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Charts for the Dynamic Properties of Counterweights

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CHARTS FOR THE DYNAMIC PROPERTIES OF COUNTERWEIGHTS

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INTRODUCTION

Dynamic analyses of reciprocating machines often require that one compute masses and moments of inertia of various shaped counterweights. Relations for the determination of these properties are not generally available in the literature. In this paper, formulas and graphs will be presented for the rapid determination of these properties for several commonly encountered shapes.

FUNDAMENTALS

Figure (2) illustrates a general counterweight of constant thickness t and having two parallel flat faces of area A oriented perpendicular to the c-c and o-o axes. The polar mass moment of inertia of this element relative to the o-o axis is

$$J = \int r^2 \rho t \, dA = t \int r^2 \rho \, dA \qquad (1)$$

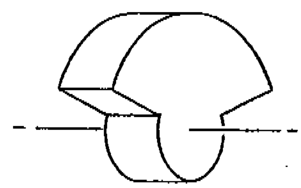
where ρ is the material density.

For counterweights, ρ is constant so that it is sufficient for us to determine only the area polar moment of inertia

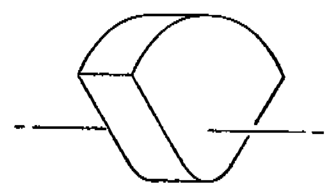
$$I = \frac{J}{\rho t} = \int r^2 \, dA \qquad (2)$$

For the dynamic analysis of compressors, one also requires the distance to the center of mass, R_{cm}, and the area, A. The counterweight mass is then

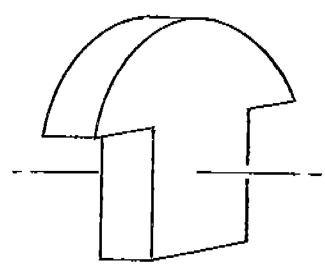
$$m = \rho t A \qquad (3)$$



Configuration #1



Configuration #2



Configuration #3

FIG. 1: Three Common Counterweight Designs

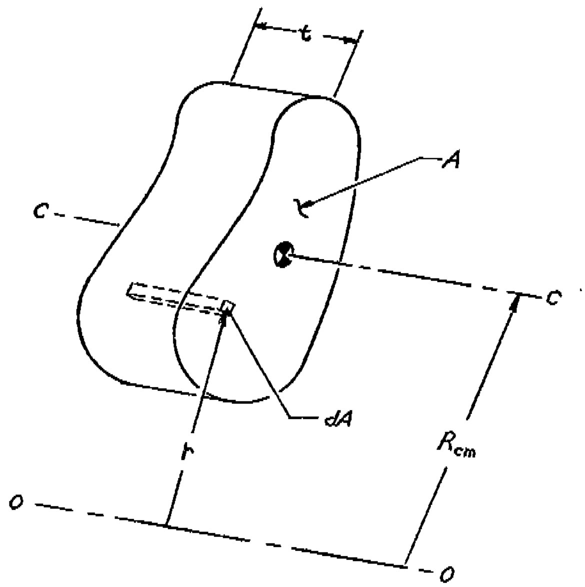


FIG. 2: General Counterweight

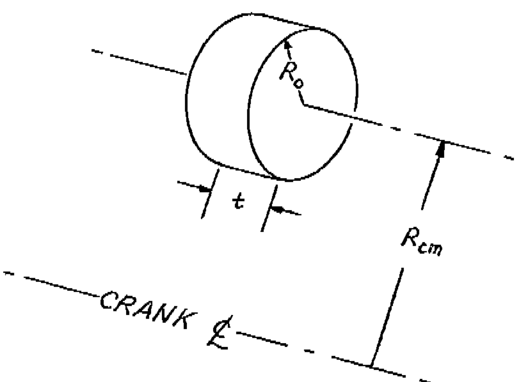


FIG. 3: Simple Disk Counterweight

For the simple disk counterweight of Figure (3), R_{cm} is easily known and

$$A = \pi R_o^2 \tag{4}$$

The moment of inertia about the crank centerline is

$$I = \int_0^{R_o} 2\pi r^3 dr + \pi R_o^2 R_{cm}^2 = \frac{\pi R_o^4}{2}\left\{1 + 2\left[\frac{R_{cm}}{R_o}\right]^2\right\} \tag{5}$$

For simple configurations such as this the necessary parameters may be determined from tables and formulas in standard texts and handbooks.

