Hydrodynamic Lubrication of Sliding Vanes

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HYDRODYNAMIC LUBRICATION OF SLIDING VANES

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INTRODUCTION

There are very few parts moving under load in a single-stage valveless, oil-flooded, sliding-vane air compressor operating at shop air pressures of 90 to 125 psi. With only one simple rotor, two rolling bearings, and a seal, it is highly reliable provided the hard-working parts-the vanes- are designed for long life.

Vanes are often made of phenolic resin laminates using various bases such as asbestos cloth. They are supposed to be strong, light weight, and non-scoring. Unfortunately they also warp, swell, delaminate, wear and suffer fatigue failures. Seeking greater reliability, Worthington turned to metallic vanes in 1961. The material selected was a hyper-eutectic aluminum-silicon alloy originally developed for engine pistons. Hardness, resistance to wear and corrosion, and low thermal expansion made it an excellent choice and brought relief from the problems associated with the plastic vanes.

Through the 1960's the vane tip speeds and loads increased as larger single-stage compressors were designed to operate at higher speeds, particularly in our engine-driven portable compressors. A disconcerting tendency to score the vane tips and cylinder under certain conditions became the stumbling-block to a new design in 1967. The search for a solution led to the following study of hydrodynamic lubrication. The requirements of new designs or unusual conditions of service can now be determined with a high degree of confidence.

Assumptions made in the analysis may yield less-than-perfect accuracy. This is also true for journal bearings. Nevertheless, in both cases the results bring design along the road from art toward science.

THE HYDRODYNAMIC WEDGE-TYPE OIL FILM

Consider the vane tip a simple form of fixed slider. Figure 1a. shows the actual case with tip radius, R_T, moving over the cylinder radius, R_c, with relative velocity U. Figure 1b. shows how the analysis may be simplified by considering a flat surface and an equivalent tip curvature equal to the difference in curvature of tip radius and cylinder wall.

$$\frac{1}{r} = \frac{1}{R_T} - \frac{1}{R_c}\ , \text{ or, } \quad r = \frac{R_c R_T}{R_c - R_T} \tag{1}$$

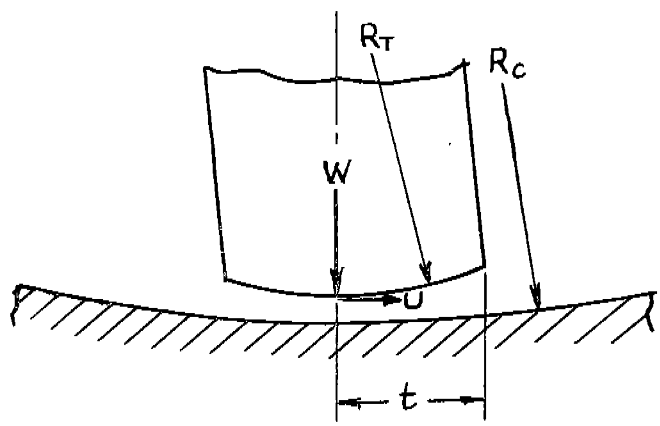

Fig. 1a Vane Tip and Cylinder

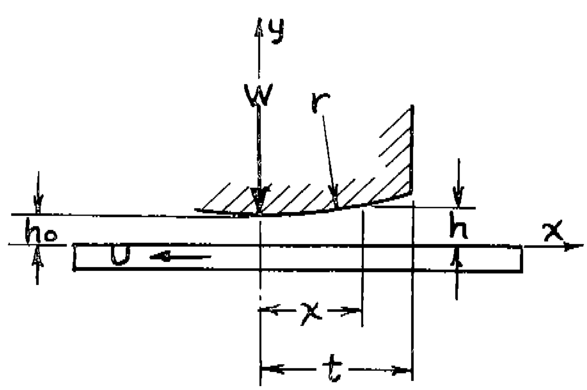

Fig. 1b Simplified Geometry for Lubrication Analysis

Reynolds equation for pressure build-up in a wedge type fluid film (1) may be written in the familiar form for a slider of infinite length:

