Publications of the Ray W. Herrick Laboratories School of Mechanical Engineering

8-28-2018

Noise Source Identification in an Under-Determined System by Convex Optimization

Tongyang Shi
Purdue University, shi247@purdue.edu

Yangfan Liu
Purdue University, liu278@purdue.edu

J Stuart Bolton
Purdue University, bolton@purdue.edu

Noise Source Identification in an Under-Determined System by Convex Optimization

Tongyang Shi[1)]

Yangfan Liu[1)]

J. Stuart Bolton[1)]

[1)] Ray W. Herrick Laboratories, Purdue University

INTER-NOISE 2018 26-29 AUGUST

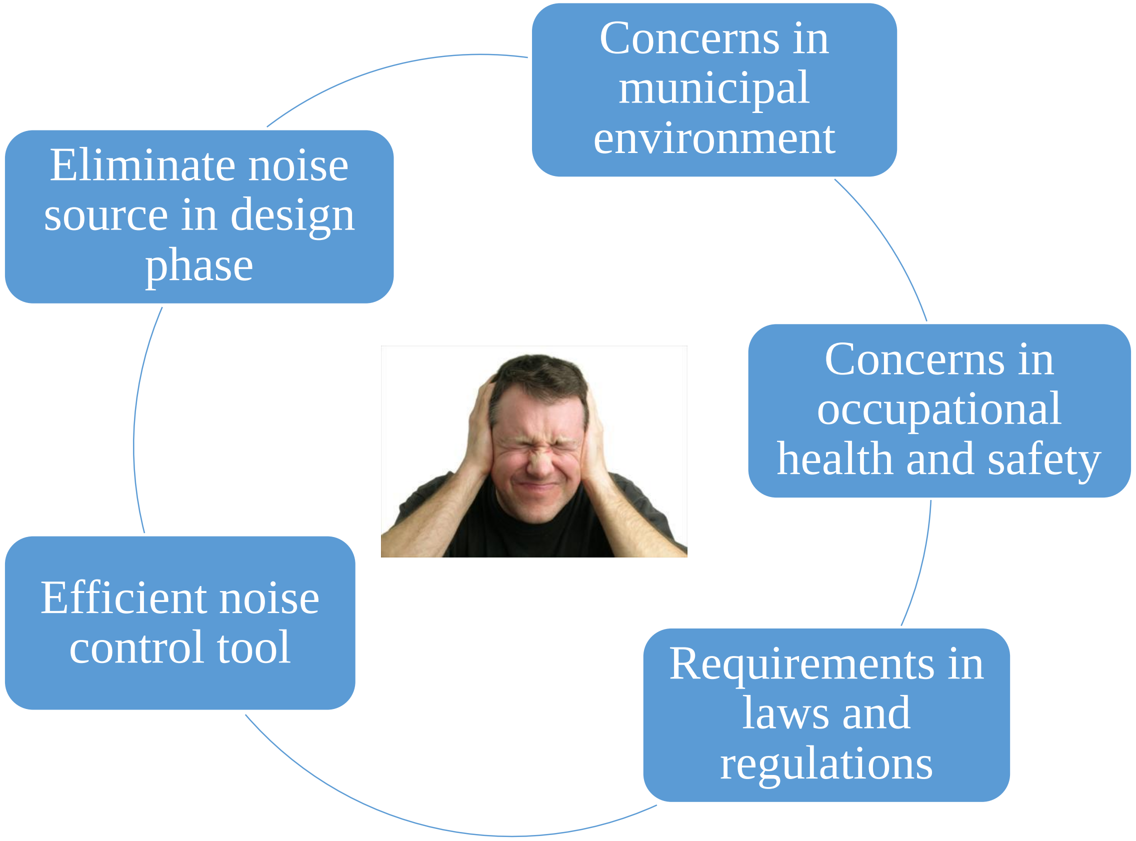
Impact of Noise Control Engineering CHICAGO, ILLINOIS

PURDUE UNIVERSITY

RAY W. HERRICK LABORATORIES

Sound Source Localization

- Importance of Sound Source Identification



- Near-field Acoustics Holography (NAH)

➢ Inverse Fourier Method

➢ Statistically Optimized Near Acoustical Holography (SONAH)

➢ Inverse Boundary Element Method (IBEM)

➢ Equivalent Source Method (ESM)

➢ Inverse Radiation Mode

  ➢ Jiawei Liu, “Noise source Identification based on an Inverse Radiation Mode Procedure”, Noise-Con 16, Providence, Rhode Island.

Motivation of Current Work

- NAH is a powerful tool to identify sound source
  - ➢ Measurements can be taken away from the source and sound field can be visualized in three-dimensional space
  - ➢ Large number of measurements is required to avoid different measurement errors: e.g., spatial aliasing, windowing errors, etc.
  - ➢ Economically costly, and hard to perform

- Motivation
  - ➢ Using a small number of microphone measurements to accurately identify major sound source locations
  - ➢ Encourage wide application of NAH in industry

