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## **An Experimental Design for the Computational Evaluation of Elliptic Partial Differential Equations Solvers**

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An Experimental Design for the Computational  
Evaluation of Elliptic Partial Differential Equation Solvers

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Purdue University

CSD-TR 264

May, 1978

1. Introduction. In this report we present a preliminary a priori experimental design of the computational evaluation of ELLPACK methods. A basic goal of this experiment is to justify from the test results a number of hypotheses made for the algorithms considered. The hypotheses are based on limited experience and the complexity of the mathematical algorithm. An attempt is made to identify the classes of problems for which the techniques were most effective (ineffective) and to use statistic sampling for drawing inferences. Our methodology of evaluation consists of selecting a population of test problems which fairly represents the domain of applicability of the methods and a set of performance indicators for measuring the efficiency of the competing techniques. We discuss the general problem selection criteria used to create a population of test problems [see 1]. A number of performance measures are presented and experimental facilities are discussed.

2. Experimental objectives. This computational evaluation is designed to test the following hypotheses for the set of ELLPACK methods. Most of these hypotheses are based on limited experience and computational complexity of the mathematical algorithms.

- H1 For "well-behaved" problems (Def.: constant coefficients and smooth solutions) the efficiency ranks are  
FFT9, HODIE9, P3C1 collocation, 5-point star
- H2 For "singular" problems (Def:  $u = O(x^\alpha)$  some place with  $\alpha < 2$ ) the ranks remain the same but the efficiency is much closer.
- H3 Extreme local variations (boundary layer, isolated peaks, sharp wave fronts) adversely affects P3C1-collocation more than other methods.
- H4 Extreme local variation adversely affects fourth order methods more than second order but not enough to consistently change rankings.
- H5 Sparse Sherman performs the same as Band Solve for modest systems (10-250 unknowns).
- H6 For "well-behaved" problems, the memory ranks are:  
A. (very low accuracy): FFT9, HODIE, P3C1, 5PT SOR, 5PTLINE, 5PT  
B. (higher accuracy): FFT9, HODIE, 5PT SOR, 5PTLINE, 5PT, P3C1
- H7 For variable coefficient problems, the efficiency ranks are P3C1, 5PT, HODIE

An attempt will be made to identify classes of problems for which the techniques were most effective (ineffective) and measure the statistical deviation of the results from the above hypotheses.

3. Population of Test Problems Two types of test problems are available for experimental testing--artificial psuedo-randomly generated and hand selected or "real-world" problems. We propose the following criteria of problem selection: The problem population should

- C1: Represent a real-world behavior
- C2: Represent mathematical theory behavior

- C3: Exhibit great variability of features
- C4: Allow variability of problem complexity and reduction of any systematic error by having parametrized problem features.
- C5: Test the necessity of the assumptions made on input data and solution to predict the a priori behavior of a method.

The population of elliptic partial differential equations presented in [1] was more or less generated on the basis of the above criteria. To reduce any systematic error, the problem complexity might be considered as random variable from a uniform distribution. We assume that the expected value of problem complexity is a function of the expected value of the complexity of the operator, boundary and solution, and thus we can determine problems with specified complexity.

The operator, boundary and solution complexity might be considered as random variables while the problem complexity as the mean (or sum) of the problem attributes complexities.

To make this population representative of the real world behavior, we systematically search a number of engineering text books and journals in various disciplines to identify problems governed by elliptic PDEs.

4. Performance Indicators. The following performance indicators are proposed for assessing algorithms and for evaluating the efficiency of competing techniques. We classify them into two groups:

Group 1: Performance Indicators per run

CPU Time: The total control processing time needed for executing the algorithm. The separate CPU times for input/output, preprocessing, discretization, indexing, and solution.

Numerical Solution Accuracy: Maximum error, least squares error

Number of Iterations: For iterative algorithms, or simple counting of the number of steps of an algorithm required to solve the problem.

Number of Equations: The size of the formulation of corresponding matrix problems.

Grid Size: Maximum mesh size

Numerical Residual Accuracy: The maximum absolute and relative residual sizes, and the  $L_2$ -norm of them.

Number of Function Evaluations: The number of times that some core subfunction is called during execution.

Storage Requirements: The memory needed for the algorithm code and variables (arrays) used.

Basic Operation Count: Estimate of arithmetic operations per module.

Group 2: Performance Indicators per problem

Accuracy vs. time

Memory vs. time

Accuracy vs. mesh size

Accuracy vs. number of iterations

Function evaluations vs. mesh size

Arithmetic operations vs. mesh size

Residual vs. time

Time for given accuracy

5. Competing Algorithms. Our approach is to assume that the test of an algorithm is its computational performance which depends on the computer implementation. We believe that ELLPACK provides a convenient and unbiased environment for implementing mathematical algorithms and applying them.

The currently existing ELLPACK and ITPACK modules are:

<u>DISCRETIZATION</u>	<u>INDEXING</u>	<u>SOLUTION</u>
1. 5-point star	9. Natural	16. Band Solve
2. P3-C1 Collocation	10. Collocate Band	17. Symmetric Band
3. P3-C1 Galerkin	11. Index Natural	18. Sparse Sherman
4. FFT 9-point	12. Index Spiral	19. Jacobi SI
5. Holr 9-point	13. Index Diagonal	20. Jacobi CG
6. Holr 27-point	14. Index Red Black	21. Compressed Jacobi CG
7. Symmetric 5-point	15. Index Line Red	22. Reduced System SI
8. Line Symmetric 5-point	Black	23. Reduced System CG
		24. Symmetric Sor CG
		25. Symmetric Sor PA
		26. Symmetric Sor SI
		27. Line Jacobi SI
		28. Line Jacobi CG
		29. Line Compressed JCG
		30. Line RSSI
		31. Line RSCG
		32. Line SSOR CG
		33. Line SSOR PA
		34. Line SSOR SI