$$\frac{dP}{dx} = 6\mu U\left(\frac{1}{h^2} - \frac{K}{h^3}\right) \tag{2}$$

where μ is the absolute viscosity of the fluid. Integration of this equation requires definition of h in terms of the variable, x, and the minimum film, h_o. A good approximation yielding an easily integrated expression is:

$$h = h_o + \frac{x^2}{2r} \tag{3}$$

Substitute in equation 2, integrate, and evaluate both the constant of integration and K by applying the knowledge that $P=0$ when $x=0$ (where the wedge diverges) and $P=0$ when $x=t$. The constant of integration is zero. K becomes simply $4h_o/3$ by dropping a term which is insignificant in the range of practical values. K physically represents the film thickness at the point of maximum pressure. The expression for the pressure at distance x from the tangent point is:

$$P = \frac{2\mu U x}{\left(h_o + \frac{x^2}{2r}\right)^2} \tag{4}$$

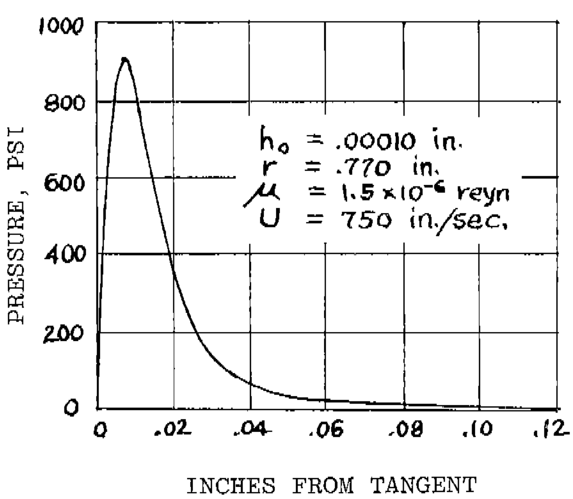

Fig. 2 Pressure Distribution in Film Under Vane Tip

Figure 2 shows a plot of the pressure distribution for some typical values. Note the rapid rise and fall of pressure. A very narrow portion of the tip carries most of the load. Note also from equation 4 that P is not zero when $x=t$ because of our assumption regarding K. Except for very small values of t it comes close enough.

For practical application, the film thickness must be found in terms of the load on the vane tip. Integrate the pressure under the tip between the limits of 0 and t to obtain the load carried by a unit length of film.

$$\frac{W}{L} = w = \int_0^t P\,dx = 2\mu U \int_0^t \frac{x\,dx}{\left(h_o + \frac{x^2}{2r}\right)^2} \tag{5}$$

This yields the expression:

$$w = 2\mu U r\left(\frac{1}{h_o} - \frac{1}{h_o + \frac{t^2}{2r}}\right) \quad \text{or;} \tag{6}$$

$$h_o = -\frac{t^2}{4r} + \sqrt{\left(\frac{t^2}{4r}\right)^2 + \frac{\mu U t^2}{w}} \tag{7}$$

Calculation of the film thickness for the given operating conditions begins the design process. However, every symbol in equation 7 is a variable during a single revolution, and each must be calculated in terms of the given values.

Vane Tip Load

A section of the type of compressor being analyzed is shown in figure 3.