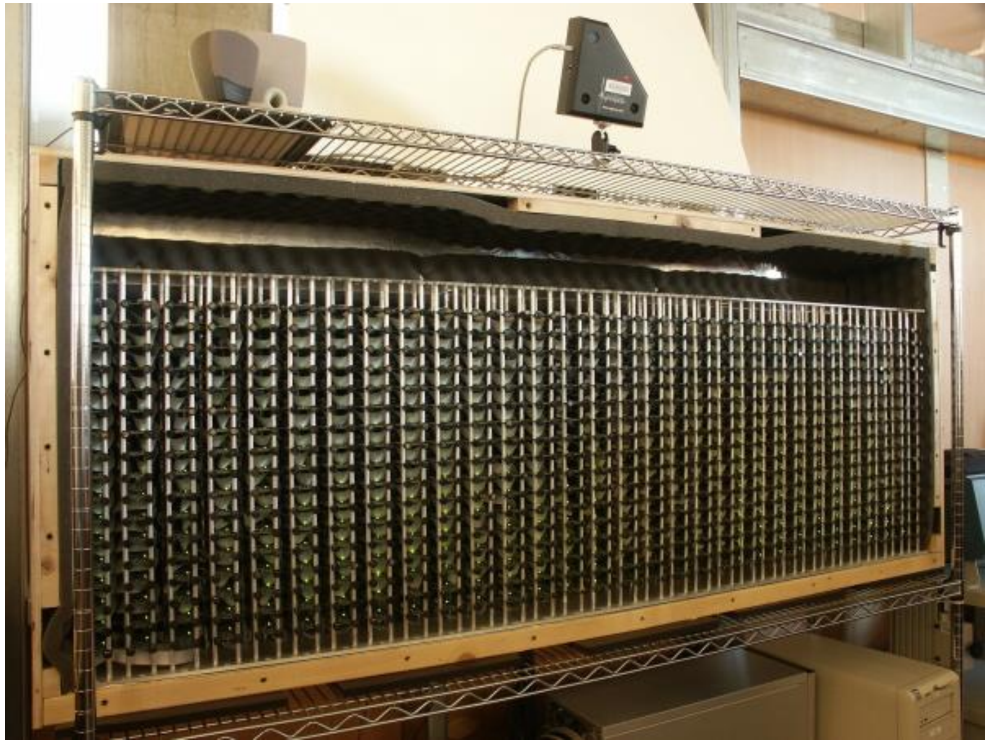
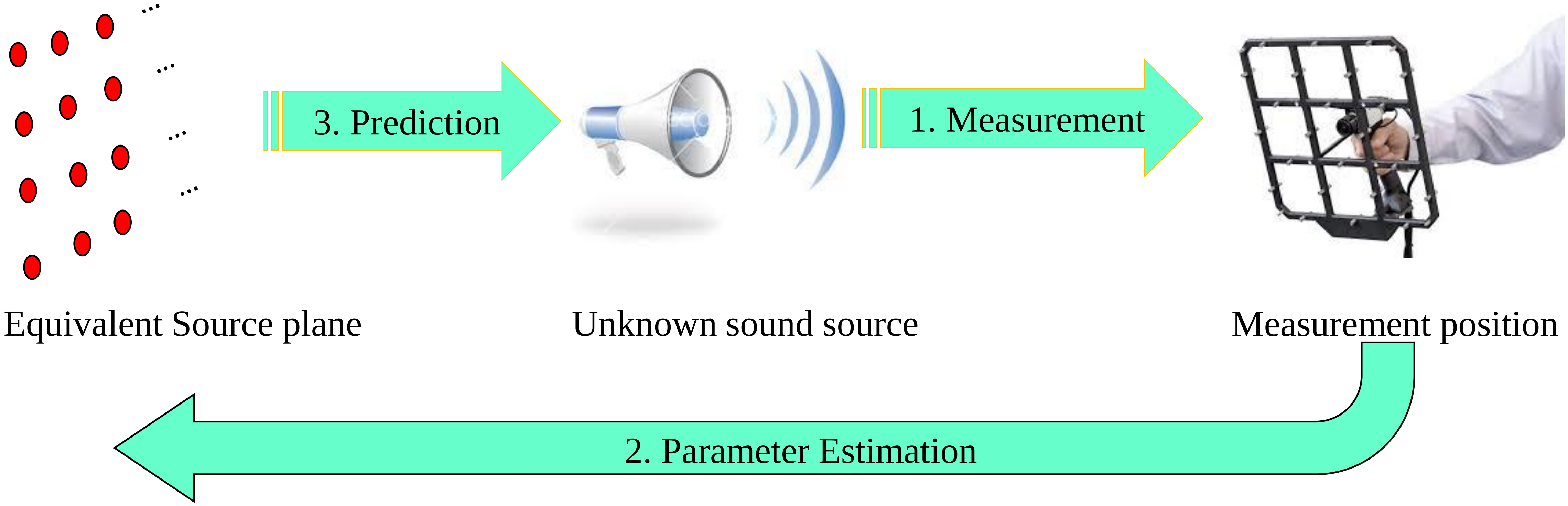