For the configurations of Figure (1), time consuming development of equations and their evaluation must be carried out by the designer. In the sections which follow these equations will be presented along with graphs for their rapid evaluation. Details of the derivations will only be presented for configuration #1.

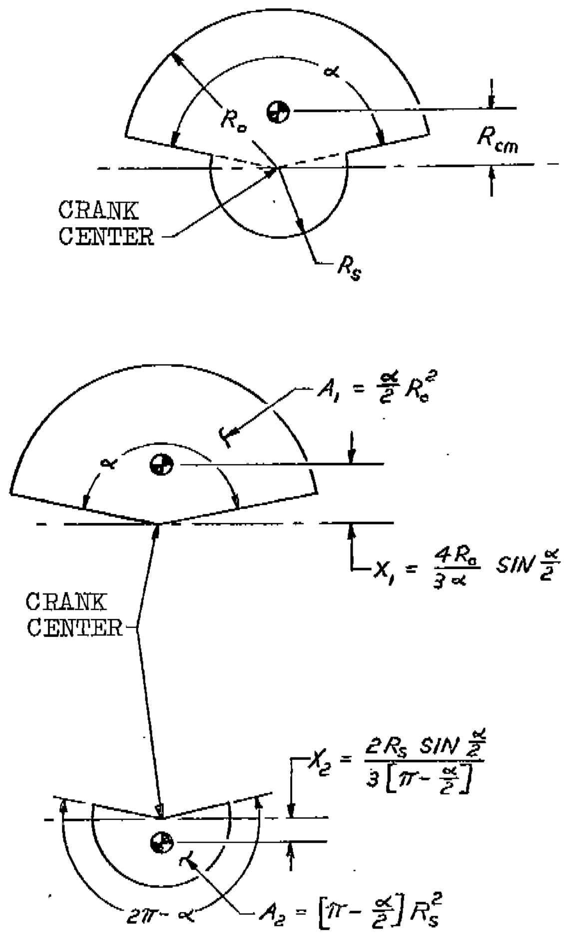


FIG. 4: Dimensions for Configuration #1

CONFIGURATION #1

The total area for this shape is

$$A = \frac{\alpha}{2} R_o^2 + \frac{2\pi - \alpha}{2} R_s^2 \tag{6}$$

and the moment of inertia about the crank center is

$$I = \frac{\alpha}{4} R_o^4 + \frac{2\pi - \alpha}{4} R_s^4 \tag{7}$$

The location of the centroids of the two parts is shown in Figure (4). To find R_{cm} we write

$$AR_{cm} = A_1 X_1 - A_2 X_2 \tag{8}$$

to obtain

$$R_{cm} = \frac{\frac{2}{3}\left[R_o^3 - R_s^3\right]\sin\frac{\alpha}{2}}{\frac{\alpha}{2}R_o^2 + \left[\pi - \frac{\alpha}{2}\right]R_s^2} \tag{9}$$

