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ADVANCES IN NUMERICAL METHODS TO SOLVE THE EQUATIONS GOVERNING UNSTEADY GAS FLOW IN RECIPROCATING COMPRESSOR SYSTEMS

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ABSTRACT

The hyperbolic partial differential equations which describe one-dimensional unsteady compressible fluid flow are presented in Conservation-law, Normal and Characteristic form and aspects of numerical methods for their solution are discussed. A comparison is made between solutions obtained for finite amplitude non homentropic flow in the pipework of a reciprocating compressor system when using (a) the well-known Method of Characteristics, (b) a composite scheme employing the Two-Step Lax-Wendroff Method within a pipe and the Method of Characteristics applied at the boundaries, (c) a composite scheme employing the Leapfrog Method within a pipe and the Method of Characteristics applied at the boundaries. Schemes (b) and (c) were simpler to apply than scheme (a), required much less computer time and gave better agreement with experimental results obtained from a single stage air compressor system.

INTRODUCTION

The general availability of digital computers has stimulated the development of numerical methods for the solution of problems in fluid dynamics. In positive displacement compressor systems pressure pulsations occur in the gas due to the inherently intermittent nature of the flow. The small-wave acoustic equation is frequently used to describe such flows but the amplitude of the pulsations can be sufficiently large to justify the use of the more complex nonlinear equations for waves of finite amplitude. The Method of Characteristics is often used to solve the hyperbolic type partial differential equations which describe one dimensional unsteady compressible flow. However, in recent years other numerical methods have been proposed; the practical merits and disadvantages of these can best be assessed by applying them to engineering problems.

PROPERTIES OF NUMERICAL METHODS

Properties of significance when using any numerical method are stability, convergence, accuracy and dispersion. When a method is stable any disturbance introduced during the calculations does not grow in subsequent time steps. Convergence implies that the solution of the finite difference equation tends to that of the differential equation as the time step tends to zero. The order of accuracy of a method is defined as the smallest power of the time step which is present in the truncation error, this error being the difference in one time step between the solution of the difference equation and the differential equation. Dispersion is the phenomenon of propagation at different speeds of the Fourier components in a solution.

FINITE DIFFERENCE METHODS

Finite difference methods may be classified as implicit or explicit. Implicit methods involve the solution of simultaneous equations generally by iterative methods. Large time steps and hence short computational times may be feasible. However the simulation of some elements within the particular physical system may require that a small time step be used for that element and hence also for the whole system. Explicit methods permit dependent quantities to be calculated directly, and the computation is free of the need to solve simultaneous equations, but time steps have to be sufficiently small to satisfy stability and convergence criteria.

A first order of accuracy explicit method was introduced in 1952 by Courant et al (1) for use with the Characteristic form of the hyperbolic type partial differential equations which govern unsteady fluid flow. In 1952 Hartree (2) proposed schemes, based on the Method of Characteristics, which had first and second order accuracy. In 1964 Benson et al (3) organised the method of Courant into a form which was suitable for solution by digital computer and used it extensively when simulating internal combustion engine and reciprocating compressor systems. The method compares well with other first order of accuracy explicit methods and has advantages over them when dealing with boundary conditions. In 1960 Lax and Wendroff (4) introduced a second order of accuracy method: a Two Step version was developed later by Richtmyer (5). Another second order of accuracy method is the Leapfrog Method (6). These and other methods were considered by Richtmyer and Morton (7) and Morton (8) concluded that the most attractive were the Two-Step Lax-Wendroff and the Leapfrog Methods. The present paper describes the use of each of these second order of accuracy methods for the solution of the equations for flow within a pipe. Each method was incorporated in a scheme which utilised a first order of accuracy Method of Characteristics approach when accounting for the boundary conditions at the pipe ends in a reciprocating gas compressor system. Comparison was made both between results predicted by these two composite schemes and those obtained using the Method of Characteristics alone and between the three schemes and experimental records from a single stage air compressor system.

FORM OF EQUATIONS

The nonlinear hyperbolic partial differential equations governing one-dimensional unsteady compressible flow may be expressed in three ways: Conservation-law, Normal and Characteristic forms.

Conservation-law form $$\frac{\partial \bar{V}}{\partial t} + \frac{\partial \bar{G}(\bar{V})}{\partial x} = \bar{B} \qquad (1)$$

Normal form $$\frac{\partial \bar{V}}{\partial t} + S\frac{\partial \bar{V}}{\partial x} = \bar{B} \qquad (2)$$

In Characteristic form the set of equations is reduced to a set of first-order linear ordinary differential equations by finding the directions in the x-t plane along which the dependent variables may be discontinuous. The set is formed by a direction condition and a compatibility relation:

Direction condition $dx/dt = L_J$

Compatibility relation $\bar{X}_J d\bar{V}/dt = \bar{X}_J \bar{B}$ (3)

where L_J is an eigenvalue of S and $\bar{X}_J$ the associated left eigenvector.

The equations of continuity, momentum, energy, and state permit a description of one-dimensional unsteady compressible flow with heat transfer, friction, and gradual area change. For computational purposes it is convenient to render the equations non-dimensional, with the variables referred to arbitrary reference values. The non-dimensional equations in Conservation-law, Normal and Characteristic form are given below and have been presented in more detail previously by MacLaren et al (9).