Figure: LOUD 1020-node microphone array

Monopole Based Equivalent Source Method

- Idea of monopoles at fixed locations



monopoles are used as low order equivalent sources in the present work

Mathematical Formulation

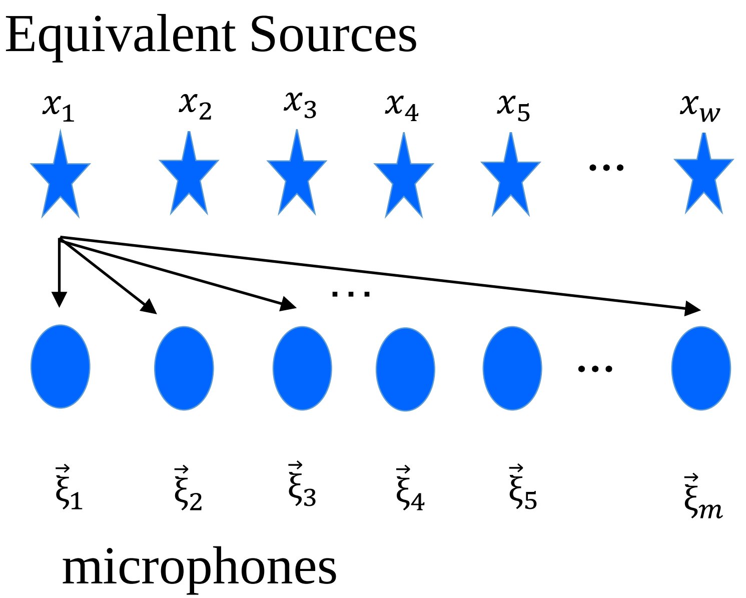
- Expression of a monopole with source strength S

$$P_{S0}\left(\vec{X}\middle|\overrightarrow{X_0},\omega\right) = S \cdot P_0\left(\vec{X}\middle|\overrightarrow{X_0},\omega\right) = \frac{Se^{-jk\left\|\vec{X}-\overrightarrow{X_0}\right\|}}{4\pi\left\|\vec{X}-\overrightarrow{X_0}\right\|},$$

Field point position ($\vec{X}$)   Monopole location ($\overrightarrow{X_0}$)

- The equation of the model-generated acoustic field at all locations can be derived in a matrix form:

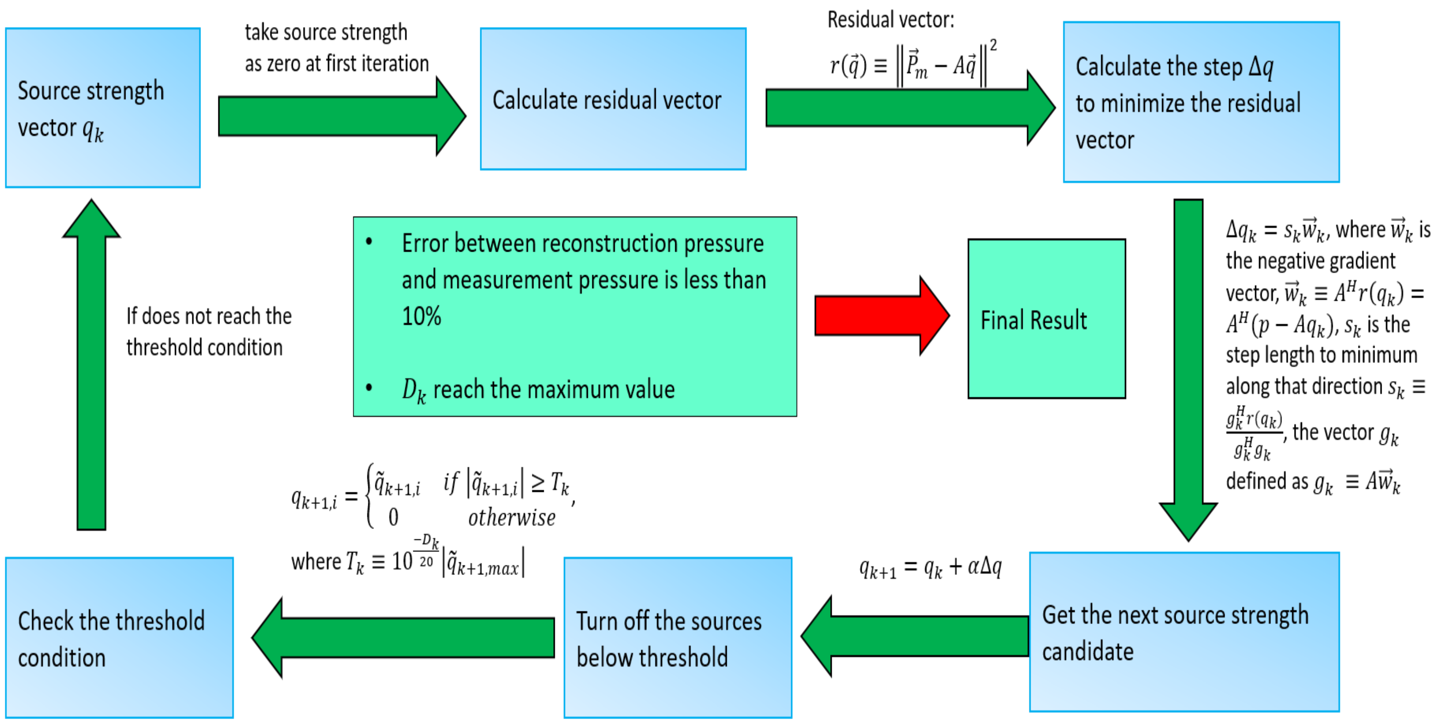
$$\begin{bmatrix} P_1(\vec{\xi}_1,\omega) \\ P_2(\vec{\xi}_2,\omega) \\ \cdots \\ P_M(\vec{\xi}_M,\omega) \end{bmatrix} = \begin{bmatrix} g_1(\vec{\xi}_1|\overrightarrow{X_1},\omega) & g_2(\vec{\xi}_1|\overrightarrow{X_2},\omega) & \cdots & g_W(\vec{\xi}_1|\overrightarrow{X_W},\omega) \\ g_1(\vec{\xi}_2|\overrightarrow{X_1},\omega) & g_2(\vec{\xi}_1|\overrightarrow{X_2},\omega) & \cdots & \cdots \\ \cdots & \cdots & & \cdots \\ g_1(\vec{\xi}_M|\overrightarrow{X_1},\omega) & g_2(\vec{\xi}_1|\overrightarrow{X_2},\omega) & \cdots & g_W(\vec{\xi}_M|\overrightarrow{X_W},\omega) \end{bmatrix} \begin{bmatrix} S_1(\omega) \\ S_2(\omega) \\ \cdots \\ S_W(\omega) \end{bmatrix}$$