For rapid evaluation, charts may be created for the ratio of A to A for a circle of radius R_o

$$\frac{A}{A_o} = \frac{A}{\pi R_o^2} = \frac{1}{2\pi}\left\{\alpha + [2\pi-\alpha]\left[\frac{R_s}{R_o}\right]^2\right\} \tag{10}$$

the ratio of I to I for a circle of radius R_o

$$\frac{I}{I_o} = \frac{2I}{\pi R_o^4} = \frac{1}{2\pi}\left\{\alpha + [2\pi-\alpha]\left[\frac{R_s}{R_o}\right]^4\right\} \tag{11}$$

and R_{cm}/R_o

$$\frac{R_{cm}}{R_o} = \frac{4\left\{1-\left[\frac{R_s}{R_o}\right]^3\right\}\sin\frac{\alpha}{2}}{3\left\{\alpha+[2\pi-\alpha]\left[\frac{R_s}{R_o}\right]^2\right\}} \tag{12}$$

These relations are plotted in Figures (7), (8), and (9).

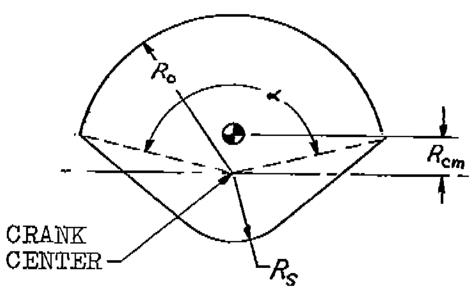


FIG. 5: Dimensions for Configuration #2

CONFIGURATION #2

For this shape the area is

$$\frac{A}{A_o} = \frac{A}{\pi R_o^2} = \frac{1}{\pi}\left\{\frac{\alpha}{2} + \beta\epsilon^2 + \epsilon\sqrt{1-\epsilon^2}\right\} \tag{13}$$

the moment of inertia about the crank center is

$$\frac{I}{I_o} = \frac{2I}{\pi R_o^4} = \frac{1}{\pi}\left\{\frac{\alpha}{2} + \beta\epsilon^4 + \frac{\epsilon}{3}[1+2\epsilon^2]\sqrt{1-\epsilon^2}\right\} \tag{14}$$

and the center of mass is at

$$\frac{R_{cm}}{R_o} = \frac{2\left[\sin\frac{\alpha}{2} - \epsilon^3\sin\beta\right] - \epsilon\cos(\beta+\gamma)\sqrt{[1+3\epsilon^2][1-\epsilon^2]}}{3\left[\frac{\alpha}{2} + \beta\epsilon^2 + \epsilon\sqrt{1-\epsilon^2}\right]} \tag{15}$$

where

$$\epsilon = \frac{R_s}{R_o} \tag{16}$$

$$\beta = \pi - \frac{\alpha}{2} - \tan^{-1}\left(\frac{\sqrt{1-\epsilon^2}}{\epsilon}\right) \tag{17}$$

$$\gamma = \tan^{-1}\left(\frac{\sqrt{1-\epsilon^2}}{2\epsilon}\right) \tag{18}$$

There is a limiting value on α for this shape which occurs when the two lines tangent to the circle of radius R_s meet. This is

$$\alpha_{max} = 2\left\{\pi - \tan^{-1}\left(\frac{\sqrt{1-\epsilon^2}}{\epsilon}\right)\right\} \tag{19}$$

Equations (13), (14), and (15) are plotted in Figures (10), (11), and (12).

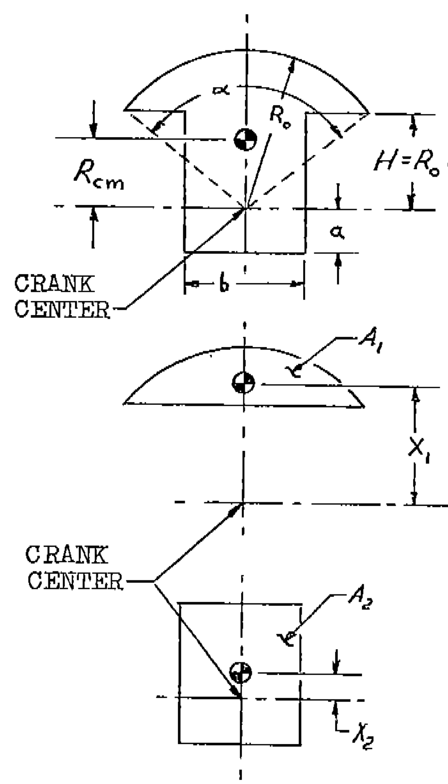


FIG. 6: Dimensions for Configuration #3

CONFIGURATION #3

For this shape, the properties may be easily determined without the need for charts. For the circular segment the area is