### 5.1 ELLPACK two-dimensional methods

<u>DISCRETIZATION</u>	<u>INDEXING</u>	<u>SOLUTION</u>	<u>PROBLEM TYPE</u>
5-point	Natural	Band Solve	$Au_{xx} + Bu_{yy} + Cu_x + Du_y + Eu = f,$ $au_x + bu_y + cu = g$
5-point		Sparse Sherman	Same
P3-C1 Collocation	Collocate Band	Band Solve	Same
P3-C1 Collocation		Sparse Sherman	Same
FFT 9-point (Order=4)			$Au_{xx} + Bu_{yy} + Cu = f$ $u=g, A, B, C = \text{constants}$
FFT 9-point (Order=6)			$u_{xx} + u_{yy} + Cu = f$ $u=g, C = \text{constant}$
HOLR 9-point (Order=40)	Natural	Band Solve	$Au_{xx} + Bu_{yy} + Cu = f$ $u=g$
HOLR 9-point (Order=40)		Sparse Sherman	Same
HOLR 9-point (Order=41)	Natural	Band Solve	Same
HOLR 9-point (Order=41)		Sparse Sherman	Same
HOLR 9-point (Order=42)	Natural	Band Solve	Same
HOLR 9-point (Order=42)		Sparse Sherman	Same
HOLR 9-point (Order=41,F)	Natural	Band Solve	$Au_{xx} + Bu_{yy} + Cu = f$ $u=g, A, B = \text{constants}$
HOLR 9-point (Order=6)	Natural	Band Solve	$u_{xx} + u_{yy} + cu = f$ $u=g$
HOLR 9-point (Order=6)		Sparse Sherman	Same
P3-C1 Galerkin		Symmetric	$(pu_x)_x + (qu_y)_y + cu = f$ $u=0 \text{ on } \Gamma, u_N=0$

5.2 Compatible ITPACK modules in ELLPACK Below is a list of 25 iterative methods formed by compatible ITPACK modules applied to two-dimensional self-adjoint operators.

<u>DISCRETIZATION</u>	<u>INDEXING</u>	<u>SOLUTION</u>
7	11	19
7	11	20
7	11	24
7	11	25
7	11	26
7	12	19
7	12	20
7	13	19
7	13	20
7	14	19
7	14	20
7	14	20
7	14	21
7	14	22
7	14	23
8	11	27
8	11	28
8	11	32
8	11	33
8	11	34
8	15	27
8	15	28
8	15	29
8	15	30
8	15	31

6. Experimental Test Facilities. A program has been written that generates an ELLPACK input program according to the specifications encoded in the following input string.

N1/N2/N3/N4/N5/N7/\$COMMENTS

where

N1 is the problem number

- N2 is the options segment number
- N3 is either M1 or M1,M2, or M1,M2,M3 giving either a grid number of the size of a 2- or 3-dimensional uniform grid, respectively
- N4 is either M1 or M1, 'parameter list,' where M1 is the module number of the discretization module and the parameters are optional parameters which are to be passed to the module
- N5 gives the module number and parameters of the indexing module as in N4
- N7 is the output segment number
- COMMENT is a comment of  $\leq 50$  characters.

After the execution of the generated ELLPACK input program, a record of the "result file" is created in the following form:

- Line 1 = Problem Number, Date, Time, Comment  
Line 2 = Prob/Method Compatibility Info. Prob. Characteristics  
Line 3 = Encoded Method Description  
Line 4,... = Table of Execution Results, Each Line Containing
1. Min number of grid lines in any direction
  2. Max grid spacing in any direction
  3. Number of unknowns
  4. Max error at nodes
  5. Discrete L2 error at nodes
  6. Max residual at midpoints of subrectangles
  7. Max relative residual at midpoints
  8. Discrete L2 residual at midpoints
  9. Max solution value at nodes
  10. Number of iterations used by iterative method
  11. Memory used
  12. Total time (sec.)
  13. List of segment times (at most 3)

Sample examples are given in Table 1 for Problem 6 from [1] and all the existing ELLPACK/ITPACK methods. To measure the performance indicators per



problem, a program was written to read the "RESULT FILE" for each problem and generate a number of plots. Plots are given in Figures 1-5 for Problem 6.

#### References

1. E. N. Houstis and J. R. Rice, A Population of Partial Differential Equations for Evaluating Methods, Computer Science Report, CSD-TR 263, Purdue University, May 15, 1978.