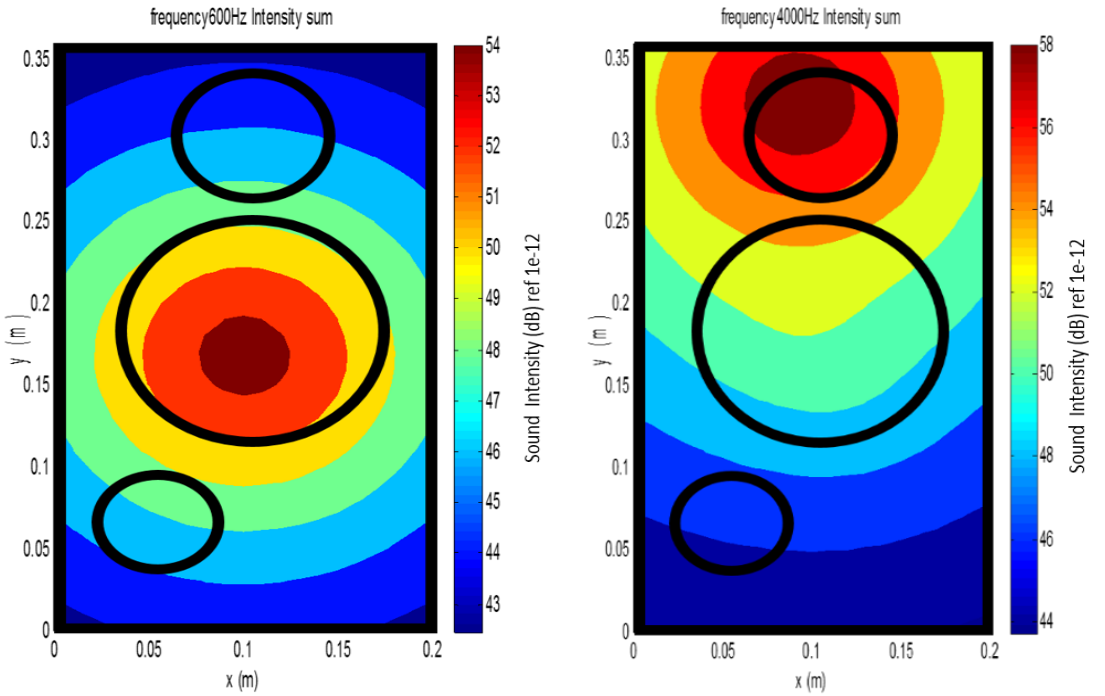


- Objective function: $\min\left\|\vec{P} - A(\vec{X})\vec{S}\right\|^2$

Previous Study

- Wideband Acoustical Holography (WBH)
  - When there is only one major sound source present, the monopole-based Wideband Acoustical Holography (WBH) can localize the sound source location and reconstruct the sound field when the system is under-determined. (T. Shi, Y. Liu, and J. Stuart Bolton. "The Use of Wideband Acoustical Holography for Noise Source Visualization." In *INTER-NOISE and NOISE-CON Congress and Conference Proceedings*, vol. 252, no. 2, pp. 479-490. Institute of Noise Control Engineering, 2016.)