(a) Conservation-law form

$$\frac{\partial}{\partial Z}\begin{bmatrix} R \\ RU \\ \frac{RU^2}{2} + \frac{P}{k(k-1)} \end{bmatrix} + \frac{\partial}{\partial X}\begin{bmatrix} RU \\ RU^2 + P/k \\ \frac{RU^3}{2} + \frac{UP}{k-1} \end{bmatrix} = \begin{bmatrix} -\frac{RU}{F}\frac{dF}{dX} \\ -R\left(\frac{U^2}{F}\frac{dF}{dX} + \frac{2fU|U|x_{ref}}{D}\right) \\ R\frac{qx_{ref}}{a^3_{ref}} - \frac{1}{F}\frac{dF}{dX}\left(\frac{RU^3}{2} + \frac{UP}{k-1}\right) \end{bmatrix} \qquad (4)$$

(b) Normal form

$$\frac{\partial}{\partial Z}\begin{bmatrix} R \\ U \\ P/k \end{bmatrix} + \begin{bmatrix} U & R & 0 \\ 0 & U & 1/R \\ 0 & A^2R & U \end{bmatrix}\frac{\partial}{\partial X}\begin{bmatrix} R \\ U \\ P/k \end{bmatrix} = \begin{bmatrix} -\frac{RU}{F}\frac{dF}{dX} \\ -\frac{2fU|U|x_{ref}}{D} \\ (k-1)R\left(\frac{qx_{ref}}{a^3_{ref}} + \frac{2fx_{ref}}{D}|U^3|\right) - \frac{A^2RU}{F}\frac{dF}{dX} \end{bmatrix} \qquad (5)$$

(c) Characteristic form

Introducing the pseudo-Riemann variables

$C_1 = A + \frac{k-1}{2}U$ and $C_2 = A - \frac{k-1}{2}U$,

the compatibility relations are:

along $dX/dZ = U \pm A$

$$dC_{1,2} = -\tfrac{1}{2}(k-1)\frac{AU}{F}\frac{dF}{dX}dZ + A\frac{dAa}{Aa}$$

$$\mp \frac{1}{2}(k-1)\frac{2 f x_{ref} U|U|}{D}\left(1 \mp (k-1)\frac{U}{A}\right) dZ + \frac{(k-1)^2}{2}\left(\frac{q\, x_{ref}}{a^3_{ref}}\right)\frac{1}{A}\, dZ \tag{6a}$$

and along the path line characteristic $dX/dZ = U$

$$dA_a = \frac{1}{2}(k-1)\frac{A_a}{A^2}\left[\frac{q x_{ref}}{a^3_{ref}} + \frac{2 f x_{ref}}{D}|U^3|\right] dZ \tag{6b}$$

The relationships between variables in the three forms are given by

$$P = \left(\frac{A}{A_a}\right)^{2k/(k-1)}, \quad R = \frac{(A/A_a)^{2k/(k-1)}}{A^2}$$

$$A = \left(\frac{P}{R}\right)^{\frac{1}{2}}, \quad A_a = \left(\frac{P^{1/k}}{R}\right)^{\frac{1}{2}} \tag{7}$$

TWO-STEP LAX-WENDROFF METHOD

By applying the chain rule to the Conservation-law form of equation (1)

$$\frac{\partial \bar{V}}{\partial t} + W\frac{\partial \bar{V}}{\partial x} = \bar{B} \tag{8}$$

Lax and Wendroff (4) devised a method of second-order accuracy for the solution of the homogeneous case (i.e. when $\bar{B} = 0$),

$$\bar{V}_j^{n+1} = \bar{V}_j^n - \frac{\Delta t}{2\Delta x} W(\bar{V}_{j+1}^n - \bar{V}_{j-1}^n) + \frac{1}{2}\left(\frac{\Delta t}{\Delta x}\right)^2 W^2(\bar{V}_{j+1}^n - 2\bar{V}_j^n + \bar{V}_{j-1}^n) \tag{9}$$

where W is the Jacobian of $\bar{G}(\bar{V})$ and $\bar{V}_j^n$ refers to the values of vector $\bar{V}$ occurring at the mesh point $j\Delta x$, $n\Delta t$. In 1972 Zehnder (10) used this method to study the flow in the cells of the Comprex type of compressor, evaluating the Jacobian W by difference approximations. The two-step version of the method due to Richtmyer (5) avoids the calculation of W. For the non-homogeneous case the two steps are

first step

$$\bar{V}_{j+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{1}{2}(\bar{V}_{J+1}^n + \bar{V}_j^n) - \frac{\Delta t}{2\Delta x}(\bar{G}(\bar{V})_{j+1}^n - \bar{G}(\bar{V})_j^n) + \frac{\Delta t}{4}(\bar{B}_{j+1}^n + \bar{B}_j^n) \tag{10}$$

second step

$$\bar{V}_j^{n+1} = \bar{V}_j^n - \frac{\Delta t}{\Delta x}(\bar{G}(\bar{V})_{j+\frac{1}{2}}^{n+\frac{1}{2}} - \bar{G}(\bar{V})_{j-\frac{1}{2}}^{n+\frac{1}{2}}) + \frac{\Delta t}{2}(\bar{B}_{j+\frac{1}{2}}^{n+\frac{1}{2}} + \bar{B}_{j-\frac{1}{2}}^{n+\frac{1}{2}}) \tag{10}$$

Figure 1(a) shows the grid points used in this method. In the first step, information from points 1 and 2 is used to produce point 4; similarly point 5 originates from points 2 and 3. During the second step, the information at grid point 6 (time $t + \Delta t$) is generated from points 2, 4 and 5.