$$A_1 = \frac{1}{2} R_o^2 \left[\alpha - \sin\alpha\right] \tag{20}$$

the center of mass is located at .

$$X_1 = \frac{4 R_o \sin^3(\alpha/2)}{3\left[\alpha - \sin\alpha\right]} \tag{21}$$

and the moment of inertia about the crank center is

$$I_1 = R_o^4 \left\{ \frac{\alpha}{4} - \frac{1}{6}\cos(\tfrac{\alpha}{2})\sin(\tfrac{\alpha}{2})\left[1 + 2\cos^2(\tfrac{\alpha}{2})\right]\right\} \tag{22}$$

For the rectangular portion

$$A_2 = b\left[a + H\right] \tag{23}$$

$$X_2 = \frac{1}{2}\left[H - a\right] \tag{24}$$

$$I_2 = \frac{b^3}{12}\left[H + a\right] + \frac{b}{3}\left[H^3 + a^3\right] \tag{25}$$

With present electronic pocket calculators, the evaluation of these equations is not difficult except for equation (22). Figure (13) may be of assistance here, being a plot of

$$\frac{I_1}{R_o^4} = \frac{\alpha}{4} - \frac{1}{6}\cos(\tfrac{\alpha}{2})\sin(\tfrac{\alpha}{2})\left[1 + 2\cos^2(\tfrac{\alpha}{2})\right] \tag{26}$$

To locate the counterweight mass center,

$$R_{cm} = \frac{A_1 X_1 + A_2 X_2}{A_1 + A_2} \tag{27}$$

which is

$$R_{cm} = \frac{\frac{2}{3} R_o^3 \sin^3(\alpha/2) + \frac{b}{2}\left[H^2 - a^2\right]}{\frac{1}{2} R_o^2\left[\alpha - \sin\alpha\right] + b\left[a + H\right]} \tag{28}$$

EXAMPLE

To illustrate the use of these charts, consider the problem of finding the thickness t of a counterweight of configuration #2 which will yield

$$m R_{cm}^2 = 10.0 \text{ gm-cm}^2$$

when the material is steel with $\rho = 7.86$ gm/cm^3,

$$R_s = 1.3 \text{ cm}$$
$$R_o = 2.0 \text{ cm}$$

and the angle α is variable.

To relate t to the parameters on the charts, write

$$m R_{cm}^2 = \rho t A R_{cm}^2 = \rho t \pi R_o^4 \left[\frac{A}{A_o}\right]\left[\frac{R_{cm}}{R_o}\right]^2 = 10.0$$

Inserting the given data then gives

$$t = \frac{0.0254}{\left[\frac{A}{A_o}\right]\left[\frac{R_{cm}}{R_o}\right]^2}$$

By taking values from Figures (10) and (12) at various values of α along the line

$$\frac{R_s}{R_o} = \frac{1.3}{2.0} = 0.65$$

the following table can be quickly created.

α (DEG)	R_{cm}/R_o	A/A_o	t (cm)
20	0.115	0.490	3.92
40	0.151	0.524	2.13
60	0.180	0.557	1.41
80	0.200	0.590	1.08
100	0.213	0.621	0.90
120	0.219	0.655	0.81
140	0.218	0.685	0.78
160	0.210	0.719	0.80
180	0.197	0.751	0.87
200	0.179	0.784	1.01
220	0.157	0.817	1.26
240	0.133	0.850	1.69
260	0.107	0.881	2.52

CONCLUSIONS

The graphs presented allow one to quickly evaluate the dynamic properties of several common counterweight shapes. The equations from which the graphs were created may be used directly or programmed for computer solution as a useful design aid.