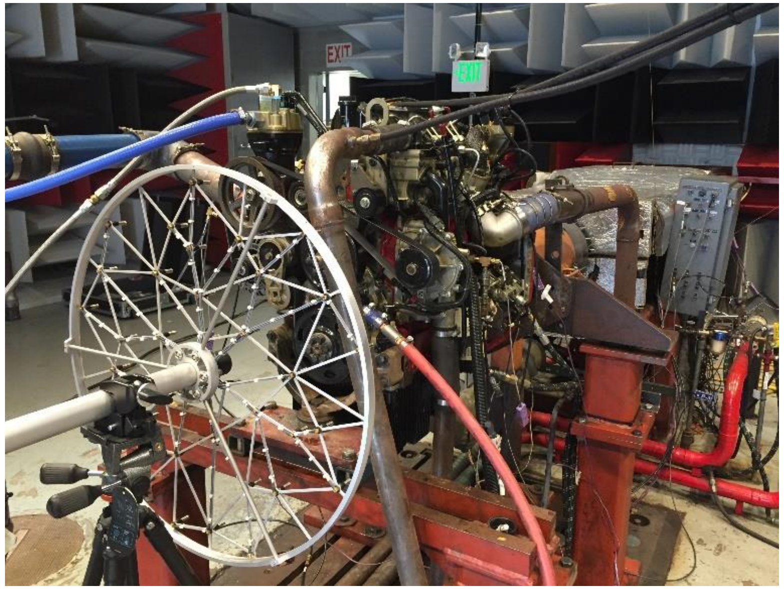




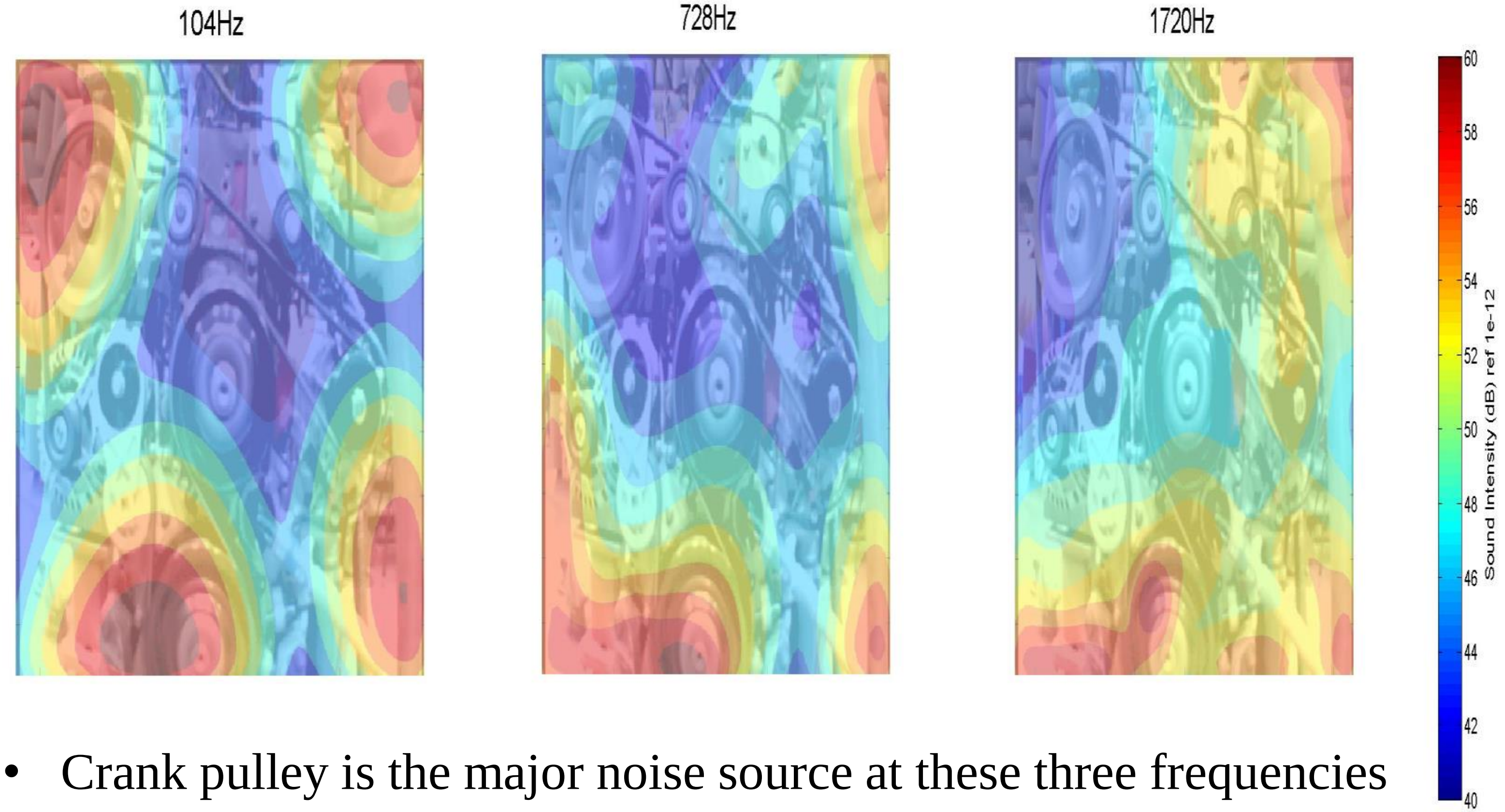
- Good results with small number of microphones

Previous Study

- Wideband Acoustical Holography (WBH)
  - Combined with Partial Field Decomposition (PFD), WBH can identify complex sound sources with a small number of measurements, e.g., diesel engine. (T. Shi, Y. Liu, J. Stuart Bolton, F. Eberhardt, and W. Frazer. ”Diesel Engine Noise Source Visualization with Wideband Acoustical Holography.” No. 2017-01-1874. SAE Technical Paper, 2017.)



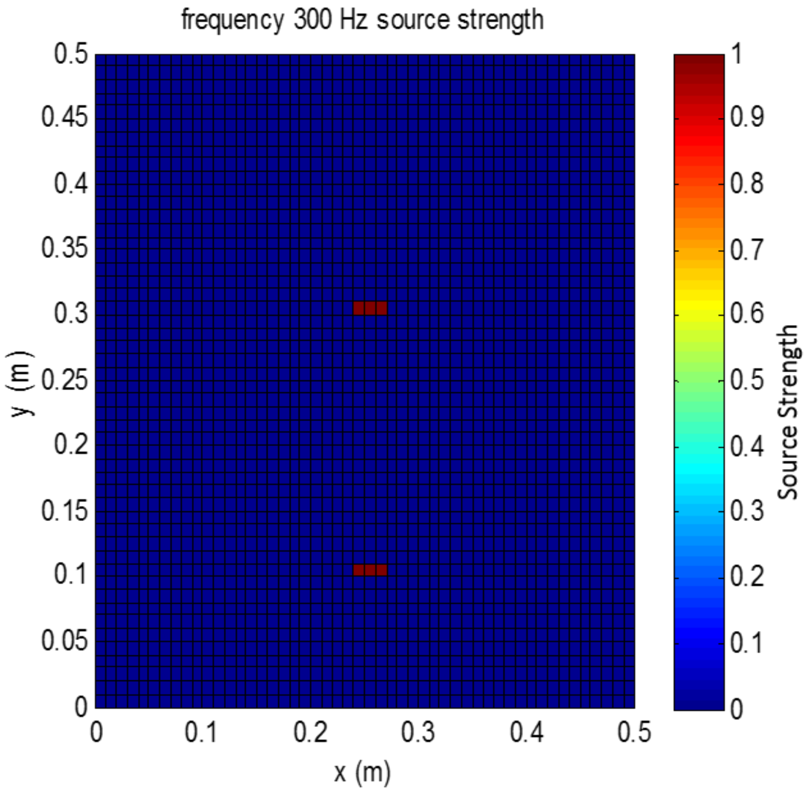
Experimental setup in Cummins Walesboro Noise and Vibration Lab, Columbus, IN