Direct application of the Lax-Wendroff method at a boundary is not possible because information is required, which is not available from points outside the boundary. We employed the Method of Characteristics to handle the end meshes in conjunction with the equations appropriate to the particular boundary. Equations were applied to compute the Characteristic form variables A and A_a at grid points 1' and 2'. (Figure 1 (a)). The theory of Characteristic directions applied at the boundaries was then used to generate the values of U, A, and A_a at point 3'. R and P at point 3' were obtained from Equation 7.

LEAPFROG METHOD

The Leapfrog Method was developed for use with the Normal form of the equations (Equation 2). However, the equations in Conservation-law form (Equation 1) may also be approximated by the Leapfrog Method,

$$\bar{V}_j^{n+1} = \bar{V}_j^{n-1} - \frac{\Delta t}{\Delta x}(\bar{G}(\bar{V})_{j+1}^n - \bar{G}(\bar{V})_{j-1}^n) + \Delta t(\bar{B}_{j+1}^n + \bar{B}_{j-1}^n) \tag{11}$$

Figure 1(b) shows the points in the x-t plane grid involved in this method. Information at time n+1 (grid point 4) is produced from information at time n-1 (grid point 1) and information at time n (grid points 2 and 3). At time t = 0 the method would require information from time $t = -\Delta t$ but this is not available. However, when the simulation of the first thermodynamic cycle of the compressor starts it is assumed that the gas in the pipes is stationary, which allows introduction of an additional condition $\partial\bar{V}/\partial t = 0$ at $t = 0$, i.e. values of the dependent variables at $t = -\Delta t$ are assumed to be those at t = 0. Grid points at the boundaries cannot be calculated directly since information from outside the boundary is required but is not available. Again we coupled the Method of Characteristics at the boundaries.

It is assumed in the Leapfrog Method that the interval of time Δt is fixed. Changes in the formulation of the method must be introduced when the time-step is allowed to vary. However, Equation (11) may be used unmodified if the values of all variables are defined at a suitable time level. Values of the dependent variables at point 1' (Figure 1(c)) can be found from a quadratic interpolation using values at points 5, 1 and 6. Quadratic interpolation preserves the second order of accuracy of the Leapfrog Method which is then applied, considering points 1', 2, 3 and 4.

STABILITY, CONVERGENCE, DAMPING AND DISPERSION

The Courant-Friedrichs-Lewy (6) condition for stability and convergence states that the ratio of the time to the space mesh size ($\Delta t/\Delta x$) should be small enough to contain the domain of dependence of the differential equation, i.e. for the nonlinear problem,

$$\frac{\Delta t}{\Delta x} \leq \frac{1}{a + |u|} \tag{12}$$

According to the necessary condition stipulated by Von-Neumann (11) any Fourier component of the solution should not be amplified in consecutive time steps. Amplification factors are shown in Figure 2 for the various schemes used, when applied to the linear equation

$$\frac{\partial u}{\partial t} + a \frac{\partial u}{\partial x} = 0 \tag{13}$$

Figure 2 (a) relates to the first order of accuracy version of the Method of Characteristics and Figure 2 (b) to the Lax-Wendroff Method. The "amplification" factor may be less than unity and hence effectively a damping factor. Note that any value of r (= a $\Delta t/\Delta x$ for the linearised problem) which satisfies the Courant-Friedrichs-Lewy condition also satisfies the Von-Neumann condition. It is preferrable to work close to a value of $r = 1$ to avoid selective damping of the Fourier components in the solution. Note also that any Fourier components present in the analytical solution would usually be more damped when using the Method of Characteristics than when using the Lax-Wendroff Method. This effect appears more clearly in Figure 3, where the amplification factor has been calculated for 360 time steps (conceivably the number of time steps needed to present a compressor cycle) over a range of values of wavelength and mesh proportion ratios. It is apparent that the high frequency components (small values of $\lambda/\Delta x$) are drastically damped when the Method of Characteristics is employed. Hence, to obtain comparable accuracy, this method requires a finer grid which, to satisfy stability criteria, implies shorter time steps and consequently longer computational time. When stability conditions are violated, rapid oscillations can develop locally. Hence, it was considered rational to apply the criteria for local stability to the present nonlinear situation.