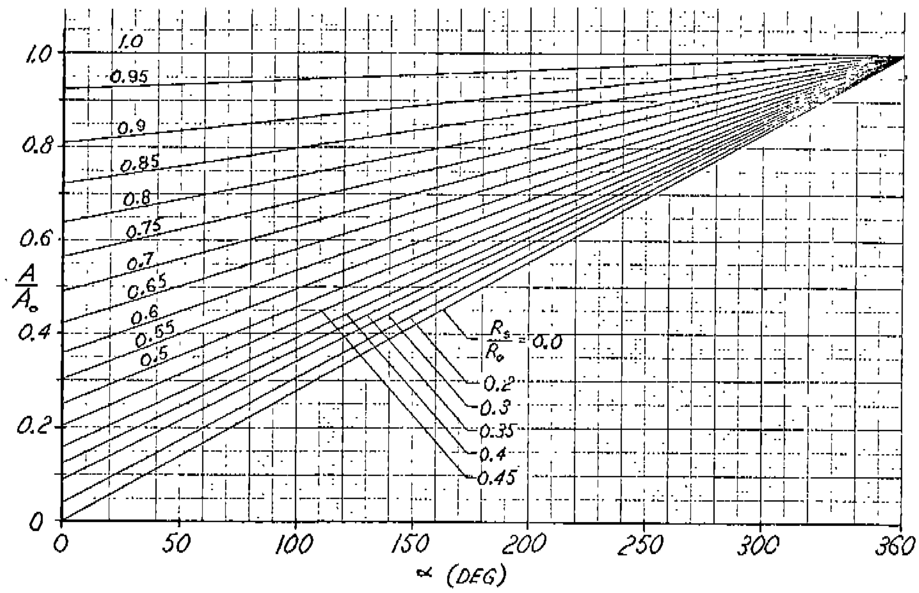


FIG. 7: A/A_o for Configuration #1

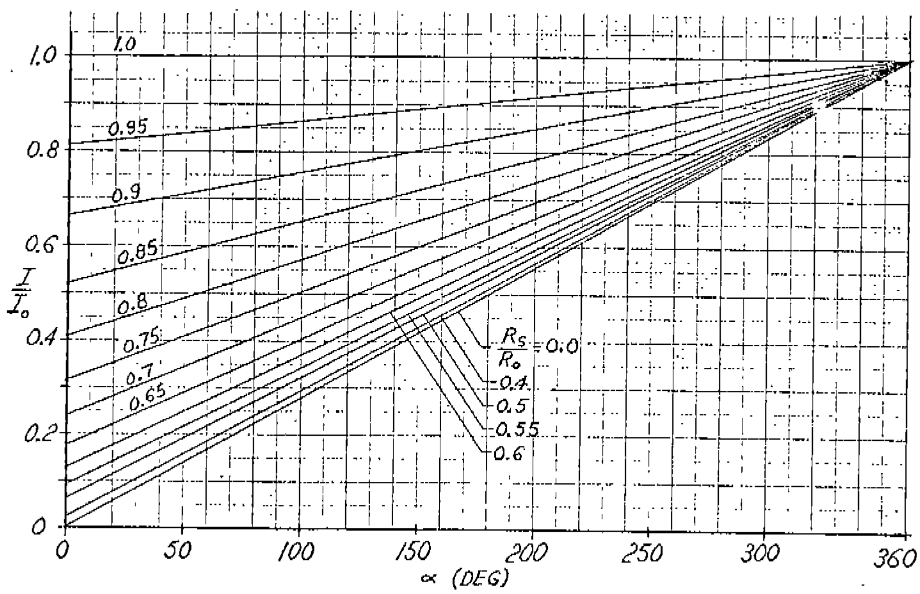


FIG. 8: I/I_o for Configuration #1