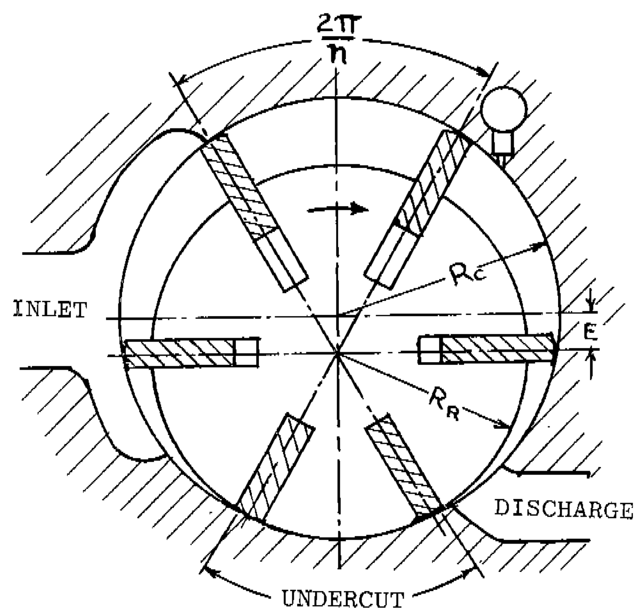

Fig. 3 Typical Compressor Section with n Vanes

Rotor slots are radial and vanes are flat-sided. The rotor clearance bore intersects the cylinder bore. The angle of this undercut is a few degrees greater than the angle between vanes. Thus, a vane is always sealing the clearance gap between suction and discharge.

The oil film calculation requires evaluation of both the load on the vane tip perpendicular to the cylinder wall tangent and the velocity of this tangent point. Refer to figure 4. The inertia force acting radially from the center of the rotor may be considered to be the sum of the centrifugal force and the force causing the vane to reciprocate in the slot. The former is simply:

$$F_1 = \omega^2 R_G M/g$$

where R_G is the varying radial distance of the vane center of gravity from the rotor center. In terms of the angle of rotation:

$$R_G = E\cos\theta - d + \sqrt{R_c^2 - E^2\sin^2\theta} \tag{8}$$

The reciprocating motion is analogous to that of the slider in a crank-slider mechanism where the crank radius is the eccentricity, E, the rod length is bore radius, R_c , and rotation is about the center of the rotor. Omitting the derivation, which is found in many handbooks, this inertia force is expressed by:

$$F_2 = \omega^2 \frac{M}{g}\left[E\cos\theta + \frac{E^2}{R_c}\cos 2\theta\right] \tag{9}$$

Summing the two forces, the total inertia force becomes:

$$W_I = \frac{\pi^2 N^2 M}{900\, g}\left(2E\cos\theta + \frac{E^2}{R_c}\cos 2\theta - d + \sqrt{R_c^2 - E^2\sin\theta}\right) \tag{10}$$

This is the component of the force parallel to the vane slot. The force perpendicular to the cylinder wall is:

$$W = \frac{W_I}{\cos\alpha}\,, \quad \cos\alpha = \sqrt{1 - \left(\frac{E\sin\theta + A - T/2}{R_c - R_T}\right)^2} \tag{11}$$

This solution is not exact, because the full effect of the vane tip radius is not considered but it is more than adequate for this analysis.

During the compression part of the cycle the vane tip load is altered by the pressure across the vane. From the leading edge of the vane to the tangent point the pressure is assumed to be increased by that in the compression cell ahead, P_2 and from the tangent point to the trailing edge by that in the following cell, P_1. If the slot beneath the vane is vented to the lower pressure of the trailing cell, the effect on the tip load per unit length is to reduce it by:

$$w_p = (P_2 - P_1)\, t \tag{12}$$

This is the favored construction; but the design must avoid a force that will lift the vane from the cylinder wall.