The change in amplitude experienced by any Fourier component of the solution determined by the Leapfrog Method can also be found by applying the Von Neumann Method (11). The factor which multiplies the amplitude of each component during one time-step, is given by the modulus of

$$L_{1,2} = -r\, i \sin \frac{2\pi}{\lambda} \Delta x \pm \sqrt{1 - r^2 \sin^2 \frac{2\pi}{\lambda} \Delta x} \tag{14}$$

which is equal to unity (i.e. $\varrho = 1$) if $r \leq 1$. Hence the condition for stability of the Leapfrog Method so derived coincides with the condition stipulated in formula (12). However, in the presence of certain non-linearities, instability may occur due to the growth of parasitic components of the solution (8). Note from equation (14) that in the Leapfrog Method there is no damping of the Fourier components of the solution in the range of stability. However, in the actual flow in pipes damping of high frequency pressure pulsations occurs due to viscous damping within the gas, a phenomenon which is not accounted for in the mathematical description of the physical situation. Hence an artificial viscosity was introduced by a filtering scheme (of second order of accuracy) to damp such frequencies in the mathematical solution

$$\bar{V}^{n+1}_{j\,\text{filt}} = \bar{V}^{n+1}_j + \frac{(1-\beta)}{4} \left(\bar{V}^{n+1}_{j+1} + 2\bar{V}^{n+1}_j + \bar{V}^{n+1}_{j-1}\right) \tag{15}$$

The amount of damping introduced is given by

$$\varrho = 1 - (1 - \beta) \sin^2 \frac{\pi \Delta x}{\lambda} \tag{16}$$

The damping which the scheme described by Equation (15) introduces to the different Fourier components is illustrated by Figure 2 (c). Short wavelength (high frequency) components are multiplied during one time step by a low amplification factor, thus decreasing in amplitude in consecutive time steps. The value of the empirical coefficient β stipulates the amount of damping which has been introduced by the user: this value of β could be estimated by experiments.

Each Fourier component might be propagated at a different speed by the different methods. This phenomenon, referred to as dispersion, is small in the Lax-Wendroff Method if the wavelength ratio $\lambda/\Delta x$ is greater than about 2.5. The Leapfrog Method introduces considerably less dispersion than the other two methods.

APPLICATION OF THE SCHEMES

Terms were included in the hyperbolic partial differential equations used to describe one dimensional finite amplitude unsteady flow in pipes to account for small changes of cross-section area, heat transfer and pipe friction. Digital computer programs were developed to solve these equations and satisfy the various boundary conditions met with in a reciprocating compressor system (12, 13). The procedure used for solution was (a) the Method of Characteristics, using a modified form of the rectangular mesh technique developed by Benson, Garg and Woollatt (3), both for the flow within the pipes and at the boundaries (b) a composite scheme employing the Two-Step Lax-Wendroff Method at the mesh points within a pipe coupled with the Method of Characteristics at the boundaries (c) a composite scheme employing the Leapfrog Method at the mesh points within a pipe coupled with the Method of Characteristics at the boundaries. In all three schemes friction and heat transfer effects were accounted for in exactly the same manner.

Predictions of the pressure variation at several points in a compressor system and of valve movement were predicted by the three schemes. It was found that convergence to an approximately repeatable analytical compressor cycle was achieved after the third or fourth simulation of the cycle. The predictions were compared with experimental records obtained from a single stage air compressor, 6 in bore x 4.5 in stroke (152 x 114 mm). The compressor installation had a simple series arrangement of inlet pipe, inlet valve, compressor cylinder, discharge valve, discharge pipe, receiver and discharge nozzle: more complex boundaries such as sudden large changes of cross-sectional area, pipe junctions and dampers were avoided. It was not easy to interpret the small differences between the analytical predictions by the three schemes and the experimental records: the mathematical equations describing the complex physical situation contain assumptions and the schemes for solution are also approximate: the experimental results are subject to experimental error (since these tests were conducted the experimental methods have been improved (14)).

Although the results obtained using each scheme showed satisfactory agreement with the experimental records the version using the Two-Step Lax-Wendroff Method had advantages over the version using only the Method of Characteristics. While the pressure predictions obtained using only the Method of Characteristics followed the general trend of the experimental traces, high-frequency components in the analytical solution were damped out (Figure 4). This diagram depicts the pressure histories in the discharge plenum chamber of the compressor operating at 404 rev/min with compression ratio 4.8, inlet pipe length 9.8 ft (3 m) and discharge pipe length 13.2 ft (4 m). In this case, when using the program which incorporated the Two-Step Lax-Wendroff Method, ten meshes were employed for the inlet pipe and fourteen for the discharge pipe. The time taken on an I.B.M. 370/158 computer to complete three compressor cycles was 47 seconds. When employing the program based on the Method of Characteristics only, the time required was 138 seconds, using sixteen meshes at inlet and twenty-four at discharge.

Figure 5 shows experimental records of pressure in the compressor cylinder, suction and discharge plenums and of valve displacement together with the analytical predictions for the fourth cycle using the Method of Characteristics only and the schemes employing the Two-Step Lax-Wendroff Method and the Leapfrog Method. These results are for the compressor operating at 612 rev/min with compression ratio 7.7, inlet pipe length 18.8 ft (5.75 m), and discharge pipe length 13.2 ft (4 m). The analytical records from the scheme which incorporated the Two-Step Lax-Wendroff Method were obtained using thirty meshes at inlet and seventeen at discharge, and required 74 seconds to reach the end of the fourth cycle. An attempt (not recorded in Figure 5) using only the Method of Characteristics with these numbers of meshes yielded poor results and required 95 seconds; the results using this method which are shown in Figure 5 were obtained with thirty-nine meshes at both inlet and discharge and required 302 seconds. However, it may be observed from Figure 5 that, even with this refined grid and large computation time, the results were less accurate than those obtained by the version which incorporated the Two-Step Lax-Wendroff Method. Results virtually identical to those obtained using the Two-Step Lax-Wendroff scheme were obtained in 59 seconds by application of the Leapfrog scheme using thirty meshes at inlet and seventeen at discharge, employing an empirical damping factor (β) equal to 0.85. This Leapfrog scheme was therefore the fastest of the two composite schemes (59 seconds c.f. 74 seconds) and had an inherent flexibility in that the amount of damping involved was at the discretion of the user and not, as in the other schemes, predetermined within the scheme.