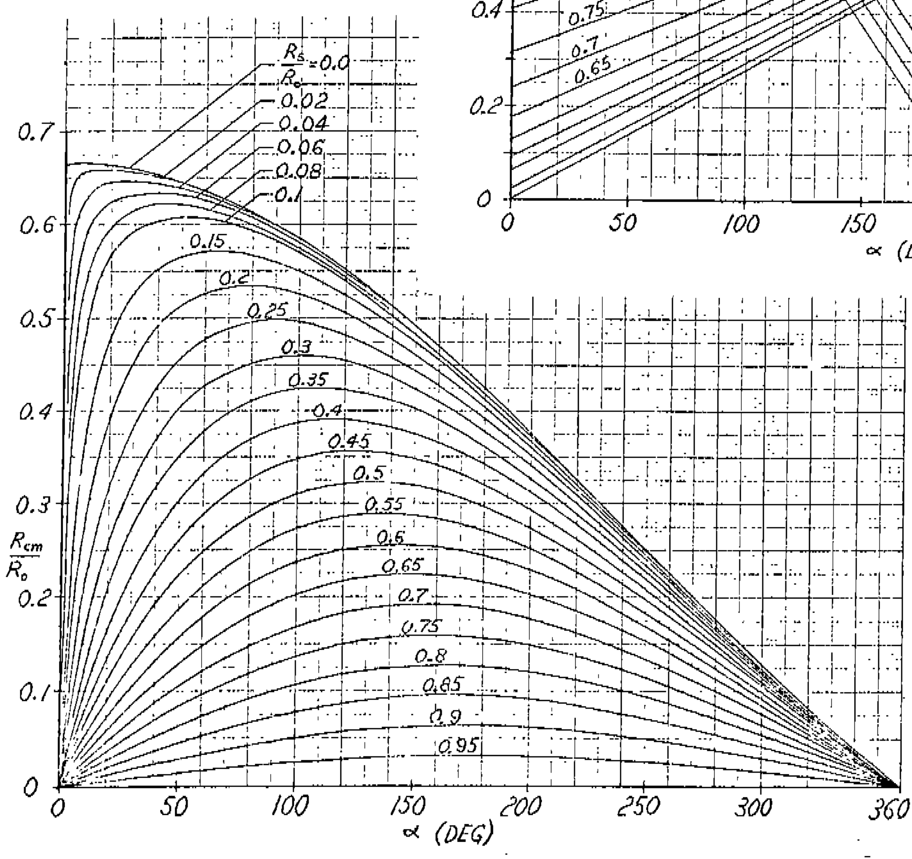


FIG. 9: R_{cm}/R_o for Configuration #1

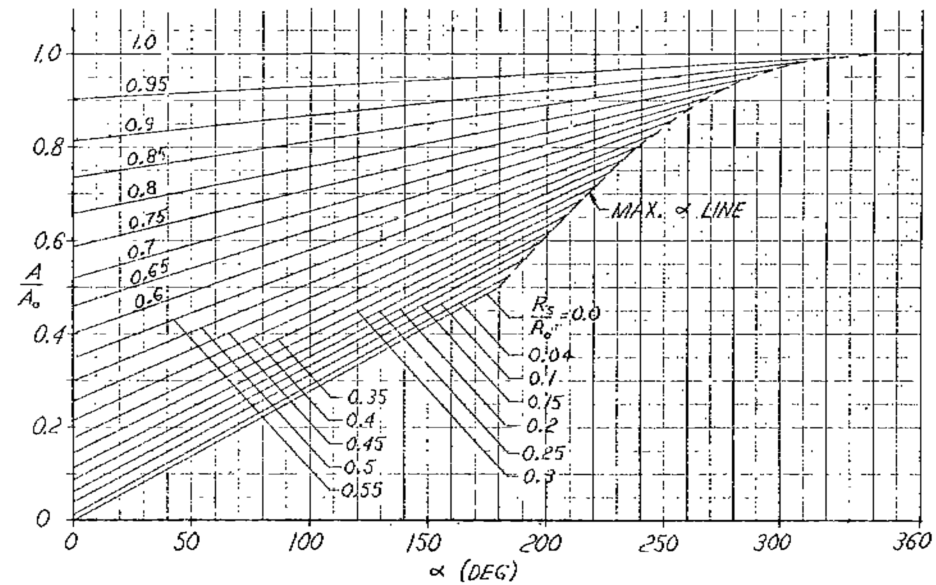