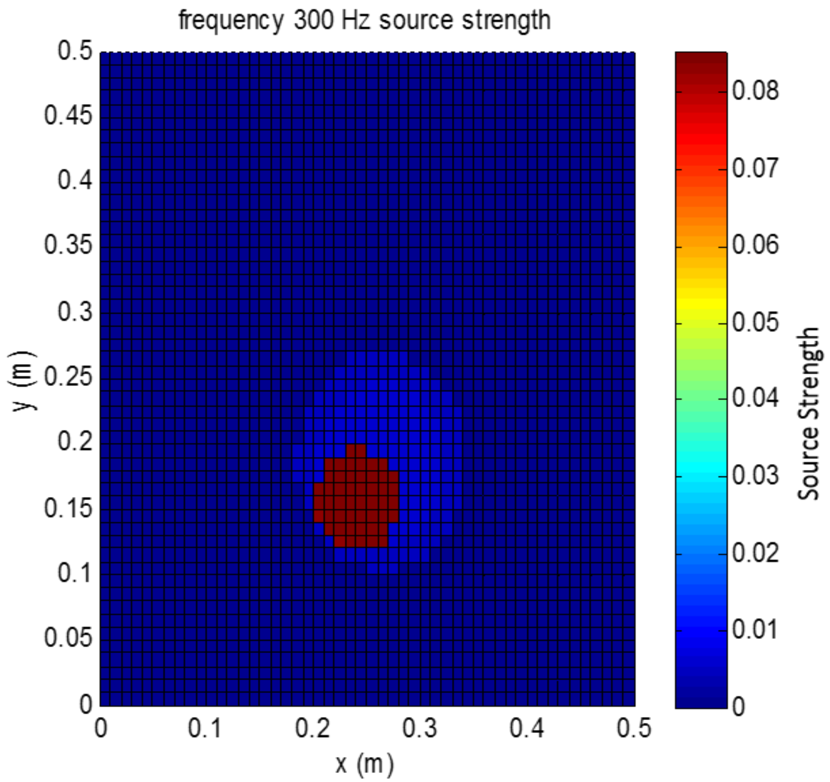
- Crank pulley is the major noise source at these three frequencies

Previous Study

- Wideband Acoustical Holography (WBH)
  - ➢ From simulation, it was found that when using WBH method, closely-positioned sources cannot be separated in space and cannot recover appropriate source strength, especially at low frequency. (T. Shi and J. S. Bolton. "Separation of closely-spaced acoustics sources in an under-determined system with convex optimization." *The Journal of the Acoustical Society of America* 143, no. 3 (2018): 1872-1872.)



True source distribution



WBH reconstructed sources

Convex Optimization

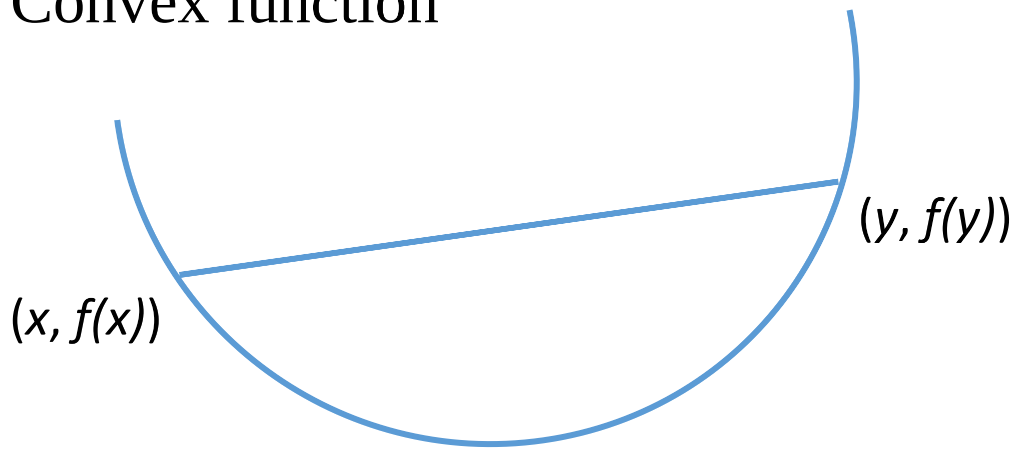
- Under-determined system
- Low spatial sampling rate