As shown in Figures 4 and 5, the main disadvantage of the Method of Characteristics was that the higher harmonics of the solution were severely damped. This can be observed (Figure 5) when comparing the predicted pressure in the discharge plenum chamber where the first harmonic was being strongly excited. According to standing-wave theory the relationship between λ/L for the fundamental mode of vibration and its harmonics for a closed/open ended pipe is $1 : \frac{1}{3} : \frac{1}{5}$ and so on. Hence, when seventeen meshes were used (Lax-Wendroff Method) in the calculations for the discharge pipe (Figure 5) the wavelength ratios $\lambda/\Delta x$ of the fundamental and first harmonic modes were 68 and 68/3; when using thirty-nine meshes (Method of Characteristics only) the corresponding values were 156 and 52. Referring to Figure 3 is can be noted that values for $\lambda/\Delta x$ of 68 and 156, corresponding to the fundamental mode when using the Methods of Lax-Wendroff and Characteristics respectively, result in a very small difference in damping factor. This is not the case when considering the first harmonic, where a value for $\lambda/\Delta x$ of 68/3 when using the Lax-Wendroff Method introduces less damping than a value of 52 when applying the Method of Characteristics. In order to obtain similar results up to the first harmonic using both schemes (see Figure 3) it would be necessary to apply the Method of Characteristics with $\lambda/\Delta x$ equal to 100, i.e. to use seventy-five meshes, thus increasing considerably the computational time.

CONCLUSIONS

The amplitude of the pressure pulsations and the magnitude of heat transfer effects in reciprocating compressor systems can be sufficiently large to justify the use of the hyperbolic type partial differential equations which describe nonhomentropic finite amplitude unsteady flow. Solution of these nonlinear equations by a first order of accuracy Method of Characteristics yielded results which, for many engineering design purposes, compared sufficiently well with experimental records from an air compressor system. However, composite schemes which used the Two-Step Lax-Wendroff Method or the Leapfrog Method for solution within the pipes of such a system, coupled with the Method of Characteristics at the boundaries yielded more accurate results. The scheme using the Method of Characteristics alone overdamped the higher frequency pulsations which were present: the composite scheme incorporating the Lax-Wendroff Method did not damp these to the same extent: in the composite scheme incorporating the Leapfrog Method the amount of damping could be introduced in an empirical manner by the user to match experimental evidence. The two composite schemes, which were of second order of accuracy, each required approximately the same amount of computer time and were faster (by an order of about 3) than the first order of accuracy scheme using the Method of Characteristics alone. This important result may appear surprising since, in general, the higher the order of accuracy, the greater the computational time required (e.g. a third order of accuracy method, while giving greater precision, required treble the time for a second order method (15)). However, since second order of accuracy methods introduce less damping of higher frequencies in the solution than first order of accuracy methods, coarser meshes may be used. Stability may be maintained with larger, consequently fewer, time steps. Hence the equations for all the mesh elements and the boundaries of the system have to be solved a fewer number of times, resulting in the saving of time. Also the interpolation procedures involved at the grid points within the pipes when using the Method of Characteristics alone, were particularly time consuming because of the need to use the non-homentropic form of the equations to account for heat transfer effects. (Such effects had to be accounted for to obtain sufficiently accurate predictions for the mass flow out of a stage (which is the inflow to the intercooler and a higher pressure stage).

NOTATION

a	Speed of sound
A	Nondimensional speed of sound (a/a_{ref})
A_a	Nondimensional form of a_A (entropy level)
$\bar{B}$	Vector of terms relating to heat transfer, area change, and friction
C_1	Pseudo Riemann variable $A + \frac{k-1}{2} U$
C_2	Pseudo Riemann variable $A - \frac{k-1}{2} U$
D	Pipe diameter
f	Friction factor $\frac{\tau_w}{\frac{1}{2}\rho u^2}$
F	Pipe cross sectional area
$\bar{G}(\bar{V})$	Vector function of V
k	Ratio of specific heats
L	Pipe length
L_J	Eigenvalue of S
P	Nondimensional pressure p/p_{ref}
q	Heat transfer per unit mass
r	Mesh proportion ratio ($a\Delta t/\Delta x$)
R	Nondimensional density (ρ/ρ_{ref})
S	Matrix whose elements depend on the elements of $\bar{V}$
t	Time
u	Particle velocity
$\|u\|$	Absolute value of u
U	Nondimensional form of the particle velocity u/a_{ref}
$\|U\|$	Absolute value of U
$\bar{V}$	Vector of dependent variables

