}_{1L} \\
S_{1R} \{x\}_{1R}
\end{bmatrix} \tag{16}
\]

Similar negative acoustic pressures exist in cavities 12, 41, and 42 while the acoustic pressure is positive in discharge cavities 21, 22, 31, and 32. Equations such as (16) can be formulated for the relative values of the other pressures. These constitute an acoustic pressure mode shape associated with the second natural frequency, and it is (normalized with respect to cavity 11)

\[
\begin{bmatrix}
1.00 \\
0.94 \\
-2.18 \\
-2.56 \\
-0.88 \\
-0.48 \\
2.27 \\
2.55 
\end{bmatrix} \tag{17}
\]

Figure 9 is a sketch meant to represent the mode shape given by equation (17). Here again, the length of the arrows correspond to the relative magnitude of the pressure in the cavities. Outward pointing arrows represent a pressure increasing with acoustic pressure in cavity 11 while an arrow pointing toward the center represents a cavity pressure which is negative when the pressure in cavity 11 is positive. For instance, if the system were excited at 267 Hz with forcing such that the acoustic pressure in cavity 11 is 2 psi, then the response in cavity 12 is 1.88 psi while in cavity 21 it will be out of phase with \(p^a_{11}\) with a magnitude of 4.36 psi. Also, if this mode was the dominant mode excited at some other frequency, one might expect that the acoustic pressure in cavities 21, 22, 41 and 42 would be larger than the fluctuating pressures in the other cavities.

The acoustic pressure mode shapes for the other frequencies are shown in Figure 10 and are
\[
\{p^a\}_2 = \begin{bmatrix}
1.00 \\
1.13 \\
0.67 \\
0.33 \\
-0.19 \\
-0.44 \\
0.06 \\
\end{bmatrix}
\quad \{p^a\}_3 = \begin{bmatrix}
1 \\
-156 \\
-99 \\
101 \\
36 \\
-16 \\
117 \\
\end{bmatrix}
\]

\[
\{p^a\}_4 = \begin{bmatrix}
1.00 \\
0.26 \\
-0.66 \\
0.28 \\
0.95 \\
0.30 \\
-1.01 \\
-0.28 \\
\end{bmatrix}
\quad \{p^a\}_5 = \begin{bmatrix}
1.00 \\
0.05 \\
-0.42 \\
1.05 \\
0.03 \\
0.28 \\
-0.40 \\
\end{bmatrix}
\]

\[
\{p^a\}_6 = \begin{bmatrix}
1.00 \\
2.14 \\
-3.57 \\
4.30 \\
3.00 \\
7.39 \\
-4.93 \\
\end{bmatrix}
\quad \{p^a\}_7 = \begin{bmatrix}
1.00 \\
-1.04 \\
1.56 \\
0.25 \\
0.62 \\
0.23 \\
0.40 \\
\end{bmatrix}
\]

HOW TO USE NATURAL FREQUENCY AND MODE INFORMATION IN DESIGN

To dramatize the use of natural frequency and mode information in design, let us not only look at compressors that run at nearly constant speed, like refrigeration compressors for appliance type applications, but rather at compressors that experience a large crank speed range, like for instance in automotive applications.

For a typical compressor of this type (exact dimensions and parameters are of no importance in this example), we will get gas oscillations that are much larger in amplitude at certain crankspeeds than normally expected. For instance, simulation results taken at three speeds show small amplitude oscillations at 1000 RPM, medium activity at 3000 RPM and very pronounced oscillations at 5500 RPM (Figure 11).

The speeds at which large oscillations occur can be predicted, without running a complete simulation model which is relatively costly and laborious, by generating from the natural frequency data a table that lists critical crankspeeds in descending order of importance as function of the discharge system natural frequency that is excited and the multiple of the crank speed that does the exciting. It can be shown from mathematical considerations, that as lower the harmonic number defining the multiple of the crankspeed is, as more pronounced in an amplitude sense will be in general the gas oscillations at that crankspeed. This applies for the frequency range that is of interest for thermodynamic performance. Especially severe are cases where there are not only low harmonic numbers, but where the mode number is low also. For the example case, the table is given as Figure 12.

The table is generated by realizing that whenever

\[
mN = \frac{fn}{60} \quad (19)
\]

we excite a natural mode. In this formula

\[
N = \text{crankspeed [RPM]}
\]

\[
m = 1, 2, \ldots, \infty \quad \text{and is}\n\]

\[
f_n = \text{natural frequency}
\]

\[
n = 0, 1, 2, \ldots \quad \text{and indicates}\n\]

\[
\text{the natural mode and frequency as subscript}
\]

To make sure that no important crankspeed gets forgotten, it is best to list natural frequencies across and harmonic numbers down. For every combination, the value of \(N\) is obtained from Equation (19). The reason that some \(N\) values are blanked out is that for the example case 6000 RPM was the upper speed limit.