FIGURE 1

RECORD 14:  $u_{xx} + u_{yy} + (100 + \cos(3\pi x) + \sin(2\pi y))u = f$

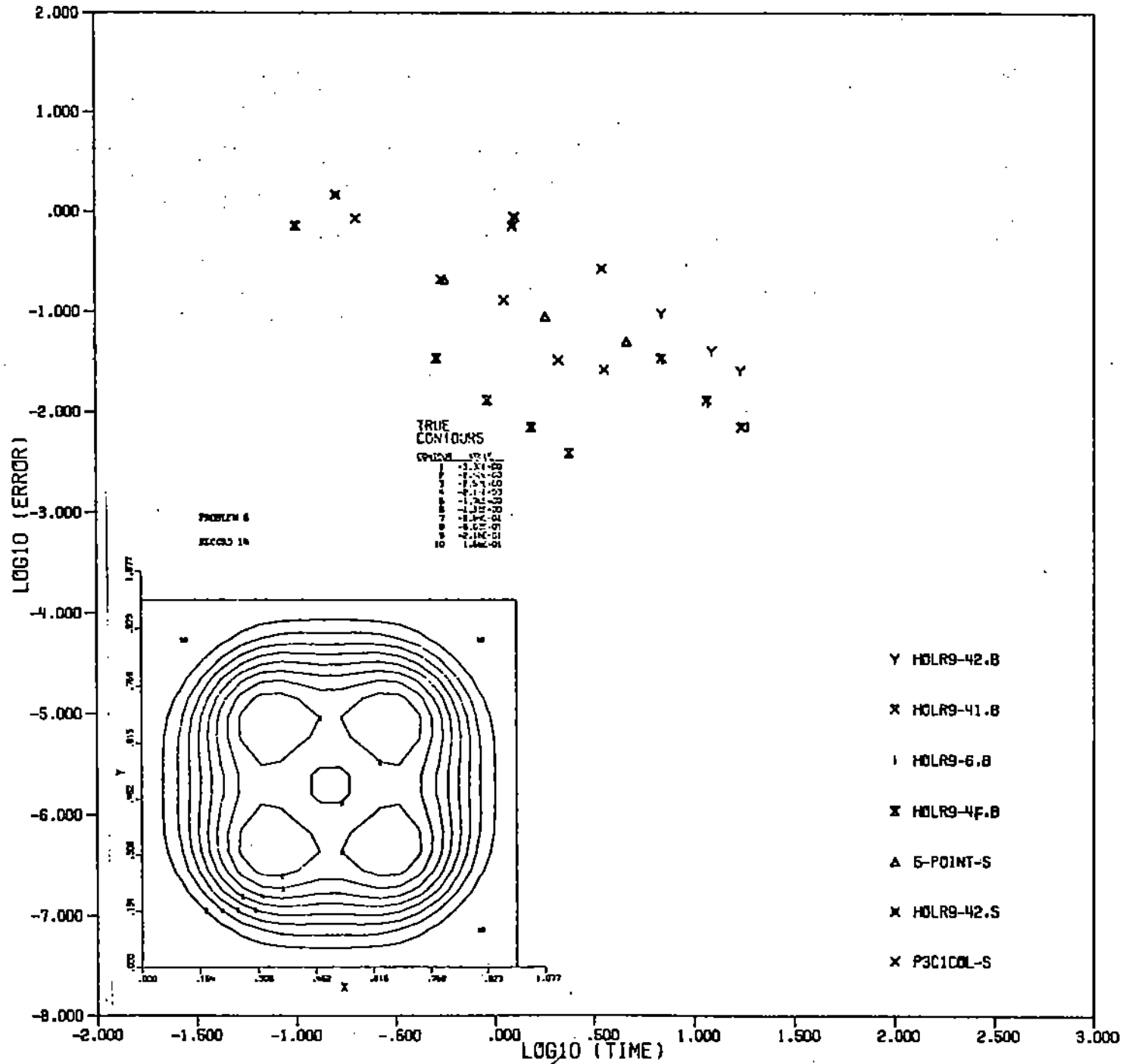


FIGURE 2

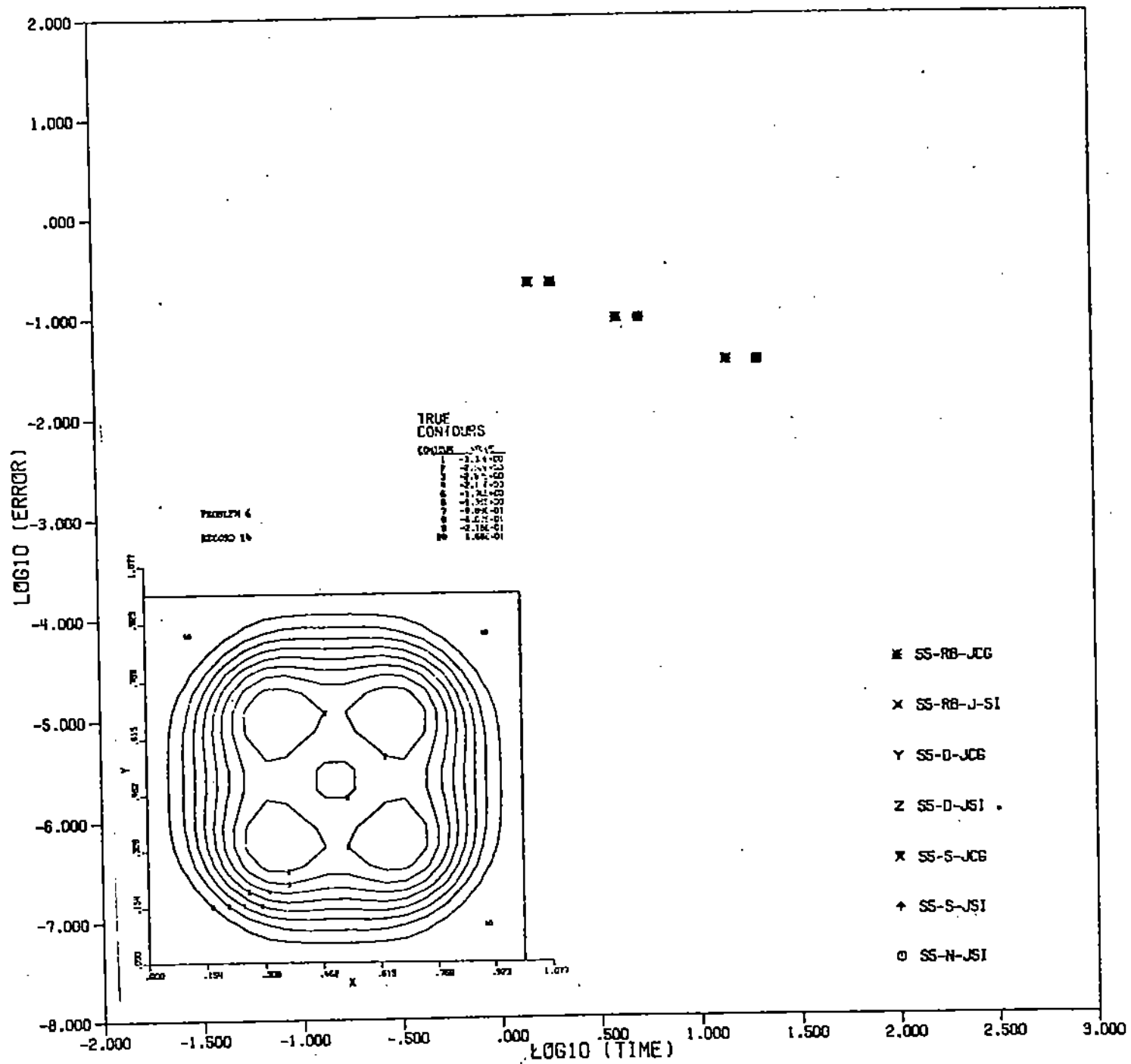


FIGURE 3

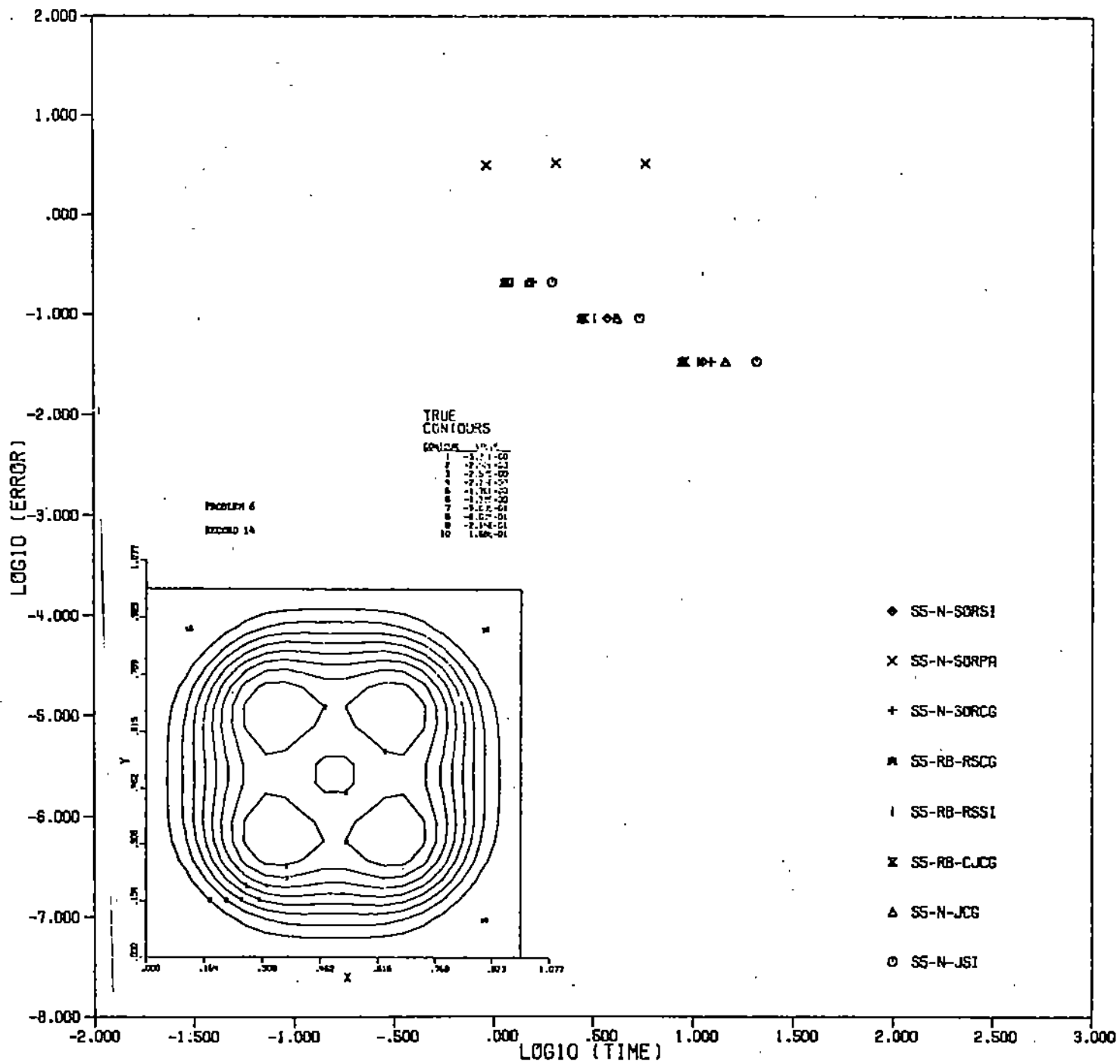


FIGURE 4

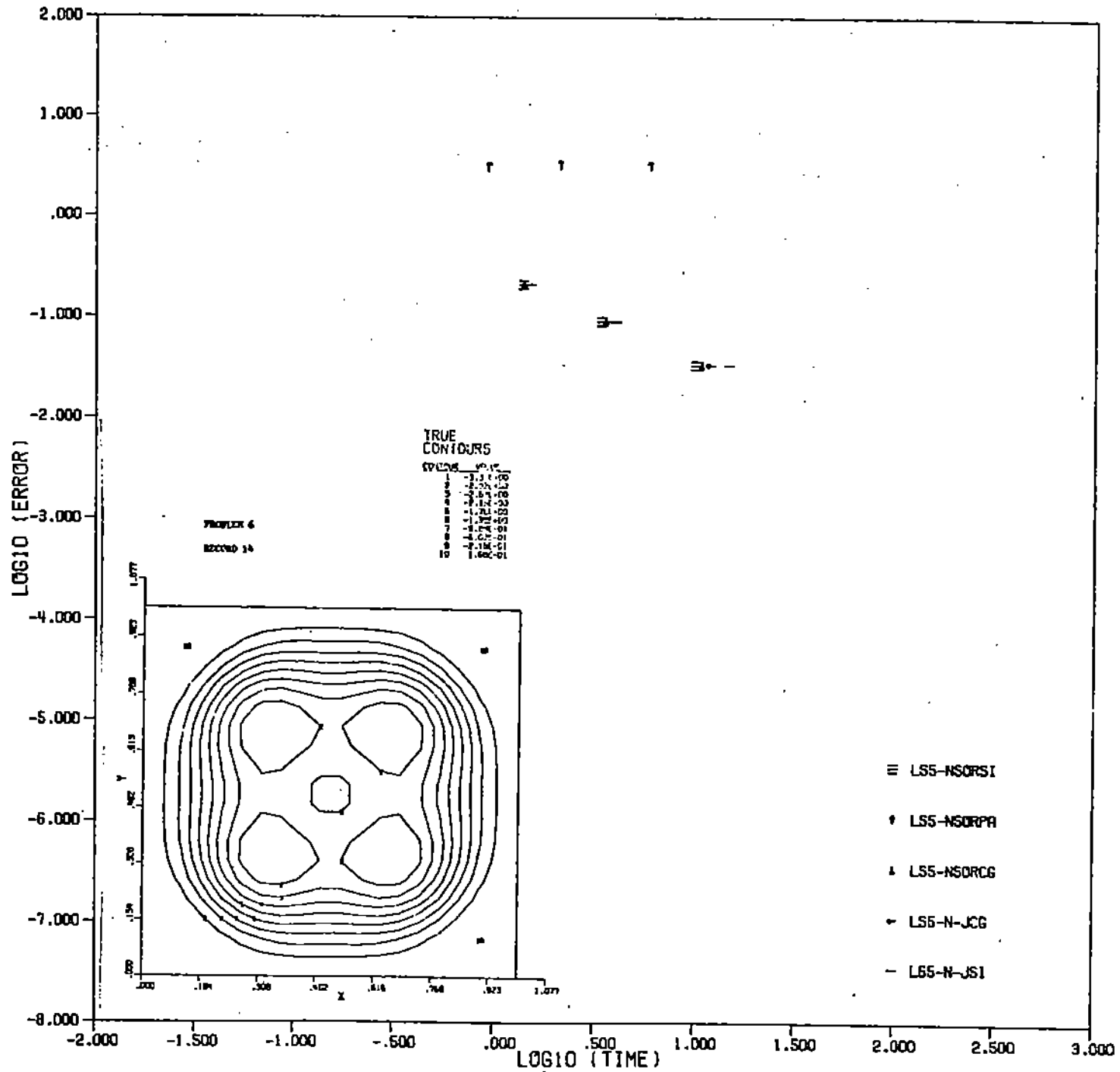


TABLE 1

7 RECORD 14

14 05/08/78 20.08.07. \$WRKSH-P3-C1-COLLOCATION  
200020100200222 002.35 090.15 000.00 010.25

2//18/

3	3	1	5.000E-01	16	8.49E-01	9.43E-02	4.43E+02	9.06E-01	2.21E+02	3.27E+00	0	7660	.20	.13	.07
4	4	1	3.333E-01	36	2.10E-01	2.63E-02	1.86E+02	1.74E+00	4.00E+01	3.54E+00	0	11512	.54	.28	.26
5	5	1	2.500E-01	64	1.31E-01	1.15E-02	7.58E+01	5.35E-01	1.02E+01	2.63E+00	0	16464	1.12	.49	.63
6	6	1	2.000E-01	100	3.31E-02	2.30E-03	7.58E+01	3.05E+00	8.63E+00	2.95E+00	0	22516	2.11	.76	1.35