$\bar{W}$	Jacobian of $\bar{G}(\bar{V})$
X_j	Left eigenvector
x	Distance
X	Nondimensional form of distance (x/x_{ref})
Z	Nondimensional form of time $t\frac{x_{ref}}{a_{ref}}$
β	Empirical coefficient
Δ	Increment
λ	Wavelength
$\lambda/_{\Delta x}$	Wavelength ratio
ξ	Amplification factor
j	Index denoting space
n	Index denoting time
ref	Subindex denoting reference conditions
filt	Sub-subindex denoting damped solutions

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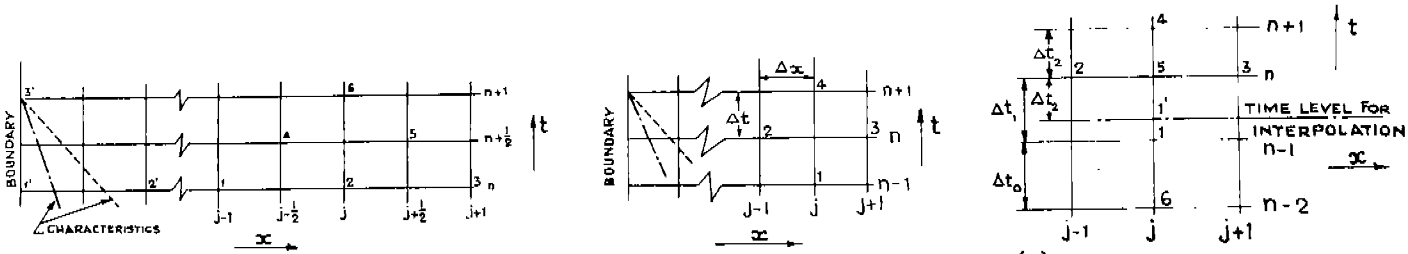
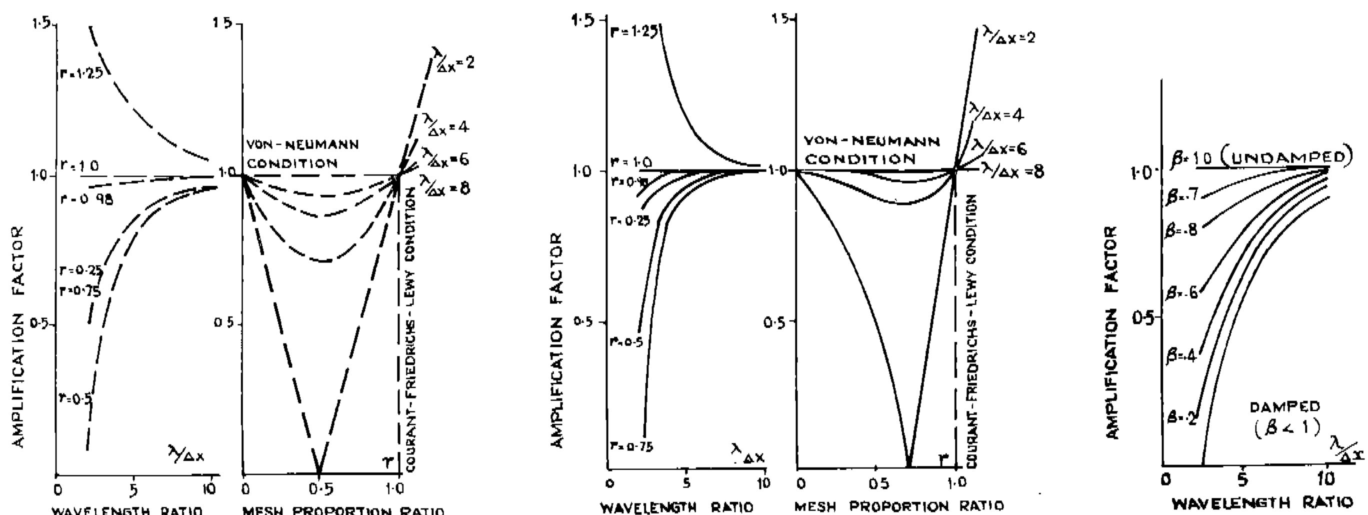


(a) GRID USED FOR THE TWO-STEP LAX-WENDROFF METHOD. (b) GRID USED FOR THE LEAPFROG METHOD. (C) VARIABLE TIME STEP GRID.

FIGURE 1 x-t PLANE GRIDS.



(a) METHOD OF CHARACTERISTICS (b) LAX-WENDROFF METHOD. (C) LEAPFROG METHOD

FIGURE 2. AMPLIFICATION FACTORS FOR THE LINEARIZED PROBLEM.