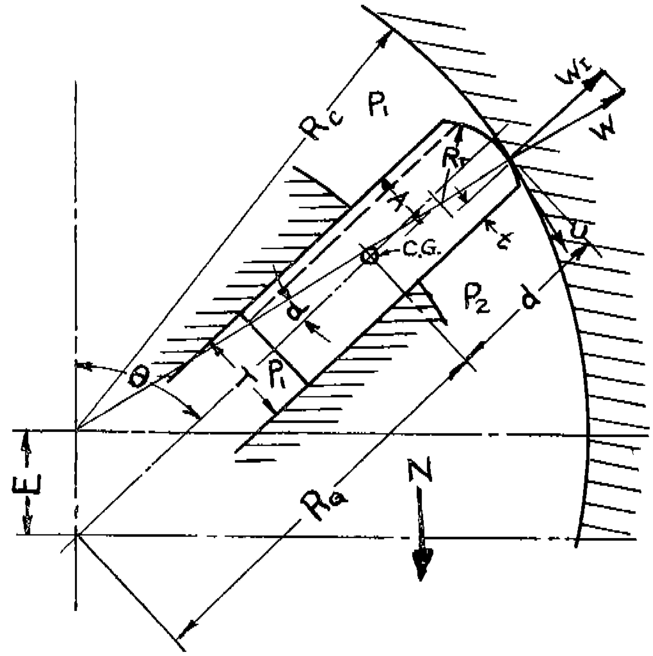

Fig. 4 Vane Tip Load and Velocity

If the slot is vented to the higher pressure of the leading compression cell, the tip load is increased by:

$$w_p = (P_2 - P_1)(T - t) \tag{13}$$

Dimension t, from tangent to leading edge, is approximated by an expression based on similar triangles:

$$t = T - A - \frac{R_T}{R_c - R_T}\left(E\sin\theta + A - \frac{T}{2}\right) \tag{14}$$

This value is also used in equation 7 for calculating film thickness.

Calculation of Gas Pressure

Perhaps it is unnecessary to review the calculation of compression cell volume but it makes the presentation complete. Figure 5 is a section showing a compression cell of a compressor with n vanes. The volume being compressed is the vane length times the area bounded by facing vane surfaces, cylinder wall and rotor, together with the volume in the slot beneath the leading vane. In this case, the latter is connected by relief ports to the cell and is assumed to be the same as that volume of the vane extending above the rotor surface.

The radius from the rotor center to the cylinder wall at the vane center for the vane at any rotor angle ϕ is:

$$X = E\cos\phi + \sqrt{R_c^2 - E^2\sin^2\phi} \tag{15}$$

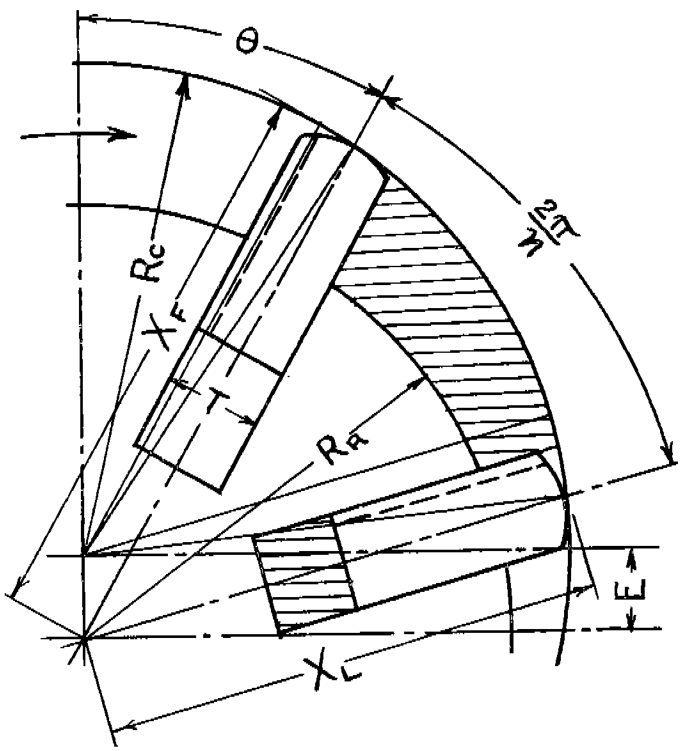

Fig. 5 Volume Between Adjacent Vanes

Denote the radius for the leading vane as X_L where $\phi = \theta + 2\pi/n$ and for the following vane as X_F where $\phi = \theta$. Then summing the area defined in the figure, but neglecting the tiny effect of the tip radius, yields:

$$A = \frac{\pi}{n}\left(R_c^2 + R_R^2\right) + \frac{X_L}{2}\left[E\sin\left(\theta + \frac{2\pi}{n}\right) + T\right] - \frac{X_F}{2}\left(E\sin\theta + T\right) + \frac{R_c^2}{2}\left(\sin^{-1}\left[\frac{E}{R_c}\sin\left(\theta + \frac{2\pi}{n}\right)\right] - \sin^{-1}\left[\frac{E}{R_c}\sin\theta\right]\right) \quad (16)$$

The maximum displacement occurs when the vanes straddle the common centerline and $\theta = -\pi/n$

The section area then simplifies to:

$$A_1 = \frac{\pi}{n}\left(R_c^2 - R_R^2\right) + X E \sin\frac{\pi}{n} + R_c^2 \sin^{-1}\left(\frac{E}{R_c}\sin\frac{\pi}{n}\right) \quad (17)$$

The pressure in the cell may be calculated by the equation for polytropic compression:

$$P = P_1\left(\frac{A_1}{A}\right)^n \quad (18)$$

The value of the exponent, n, is best determined experimentally. For us, a value of 1.45 to 1.5 seems to account for initial heating, oil injection, and leakage. Great accuracy is not justified for this purpose.

Friction Between Vane and Rotor Slot

Pressure ahead of the vane in the compression portion tends to cock the vane in the slot. If those portions of the slot against which the vane bears heavily are not well finished, the friction between the vane and the slot may produce a high load on the tip. In well finished slots with reasonable projection of the vane, the friction force is small compared to inertia and pressure forces and may be neglected.

Vane Tip Tangent Velocity

The velocity of the tangent point at any vane angle, θ, with reference to the notation in Fig. 4 is:

$$U = R_c \frac{d(\theta + \alpha)}{dt} = R_c\left(\frac{d\theta}{dt} + \frac{d\alpha}{dt}\right) \quad (19)$$

but

$$\alpha = \sin^{-1}\left(\frac{E\sin\theta + A - T/2}{R_c - R_T}\right) \quad (20)$$

and

$$\frac{d\alpha}{dt} = \frac{E\cos\theta}{\sqrt{(R_c - R_T)^2 - (E\sin\theta + A - T/2)^2}} \frac{d\theta}{dt} \quad (21)$$

Since the angular velocity of the rotor is: $\frac{d\theta}{dt} = 2\pi N$

substituting these values in equation 19 yields:

$$U = \frac{\pi N R_c}{30}\left(1 + \frac{E\cos\theta}{\sqrt{(R_c - R_T)^2 - (E\sin\theta + A - T/2)^2}}\right) \quad (22)$$

Lubricant Viscosity

The absolute viscosity of the lubricant must be known for the film thickness calculation. Usually a chart is available showing kinematic viscosity versus temperature together with the API gravity or the specific gravity, 60°F/60°F. This must be converted to absolute viscosity, the kinematic viscosity multiplied by the density.

Viscosity of petroleum oils given in Saybolt Seconds Universal (SSU) may be converted to centistokes (2) by one of the following:

$$\nu = .226\ SSU - 195/SSU \quad \text{for } 32 < SSU < 100 \quad (23a)$$

$$\nu = .220\ SSU - 135/SSU \quad \text{for } SSU > 100 \quad (23b)$$

The density at 60°F in g/cm^3 is related to °API and specific gravity at 60°F by the following, (3):

$$\rho_{60} = .999\ Sp.Gr.\ 60/60°F = 141.4/(°API + 131.5) \quad (24)$$

At a temperature other than 60°F, the density may be calculated from:

$$\rho_T = \rho_{60}/[1 + \alpha(T+60)] \qquad (25)$$

where the coefficient of expansion, α, averages about .00045 for the petroleum base lubricants normally used. The absolute viscosity in centipoises becomes:

$$Z = \rho_T \nu \qquad (26)$$

For viscosity in Reyns:

$$\mu = Z \times 1.45 \times 10^{-7} \qquad (27)$$

These are (lbf) (sec)/(in^2) units to agree with the derivation in English units.