7	7	1	1.667E-01	144	2.69E-02	1.78E-03	7.30E+01	4.61E-01	5.68E+00	3.35E+00	0	29668	3.59	1.08	2.51

14 05/08/78 20.16.34. \$WRKSH-4TH-ORDER-HODIE  
200020100200222 002.35 090.15 000.00 010.25

5.IORDER=42//18/

3	3	1	5.000E-01	1	1.48E+00	1.65E-01	4.03E+02	8.25E-01	2.01E+02	9.36E-01	0	6767	.16	.16	.01
5	5	1	2.500E-01	9	8.88E-01	7.41E-02	4.59E+02	4.70E+00	7.18E+01	3.28E+00	0	7051	1.27	1.25	.03
7	7	1	1.667E-01	25	2.69E-01	1.41E-02	4.84E+02	5.01E+00	3.07E+01	3.42E+00	0	7615	3.51	3.41	.09

14 05/08/78 20.37.53. \$WRKSH-5-POINTSTAR  
200020100200222 002.35 090.15 000.00 010.25

1//18/

9	9	1	1.250E-01	49	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	0	7177	.56	.36	.20
13	13	1	8.333E-02	121	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	0	12025	1.81	.86	.95
17	17	1	6.250E-02	225	5.10E-02	1.09E-03	1.89E+02	1.73E+01	3.39E+00	3.34E+00	0	25830	4.63	1.61	3.02

14 05/10/78 11.48.20. \$.....HODIE.4TH.ORDER....FAST.VERSION  
200020100200222 002.35 090.15 000.00 010.25

5.IORDER=0/9/16/

5	5	1	2.500E-01	9	7.13E-01	6.08E-02	4.31E+02	4.31E+00	6.76E+01	3.11E+00	0	7042	.10	.07	.01	.01
9	9	1	1.250E-01	49	3.43E-02	1.52E-03	3.03E+02	7.68E+01	1.44E+01	3.16E+00	0	8802	.51	.27	.10	.14
11	11	1	1.000E-01	81	1.32E-02	5.89E-04	2.74E+02	5.09E+00	9.54E+00	3.21E+00	0	10474	.92	.41	.18	.33
13	13	1	8.333E-02	121	7.10E-03	2.32E-04	2.11E+02	1.46E+01	6.51E+00	3.34E+00	0	12802	1.53	.57	.28	.67
15	15	1	7.143E-02	169	3.99E-03	1.02E-04	1.98E+02	2.53E+01	4.64E+00	3.27E+00	0	15882	2.38	.76	.40	1.22