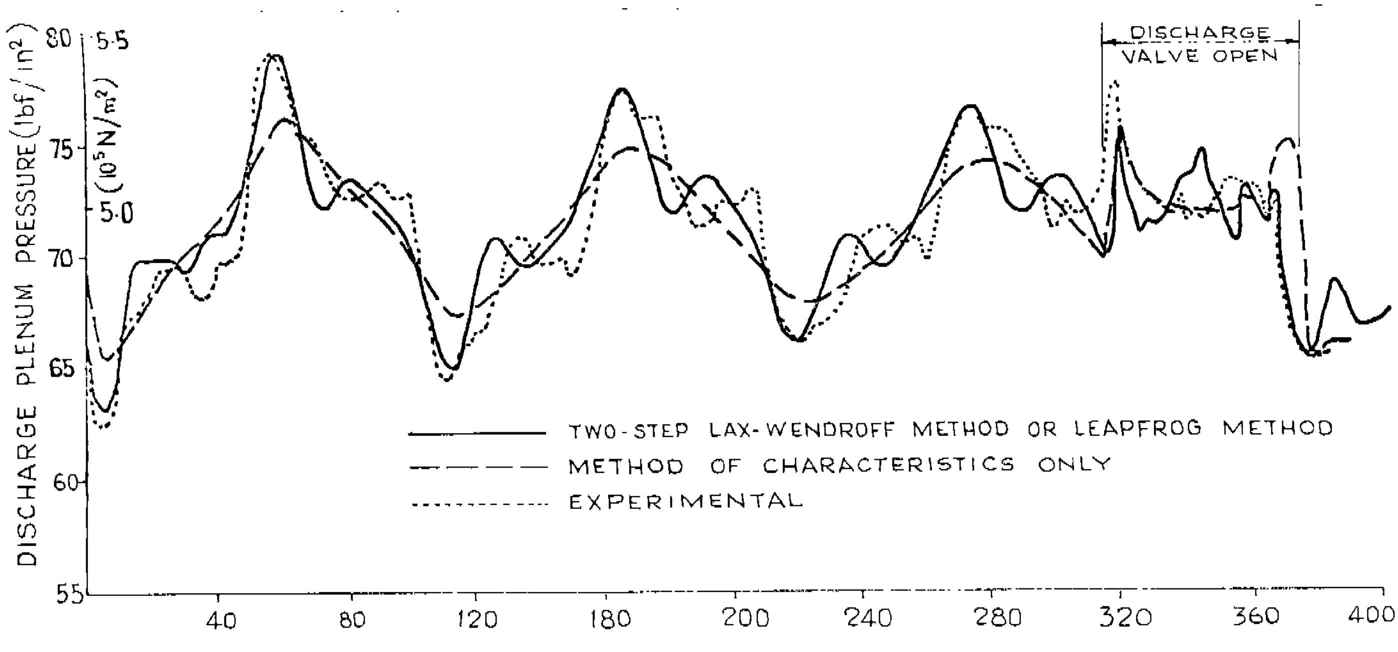


FIG 4 COMPARISON OF ANALYTICAL PREDICTIONS WITH EXPERIMENTAL RECORDS OF PRESSURE IN COMPRESSOR DISCHARGE PLENUM.