- Nyquisit-Shannon sampling theorem
- Compressive Sampling (CS)

$$\min \left\| \vec{P} - A(\vec{X})\vec{S} \right\|^2$$

- Convex function
- Convex Optimization

➢ Convex function



- $f(\theta x + (1-\theta)y) \leq \theta f(x) + (1-\theta)f(y)$

➢ Convex optimization problem

$$\begin{aligned} &\text{minimize} \quad f_0(x) \\ &\text{subject to} \quad f_i(x) \leq 0, \qquad i = 1, \ldots, m \\ &\qquad\qquad\quad h_i(x) = 0, \qquad i = 1, \ldots, m \end{aligned}$$

Objective Function Formulation

- Objective function

Weighting parameter

$$\text{minimize } \left\|\vec{S}\right\|_1 + \lambda \left\|\boldsymbol{A}\vec{S} - \vec{P}_m\right\|_2$$

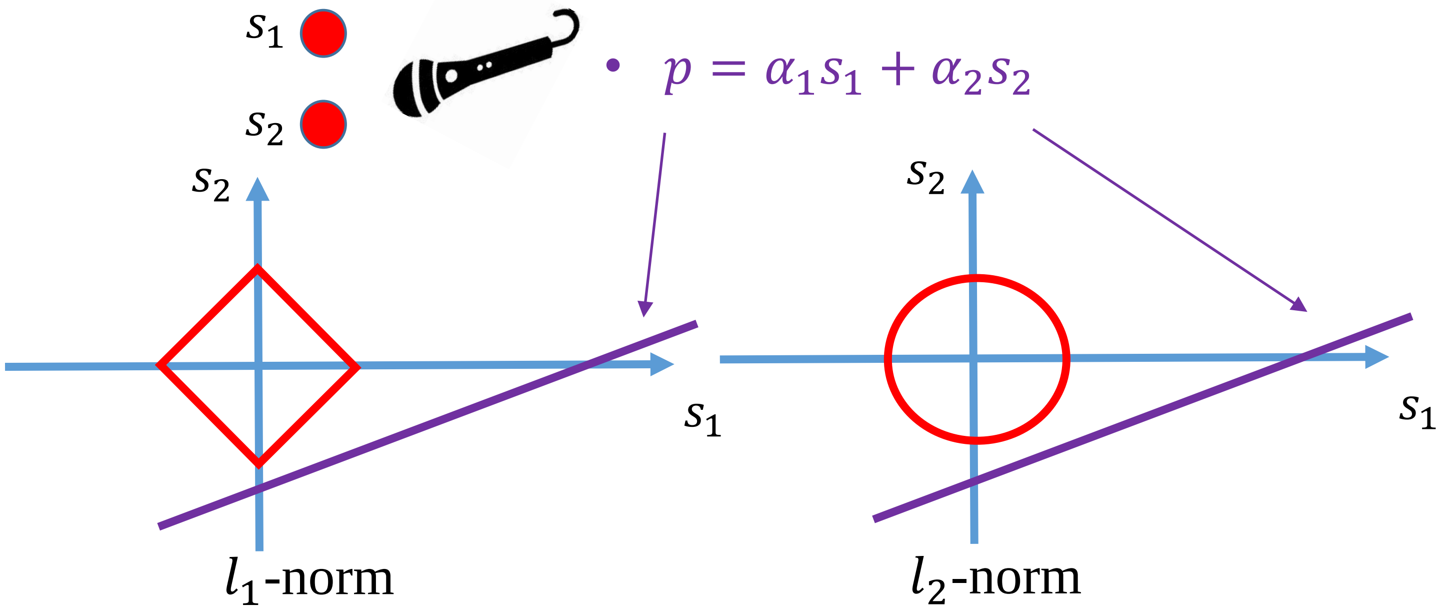
Solution sparsity

Solution accuracy

- Careful choosing of the weighting parameter
- l_1-norm for source strength and l_2-norm for residual

- M. Grant, S. Boyd, and Y. Ye *CVX: software for disciplined convex programming*

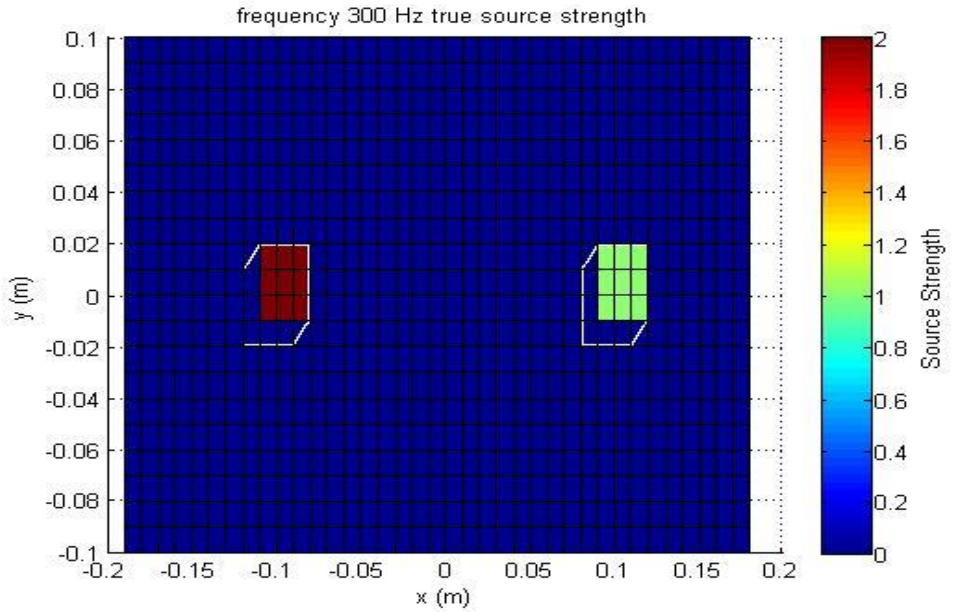
- l_1-norm and l_2-norm
  - l_1-norm: $\left\|\vec{S}\right\|_1 = \sum_{i=1}^{m} |s_i|$
  - l_2-norm: $\left\|\vec{S}\right\|_2 = \sqrt{s_1^2 + s_1^2 + \cdots + s_m^2}$
  - Choice of l_1-norm ensures solution sparsity