A simple mathematical expression for absolute viscosity at any temperature in the range of normal operation is desirable for ease in calculation. Over such a small range of perhaps 200°F, it is assumed that the relation of temperature and viscosity is a straight line on a log-log plot. Determine absolute viscosity at two temperatures as outlined above, μ_1 at T_1 and μ_2 at T_2, then at temperature T:

$$\mu = K_2/T^{K_3} \qquad (28)$$

where $$K_3 = \log\left(\frac{\mu_1}{\mu_2}\right) / \log\left(\frac{T_2}{T_1}\right) \qquad (29)$$

and $$K_2 = \mu_1 T_1^{K_3} = \mu_2 T_2^{K_3} \qquad (30)$$

The lubricant entering the squeeze area under the vane tip is assumed to be at the temperature of the cylinder wall. This temperature and its variation around the cylinder depends on design and should be determined experimentally. Oil films calculated with viscosity at this temperature yield operating results which are not in harmony with laboratory tests. Such agreement seems to be reached when the effective viscosity is that at the average of the cylinder wall temperature and the temperature of the oil leaving the tip after having been heated by friction.

Calculating the Effect of Frictional Heating

The lubricant passing under the vane tip is heated by friction caused by the viscous shearing of the film. The friction is readily calculated if film thickness is known. Without giving the derivation, which is available in text books (1), the differential friction for a unit length of vane is:

$$dF = \left[\frac{1}{2}\frac{dP}{dx}(2y-h) + \frac{\mu U}{h}\right]dx \qquad (31)$$

Integration between the limits of $x=0$ and $x=t$ yields two values of friction force: at the cylinder surface where $y=0$, and at the vane surface, $y=h$. The larger, at $y=0$, includes the component of the film pressure against the tip parallel to the cylinder surface. The term $\frac{dP}{dx}$ for equation 30 is obtained by differentiating equation 4, and h comes from equation 3. Substituting these values in equation 30 and integrating yields:

$$F = 4\mu U r\left(\frac{1}{\sqrt{2rh_0}}\tan^{-1}\frac{t}{\sqrt{2rh_0}} - \frac{t}{2rh_0+t^2}\right) \qquad (32)$$

From this friction force and the tip velocity, U, the rate at which energy enters the lubricant film per unit of vane length becomes:

$$q = FU/778\times 12 \quad \text{BTU/sec.} \qquad (33)$$

The volume of lubricant passing under the vane tip may be found from our earlier disclosure that the film thickness at the point of maximum pressure is $4h_0/3$. Here the velocity of the fluid at the cylinder wall is zero and at the vane tip is U, with no velocity due to a pressure difference. The mean velocity becomes $U/2$, so the volume passing beneath a unit of vane length is:

$$Q = \frac{2}{3} h_0 U \quad \text{in}^3/\text{sec.} \qquad (34)$$

The mass flow in lbs. per sec. per inch of vane length is obtained by multiplying Q by the density from equation 25 and by 0.03613 to convert to pounds per cubic inch. An approximation of the specific heat (3) of an average petroleum lubricant is:

$$C_p = (0.388 + 0.00045\,T)/\sqrt{\rho_{60}} \qquad (35)$$

The increase in oil temperature, °F, with substitutions for q and Q using equations 33 and 34 becomes:

$$\Delta T = \frac{q}{Q\rho_T C_p} = \frac{F}{224.86\, h_0 \rho_T C_p} \qquad (36)$$

The new trial temperature to determine the viscosity is the sum of cylinder temperature plus half of this increase. The process is repeated until a stable temperature and oil film thickness is attained. Several iterations are usually enough, extreme accuracy serves no useful purpose.