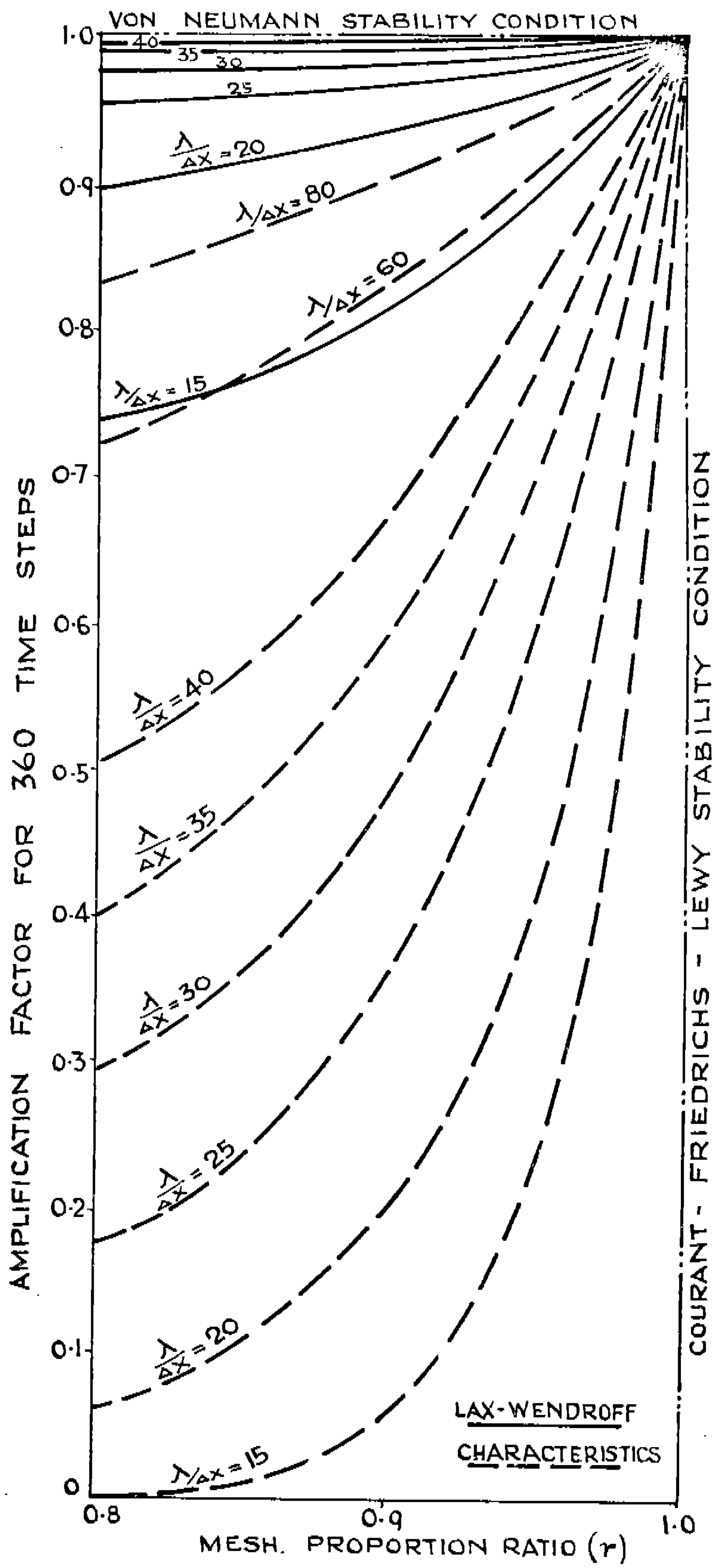


FIG. 3. COMPARISON OF AMPLIFICATION FACTORS.

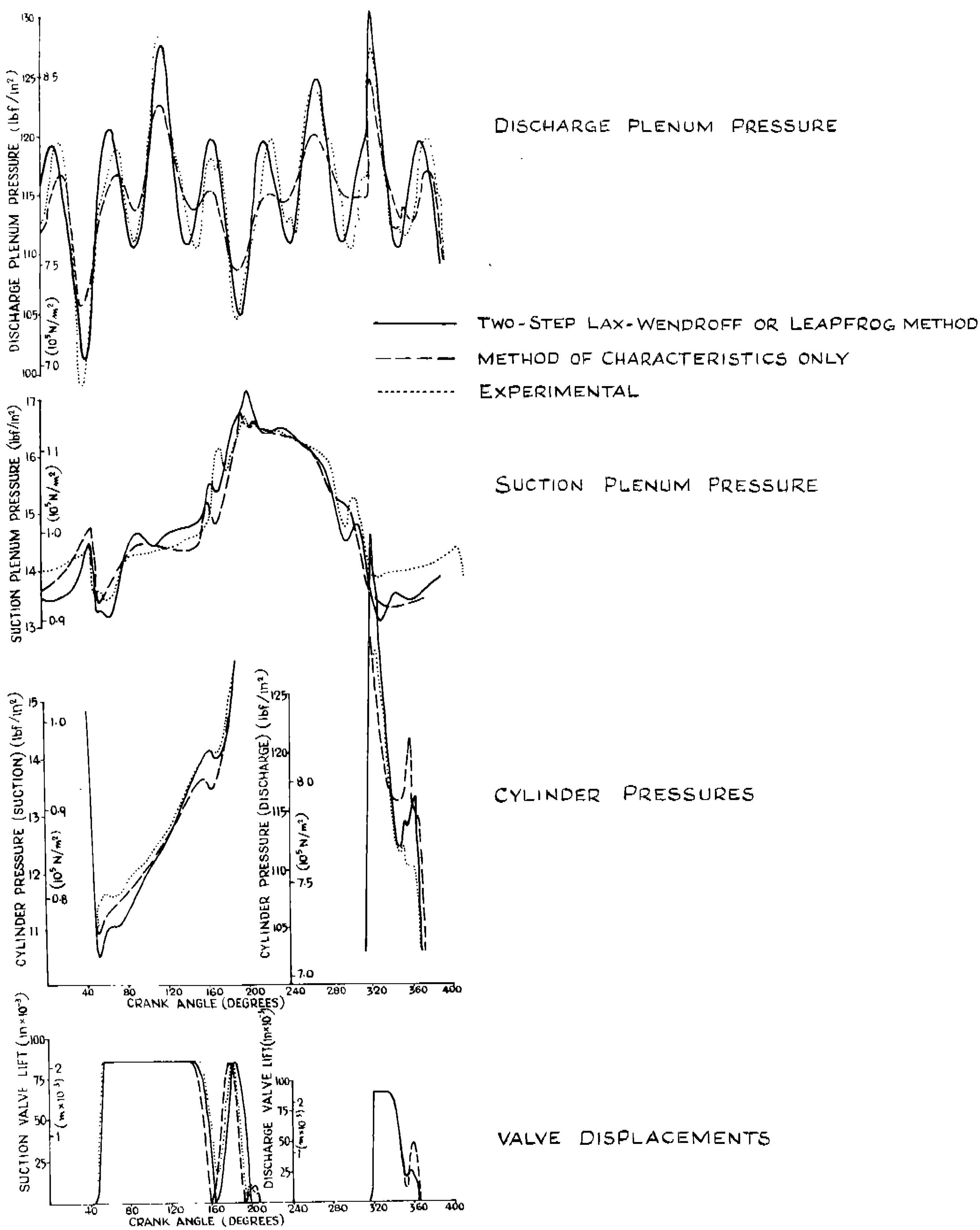


FIG. 5. COMPARISON OF ANALYTICAL PREDICTIONS WITH EXPERIMENTAL RECORDS