FIG. 10: A/A$_o$ for Configuration #2

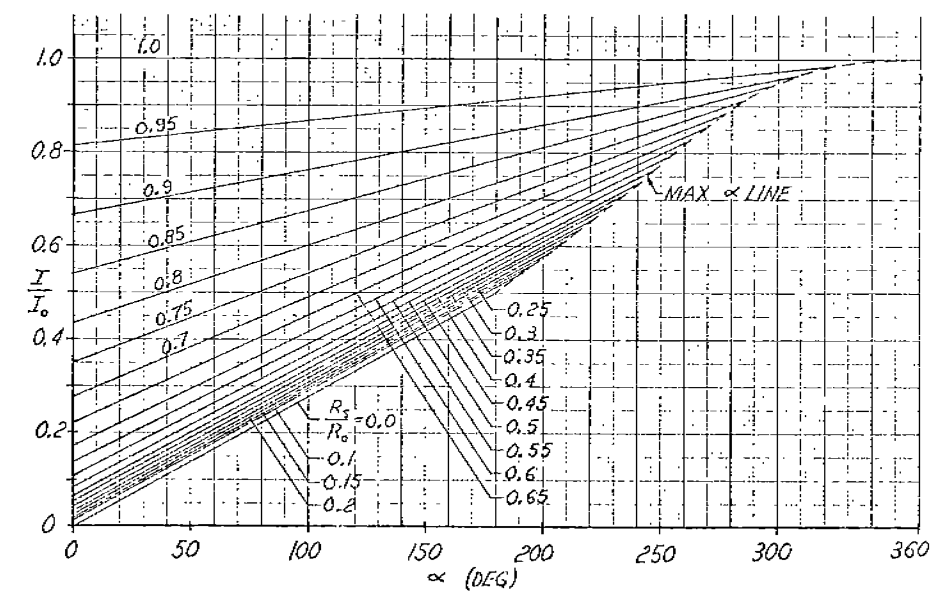


FIG. 11: I/I$_o$ for Configuration #2

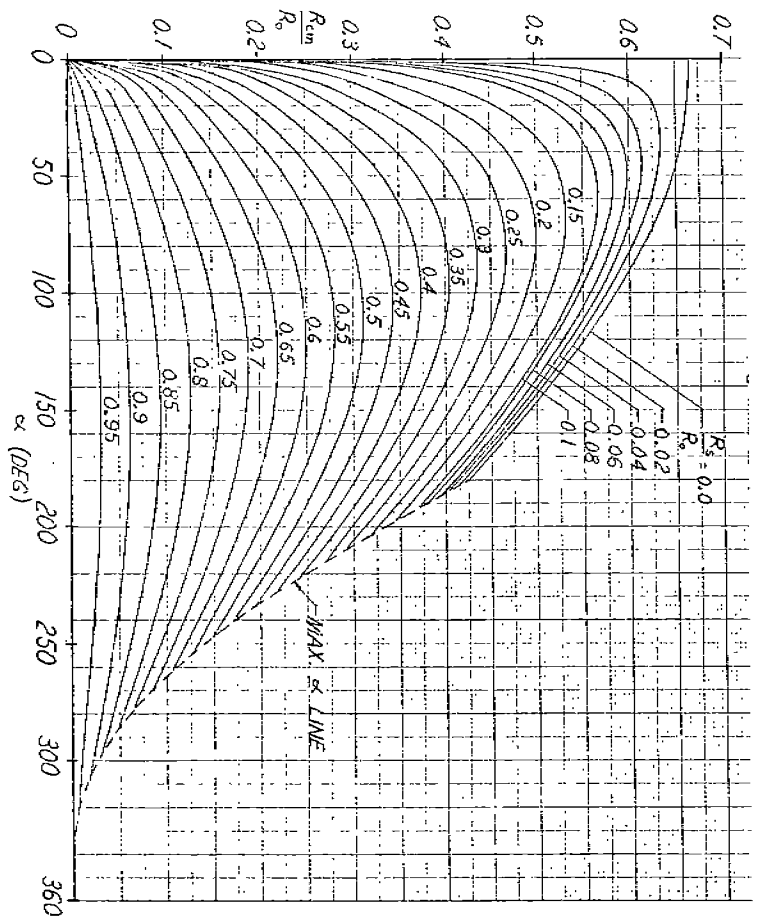