Let us now use the table. According to it we indeed expect large oscillation amplitudes in the vicinity of 5500 RPM. To be exact, at: 5380 and 5802 RPM. The example illustrates that one does not necessarily have to hit the critical speeds exactly. Coupling of the natural modes due to damping effects will spread the area in which to expect large amplitude oscillations.

Natural modes are another important piece of information for the designer. Those modes whose shape is encouraged by the particular phasing of the pistons will be excited stronger than mode shapes that are not encouraged by the piston phasing. This mode shape encouragement can override the harmonic number criteria outlined before in certain cases. Thus, in the four cylinder piston case each pair of opposing cylinders discharges out of phase while the two pairs are \(\pi\) radians crank angle out of phase, the modes associated with \(f_1, f_2, \text{and} f_5\) will be encouraged that is, excited more, whereas the modes associated with \(f_3, f_4, f_6\) and \(f_7\) will be discouraged, that is, excited less.

Thus, the recommended procedure is to:

1. Obtain natural frequencies and modes of discharge and suction systems either theoretically as shown in this
2. Generate a table as discussed before. (Note that if the compressor runs more or less at a constant speed, you have to generate the table only for a relatively narrow speed range.)

3. Investigate if a low harmonic number occurs.

4. Change natural frequencies of system, if necessary, by changing neck dimensions or volume sizes.

5. Investigate the piston phasing of the natural mode shapes. Change phasing, if necessary and feasible.

SUMMARY

A case study of a discharge manifold system for a four cylinder compressor was presented where particular attention was given to the role of natural frequencies and modes in the gas oscillation behavior.

The paper presented the equations of motion and showed how natural frequencies and modes were found for the example case.

It discussed the use of this information in design.

REFERENCES


NOMENCLATURE

\( K \) = stiffness matrix terms \([N/m]\)

\( \xi \) = "plug" displacements \([m]\)

\( S \) = effective "plug" area \([m^2]\)

\( \Delta p \) = pressure differential across "plug" \([N/m^2]\)

\( \eta \) = damping coefficient \([1/sec]\)

\( \omega \) = frequency \([\text{rad/sec}]\)

\( \Delta L \) = end correction \([m]\)

\( L \) = "plug" length

\( f_n \) = natural frequency \([1/sec]\)

\( X \) = natural mode \([m]\)

\( \rho_a \) = pressure with respect to mean pressure \([N/m^2]\)

\( c_d \) = speed of sound at mean discharge pressure \([m/sec]\)

\( \rho_d \) = mean discharge density \([N sec^2/m^4]\)

\( N \) = crankspeed \([\text{RPM}]\)

\( m \) = harmonic number

\( n \) = natural frequency and mode number

\( V \) = volumes \([m^3]\)

\( M \) = mass matrix terms \([N sec^2/m]\)

\( C \) = damping matrix terms \([N sec/m]\)
Figure 1 Discharge System
<table>
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<th>$L_{1L}S_{1L}$</th>
<th>$L_{1R}S_{1R}$</th>
<th>$L_{2L}S_{2L}$</th>
<th>$L_{2R}S_{2R}$</th>
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</table>

**Figure 2** Mass Matrix
$\delta C^2_d$

$$
\begin{array}{cccccccc}
\frac{S_0}{2c_0} & \frac{S_0}{2c_0} & \frac{S_0}{2c_0} & \frac{S_0}{2c_0} & \frac{S_0}{2c_0} & \frac{S_0}{2c_0} & \frac{S_0}{2c_0} & \frac{S_0}{2c_0} \\
\frac{S_0}{2c_0} & \frac{S_0}{2c_0} & \frac{S_0}{2c_0} & \frac{S_0}{2c_0} & \frac{S_0}{2c_0} & \frac{S_0}{2c_0} & \frac{S_0}{2c_0} & \frac{S_0}{2c_0} \\
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\end{array}
$$

Figure 3  Stiffness Matrix
Figure 4  Damping Matrix
$\{ \Delta p \} = \begin{bmatrix}
S_1 (p_{4L} - p_{4R}) \\
S_2 (p_{3R} - p_{3L}) \\
S_3 (p_{2L} - p_{2R}) \\
S_4 (p_{1L} - p_{1R}) \\
S_5 (p_{0L} - p_{0R})
\end{bmatrix}$

$\{ \xi \} = \begin{bmatrix}
\xi_{1L} \\
\xi_{1R} \\
\xi_{2L} \\
\xi_{2R} \\
\xi_{3L} \\
\xi_{3R} \\
\xi_{4L} \\
\xi_{4R} \\
\xi_{5L}
\end{bmatrix}$

Figure 5  Force and Displacement Vectors
Figure 6 Matrix \([A]\)
Figure 7. Gas Displacement Mode Associated with $f_1$

Figure 8. Gas Volume Displacement Mode Associated with $f_1$

Figure 9. Pressure Mode Associated with $f_1$
Figure 10 The Other Pressure Modes
Harmonic of Running Speed Which Excites Mode

Natural Frequency of Mode Excited

<table>
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Figure 12 Table of Critical Crank Speeds