14 05/10/78 11.52.31. \$.....HODIE.4TH.ORDER....40.VERSION  
200020100200222 002.35 090.15 000.00 010.25

5.IORDER=40/9/16/

5	5	1	2.500E-01	9	7.29E-01	5.87E-02	4.33E+02	4.33E+00	6.64E+01	3.12E+00	0	7042	1.27	1.24	.01	.01
9	9	1	1.250E-01	49	3.42E-02	1.46E-03	3.03E+02	7.68E+01	1.44E+01	3.17E+00	0	8802	7.02	6.78	.10	.14
11	11	1	1.000E-01	81	1.26E-02	5.73E-04	2.74E+02	5.09E+00	9.54E+00	3.21E+00	0	10474	11.78	11.27	.18	.34
13	13	1	8.333E-02	121	7.22E-03	2.28E-04	2.11E+02	1.46E+01	6.51E+00	3.34E+00	0	12802	18.55	17.58	.29	.68

14 05/10/78 11.55.12. \$.....HODIE.4TH.ORDER....41.VERSION  
200020100200222 002.35 090.15 000.00 010.25

5.IORDER=41/9/16/

5	5	1	2.500E-01	9	7.13E-01	6.08E-02	4.31E+02	4.31E+00	6.76E+01	3.11E+00	0	7042	1.24	1.22	.01	.01
9	9	1	1.250E-01	49	3.43E-02	1.52E-03	3.03E+02	7.68E+01	1.44E+01	3.16E+00	0	8802	6.93	6.69	.11	.14
11	11	1	1.000E-01	81	1.32E-02	5.89E-04	2.74E+02	5.09E+00	9.54E+00	3.21E+00	0	10474	11.60	11.09	.18	.33
13	13	1	8.333E-02	121	7.10E-03	2.32E-04	2.11E+02	1.46E+01	6.51E+00	3.34E+00	0	12802	17.31	16.37	.27	.67

14 05/10/78 11.58.46. \$.....HODIE.4TH.ORDER....42.VERSION  
200020100200222 002.35 090.15 000.00 010.25

5.IORDER=42/9/16/

5	5	1	2.500E-01	9	8.88E-01	7.41E-02	4.59E+02	4.70E+00	7.18E+01	3.28E+00	0	7042	1.25	1.22	.01	.01
9	9	1	1.250E-01	49	9.62E-02	4.09E-03	3.32E+02	8.42E+01	1.47E+01	3.16E+00	0	8802	6.92	6.68	.10	.14
11	11	1	1.000E-01	81	4.07E-02	1.50E-03	2.78E+02	5.41E+00	9.63E+00	3.23E+00	0	10474	12.32	11.80	.18	.33
13	13	1	8.333E-02	121	2.60E-02	6.40E-04	2.08E+02	1.51E+01	6.54E+00	3.34E+00	0	12802	17.23	16.28	.28	.67