FIG. 12: R$_{cm}$/R$_o$ for Configuration #2

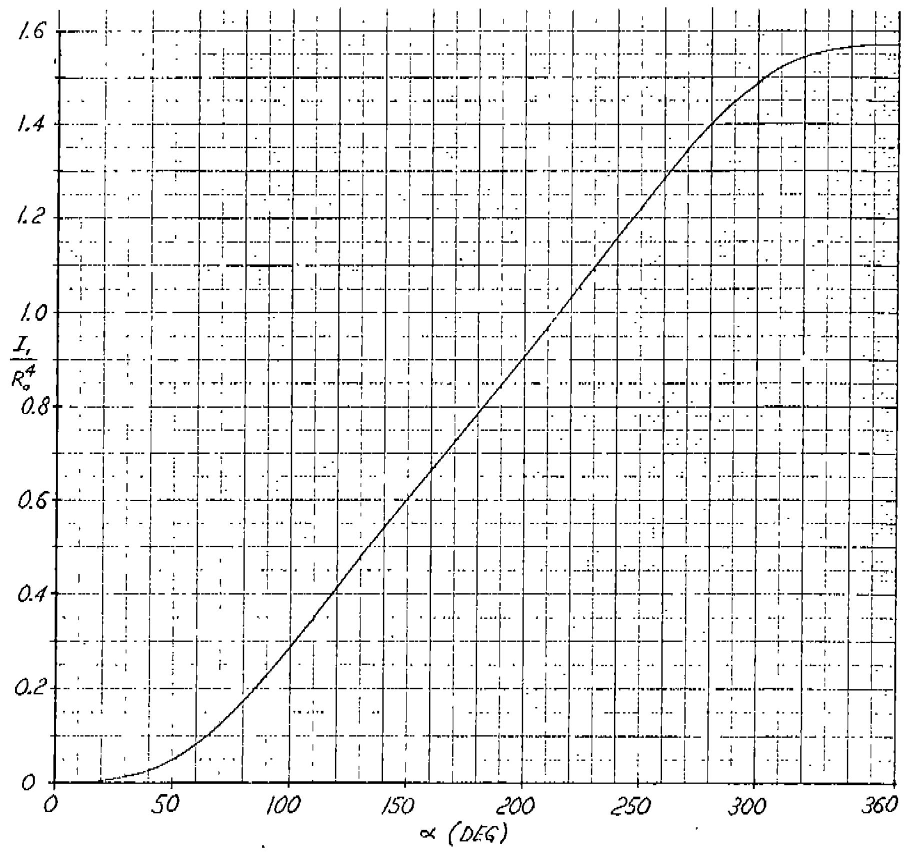


FIG. 13: I_1/R_o^4 <u>for</u>
<u>Configuration #3</u>