APPLYING THE DESIGN PROCEDURE

The calculation of tip speed, gas pressure, tip load viscosity, friction, temperature rise, and oil film thickness is simple but lengthy. It is best set up in a computer program. Repeating the calculations at regular intervals about the cylinder defines the areas of critical oil film thickness and temperature increase. Summing the product of friction and tip speed establishes the tip friction power, normally the largest friction loss in the compressor.

Specific Film Thickness

Calculated film thickness serves no purpose until it is related to adequate separation of the surfaces. The required thickness depends on the surface roughness of the parts. To define this relationship the concept of specific film thickness was adapted from work done on elasto-hydrodynamic lubrication of gears and rolling bearings (4), (5). Surface roughness is usually measured as either root-mean-square or arithmetic mean deviation, S, from a plane surface. See figure 6. Assume that the oil film is bounded by those median plane surfaces. Complete separation will depend on how high the peak asperities extend above the plane. This in turn depends on the texture of the finish, which is a function of the machining process. The ratio of peak asperity height to measured roughness is a machining factor, C. To separate two surfaces, the oil film must be at least the thickness:

$$h_o = C_1 S_1 + C_2 S_2 \approx C(S_1 + S_2) \qquad (37)$$

The specific oil film is defined as the ratio of film thickness to peak surface roughness.

$$\lambda = h_o / C(S_1 + S_2) \qquad (38)$$

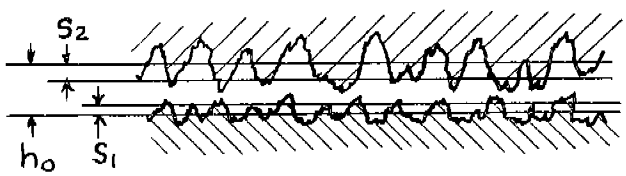

Fig. 6 Fluid Film Thickness and Surface Roughness

Testing indicates that cylinder roughness, S_1, is of greatest importance. Machined finish on the vane tips is not particularly fine. With a smooth cylinder, however, the high spots quickly run in to an excellent finish, usually 8 to 10 microinch RMS, without noticeably changing the cylinder finish. An oil film is established and wear ceases unless the cylinder is too rough. Equation 38 may be rewritten for a finely honed cylinder with a circumferential lay:

$$\lambda = h_o / C(S_1 + 8) \qquad (39)$$

Acceptable Results

The surfaces have waviness and other variations from true straightness so that separation requires an oil film greater than the sum of the nominal mean-to-peak distances. Assuming that C is about 2, the minimum practical value for λ is about 1.5. Oil film thickness is plotted against cylinder finish in figure 7 for various values of λ, using equation 39. An oil film of less than 50 microinches is probably not practical, although compressors have run with lesser films over short distances without distress on highly finished cylinder surfaces. Such a situation might occur where the vanes pass ports in the cylinder wall and load per unit length is high due to the loss of support.