FIGURE 5

RECORD 14 IT PACK METHODS GROUP 4

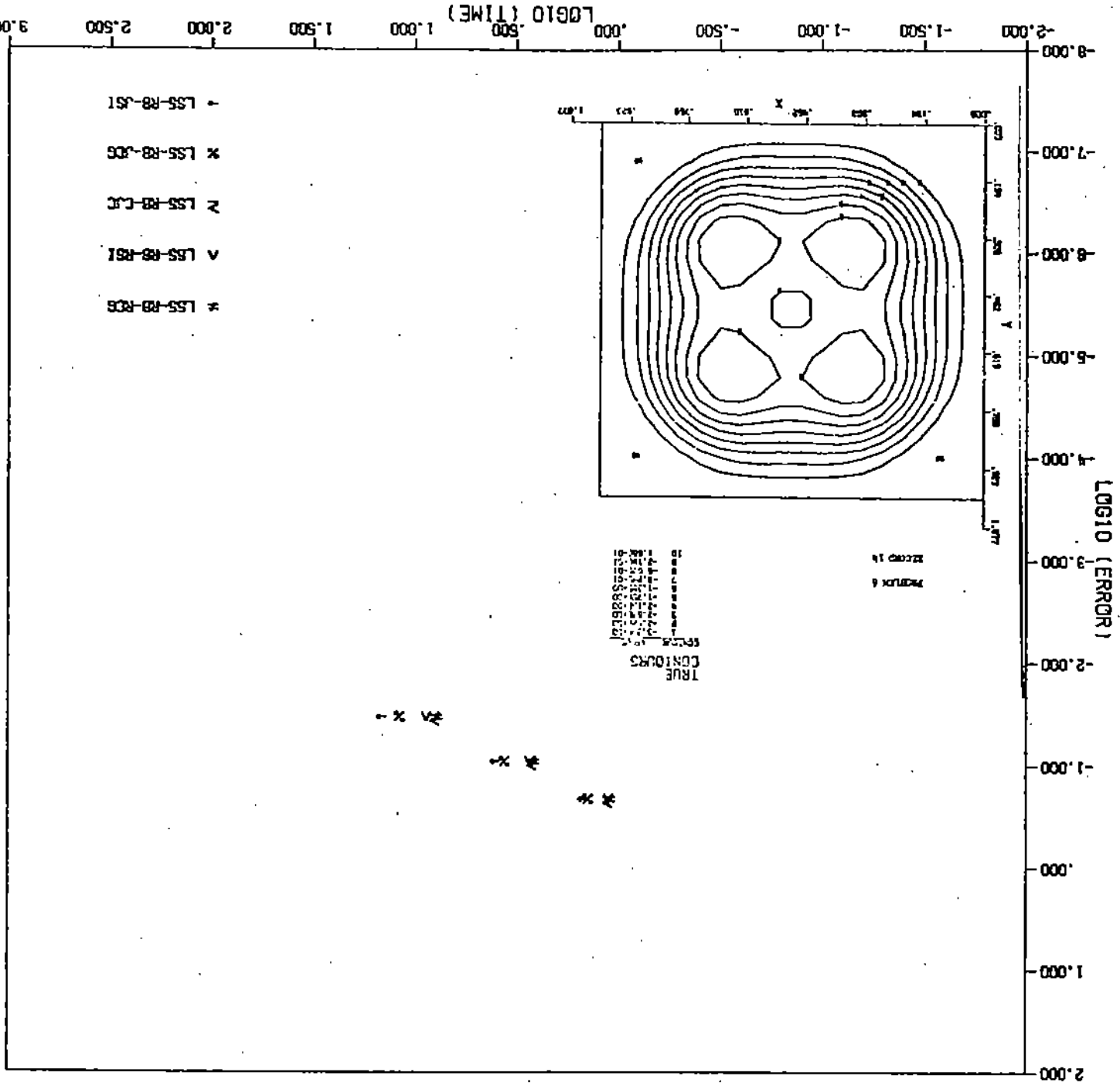


TABLE 2

7 RECORD 14 ITPACK MODULES GROUP1

14 05/14/78 11.53.21. \$WRKSH-5-POINT-NAT-JS  
200020100200222 002.35 090.15 000.00 010.25

7/11/19/

9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	19	4016	2.01	.88	.00	1.13
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	29	5432	5.50	1.96	.00	3.53
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	49	9800	21.02	5.52	.00	15.50

14 05/14/78 11.55.11. \$WRKSH-5-POINT-SPIR-JS  
200020100200222 002.35 090.15 000.00 010.25

7/12/19/

9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	19	4016	2.01	.88	.00	1.13
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	29	5432	5.54	1.97	.00	3.56
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	49	9800	21.21	5.57	.01	15.62

14 05/14/78 11.57.38. \$WRKSH-5-POINT-SPIR-J-CG  
200020100200222 002.35 090.15 000.00 010.25

7/12/20/

9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	9	4297	1.57	.87	.00	.69
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	15	5801	4.26	1.98	.01	2.27
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	24	10441	14.78	5.50	.01	9.27

14 05/14/78 11.59.13. \$WRKSH-5-POINT-DIAG-JS  
200020100200222 002.35 090.15 000.00 010.25

7/13/19/

9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	19	4016	1.99	.86	.00	1.12
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	29	5432	5.49	1.96	.01	3.52
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	49	9800	21.10	5.54	.01	15.56

14 05/14/78 12.00.54. \$WRKSH-5-POINT-DIAG-J-CG  
200020100200222 002.35 090.15 000.00 010.25

7/13/20/

9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	9	4297	1.56	.88	.00	.69
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	15	5801	4.26	1.98	.00	2.27
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	24	10441	14.88	5.56	.01	9.32

14 05/14/78 12.02.39. \$WRKSH-5-POINT-REDBL-JS  
200020100200222 002.35 090.15 000.00 010.25

7/14/19/

9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	19	4016	2.02	.88	0	1.14
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	29	5432	5.54	1.98	.00	3.56
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	49	9800	21.04	5.55	.00	15.49

14 05/14/78 12.04.28. \$WRKSH-5-POINT-REDBL-J-CG  
200020100200222 002.35 090.15 000.00 010.25

7/14/20/

9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	9	4297	1.55	.87	.00	.68
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	15	5801	4.24	1.97	.00	2.27
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	24	10441	14.76	5.52	.00	9.24

8 RECORD 14 ITPACK METHODS GROUP2

14 05/14/78 12.16.55. \$WRKSH-5-POINT-NAT-JS  
200020100200222 002.35 090.15 000.00 010.25

7/11/19/

9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	19	4016	2.02	.88	0	1.14
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	29	5432	5.55	1.98	.00	3.57
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	49	9800	21.14	5.57	.00	15.57

14 05/14/78 12.18.51. \$WRKSH-5-POINT-NAT-J-CG



200020100200222 002.35 090.15 000.00 010.25

7/11/20/																
9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	9	4297	1.55	.87	.00	.68
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	15	5801	4.26	1.97	.00	2.29
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	24	10441	14.84	5.57	.00	9.27

14 05/14/78 12.20.23. \$WRKSH-P-SYM-5-POINT-REDBL-CJ-CG

200020100200222 002.35 090.15 000.00 010.25

7/14/21/																
9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	5	4216	1.19	.87	.00	.32
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	8	5632	2.92	1.97	.00	.94
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	13	10000	9.31	5.56	.00	3.75

14 05/14/78 12.21.47. \$WRKSH-P-SYM-5-POINT-REDBL-RS-SI

200020100200222 002.35 090.15 000.00 010.25

7/14/22/																
9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	9	3935	1.27	.86	0	.41
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	15	5263	3.29	1.97	.00	1.32
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	24	9359	10.83	5.53	.00	5.30

14 05/14/78 12.23.05. \$WRKSH-P-SYM-5-POINT-REDBL-RS-CG

200020100200222 002.35 090.15 000.00 010.25

7/14/23/																
9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	5	4297	1.16	.87	0	.29
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	8	5801	2.84	1.97	.00	.87
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	13	10441	9.05	5.58	.00	3.47

14 05/14/78 12.24.30. \$WRKSH-P-SYM-5-POINT-NAT-SSOR-CG

200020100200222 002.35 090.15 000.00 010.25

7/11/24/																
9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	5	4459	1.61	.88	.00	.73
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	7	6139	4.12	1.99	.00	2.13
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	9	11323	12.52	5.54	.00	6.98

14 05/14/78 12.26.34. \$WRKSH-P-SYM-5-POINT-NAT-SSOR-PA

200020100200222 002.35 090.15 000.00 010.25

7/11/25/																
9	9	1	1.250E-01	81	3.16E+00	1.77E-01	5.84E+02	1.00E+00	3.27E+01	0	0	4259	.94	.88	0	.07
13	13	1	8.333E-02	169	3.33E+00	1.27E-01	5.95E+02	1.00E+00	2.17E+01	0	0	5939	2.12	1.97	.00	.15
21	21	1	5.000E-02	441	3.30E+00	8.13E-02	5.81E+02	1.00E+00	1.30E+01	0	0	11123	5.96	5.58	.00	.38

14 05/14/78 12.28.06. \$WRKSH-P-SYM-5-POINT-NAT-SSOR-SI

200020100200222 002.35 090.15 000.00 010.25

7/11/26/																
9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	5	4178	1.55	.88	0	.68
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	7	5770	3.81	1.99	.00	1.82
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	9	10682	11.42	5.56	.00	5.86

5 RECORD 14 ITPACK METHODS GROUP3

14 05/14/78 12.24.00. \$WRKSH-P-SYM-5-POINT-NAT-LJ-SI

200020100200222 002.35 090.15 000.00 010.25

8/11/27/																
9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	13	4349	1.52	.87	.00	.64
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	20	6121	4.07	2.01	.00	2.05
21	21	1	5.000E-02	441	3.35E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	34	11585	14.85	5.65	.00	9.20

14 05/14/78 12.28.32. \$WRKSH-P-SYM-5-POINT-NAT-LJ-CG

200020100200222 002.35 090.15 000.00 010.25

8/11/28/																
9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	9	4630	1.43	.88	0	.54

13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	13	8490	3.68	2.01	.00	1.66
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	19	12226	12.02	5.63	.00	6.39
14 05/14/78 12.28.15. \$WRKSHF-SYM-5-POINT-NAT-L-SSOR-CG																
200020100200222 002.35 090.15 000.00 010.25																
8/11/32/																
9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	4	4792	1.41	.88	.00	.53
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	6	6628	3.58	2.00	.00	1.58
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	7	13108	10.84	5.64	.00	5.20
14 05/14/78 12.30.00. \$WRKSHF-SYM-5-POINT-NAT-L-SSOR-PA																
200020100200222 002.35 090.15 000.00 010.25																
8/11/33/																
9	9	1	1.250E-01	81	3.16E+00	1.77E-01	5.84E+02	1.00E+00	3.27E+01	0	0	4532	.93	.88	0	.05
13	13	1	8.333E-02	169	3.33E+00	1.27E-01	5.95E+02	1.00E+00	2.17E+01	0	0	6628	2.13	2.00	.00	.12
21	21	1	5.000E-02	441	3.30E+00	8.13E-02	5.81E+02	1.00E+00	1.30E+01	0	0	12308	5.97	5.63	.00	.34
14 05/14/78 12.31.07. \$WRKSHF-SYM-5-POINT-NAT-L-SSOR-SI																
200020100200222 002.35 090.15 000.00 010.25																
8/11/34/																
9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	4	4511	1.39	.88	.00	.50
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	6	6459	3.43	1.99	.00	1.44
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	7	12467	10.13	5.62	.00	4.51
5 RECORD 14 ITPACK METHODS GROUP4																
14 05/14/78 12.11.49. \$WRKSHF-SYM-5-POINT-NAT-LJ-SI																
200020100200222 002.35 090.15 000.00 010.25																
8/15/27/																
9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	13	4349	1.51	.88	0	.63
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	20	6121	4.06	1.99	.00	2.06
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	34	11585	14.78	5.65	.00	9.13
14 05/14/78 12.14.02. \$WRKSHF-SYM-5-POINT-NAT-LJ-CG																
200020100200222 002.35 090.15 000.00 010.25																
8/15/28/																
9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	9	4630	1.43	.88	0	.54
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	13	6490	3.67	2.01	0	1.66
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	19	12226	12.04	5.60	0	6.44
14 05/14/78 12.15.59. \$WRKSHF-SYM-5-POINT-LREDBL-LJ-CJCG																
200020100200222 002.35 090.15 000.00 010.25																
8/15/29/																
9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	5	4549	1.14	.89	.00	.25
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	7	6321	2.69	2.00	0	.69
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	10	11785	8.11	5.64	.00	2.47
14 05/14/78 12.17.34. \$WRKSHF-SYM-5-POINT-LREDBL-L-RSSI																
200020100200222 002.35 090.15 000.00 010.25																
8/15/30/																
9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	7	4263	1.13	.89	0	.25
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	10	5662	2.74	2.00	.00	.74
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	18	11144	8.89	5.63	0	3.26
14 05/14/78 12.18.50. \$WRKSHF-SYM-5-POINT-LREDBL-L-RSSS																
200020100200222 002.35 090.15 000.00 010.25																
8/15/31/																
9	9	1	1.250E-01	81	2.11E-01	8.20E-03	3.50E+02	8.86E+01	1.48E+01	3.21E+00	5	4630	1.11	.89	0	.23
13	13	1	8.333E-02	169	9.09E-02	2.54E-03	2.12E+02	1.54E+01	6.60E+00	3.40E+00	7	6490	2.63	1.99	.00	.64
21	21	1	5.000E-02	441	3.36E-02	5.67E-04	1.49E+02	6.19E+01	2.24E+00	3.32E+00	10	12226	7.91	5.63	0	2.27