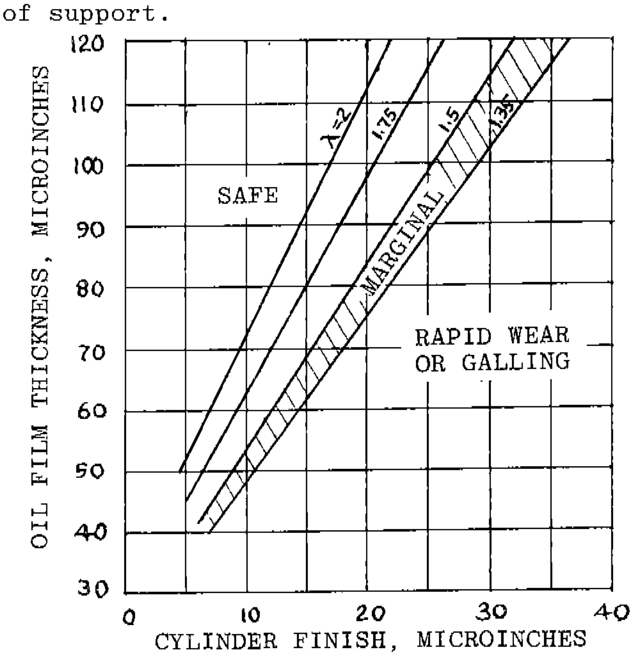

Fig. 7 Film Thickness vs. Cylinder Finish for Hydrodynamic Lubrication

The chart makes it apparent that operation with very thin films also requires excellent cylinder finishes such as may be obtained by a super-finishing process. For obvious reasons of cost, it is desirable to manipulate tip radius and oil viscosity so that bore finishes better than 16 microinch are not required. For some applications, combining high speed and load, 6 to 10 microinch finishes have been obtained regularly in production. An example of the latter was our 900 CFM vane compressor in which normal tip speed reached 100 ft./sec. with loads as high as 53 lbs. per linear inch of vane on an oil film averaging about 100 micoinches.

Attesting to the validity of these design criteria are many laboratory tests together with field experience with thousands of compressors, both portable and stationary. Perhaps as many as twenty new designs have been successfully produced, here and abroad, and modifications made to a number of older designs produced before the study was made.

One compressor, called to our attention, showed no measurable wear of cylinders or vane tips after 43,000 hours of operation. Another operated for 30,500 hours on oil one grade lighter than recommended. Again there was no measurable wear although the vane tips traveled over a million miles on an oil film averaging perhaps less than .0001 inches. Quite surely some polishing of the cylinder took place, however.

These compressors were obviously well maintained as regards filtering of air and oil. Abrasive dirt is the bugaboo for the minute clearances at vane tips and in the roller bearings. It is remarkable that so little wear takes place.

CONCLUSION

The ability to effectively tie vane tip design to required cylinder finish and lubricant viscosity is of great benefit. The methods used are not limited to air compressors of the type described. The basic derivation is applicable to many other designs where a vane or seal must operate under load with a rounded tip against a curved surface in the presence of a fluid lubricant.

NOTATION

a -- coefficient of thermal expansion
A -- Area, or vane dimension from trailing edge to centerline of tip radius
c -- surface finish machine constant
C_P -- specific heat of lubricant
d -- vane dimension from tip to C.G.
E -- eccentricity of rotor in cylinder bore
F -- friction per unit length of vane
F_1, F_2 - inertia forces on vane
g -- acc. of gravity, 386 in./sec.^{2}
h -- fluid film thickness
h_o -- minimum fluid film thickness
K -- film thickness where pressure is max.
K_2, K_3 - constants in viscosity calculation
L -- length of vane tip support
M -- vane weight
n -- no. of vanes, or polytropic exponent
N -- speed, rpm
P -- pressure
q -- power, BTU/sec.
Q -- volume flow rate, in.^{3}/sec.
r -- equivalent vane tip radius
R_T -- vane tip radius
R_C -- cylinder bore radius
R_R -- rotor radius
R_G -- radius, center of rotor to C.G.
S -- measured surface roughness
t -- vane tip tangent to leading edge, in.
T -- vane thickness or temperature, ^oF
U -- vane tip tangent point velocity, in./sec.
W -- total load on vane tip, lbs.
w -- vane tip load per unit of supported length lbs./in.
x -- coordinate, oil film
X -- radius from center of rotor to vane tip center at cylinder wall
y -- coordinate, oil film
Z -- viscosity, centipoise
α -- angle between vane centerline and radius from center of cylinder bore to tangent point of tip
θ -- rotor rotation angle from common centerline
λ -- specific oil film thickness
μ -- viscosity, reyns (lbf) (sec)/(in^2)
ν -- viscosity, centistokes
ρ_T -- lubricant density at temp. T, g/cm^3
ω -- angular velocity, radians/sec.

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