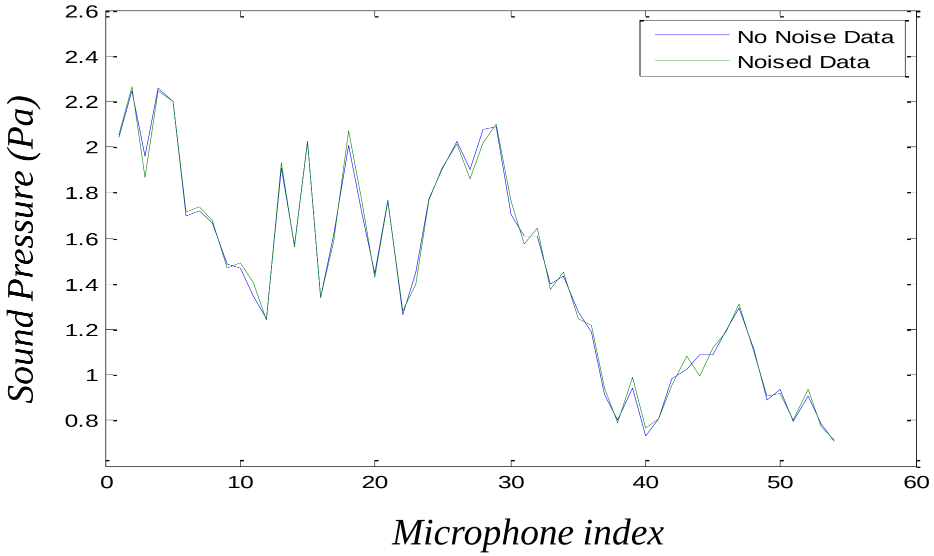


- $p = \alpha_1 s_1 + \alpha_2 s_2$

Simulated Closely-Positioned Sources

- Simulation set up
  - Two simulated sources were placed 0.2 m from each other, source at left was composed of nine monopoles with source strength two, and source at right composed of nine unit source strength monopoles.
  - 54 virtual microphones measurement 0.23 m in front of virtual sources
  - 300Hz, wavelength λ=1.14m; 2000Hz, wavelength λ=0.17m
- Equivalent source plane
  - -0.19 - 0.18 m, in *x*-direction
  - -0.1 - 0.1 m, in *y*-direction
  - 0.01 m spacing in both *x*- and *y*-direction, 798 monopoles.
  - 0.02 m in *z*-direction
- White Gaussian Random Noise added into the virtual measurement, SNR = 30 dB

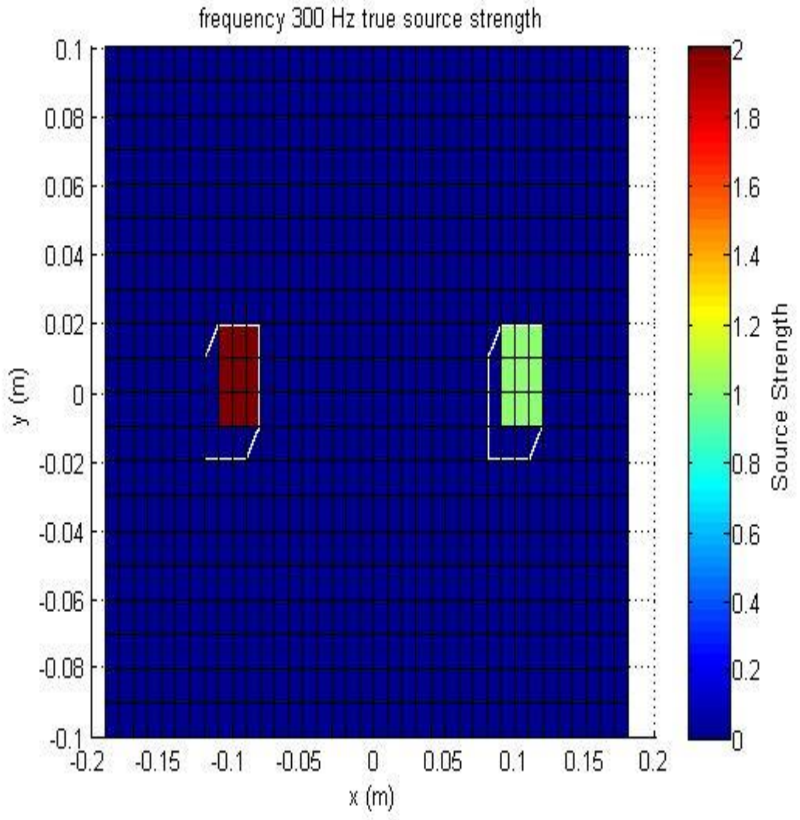




Reconstruction on Equivalent Source Plane at 300 Hz

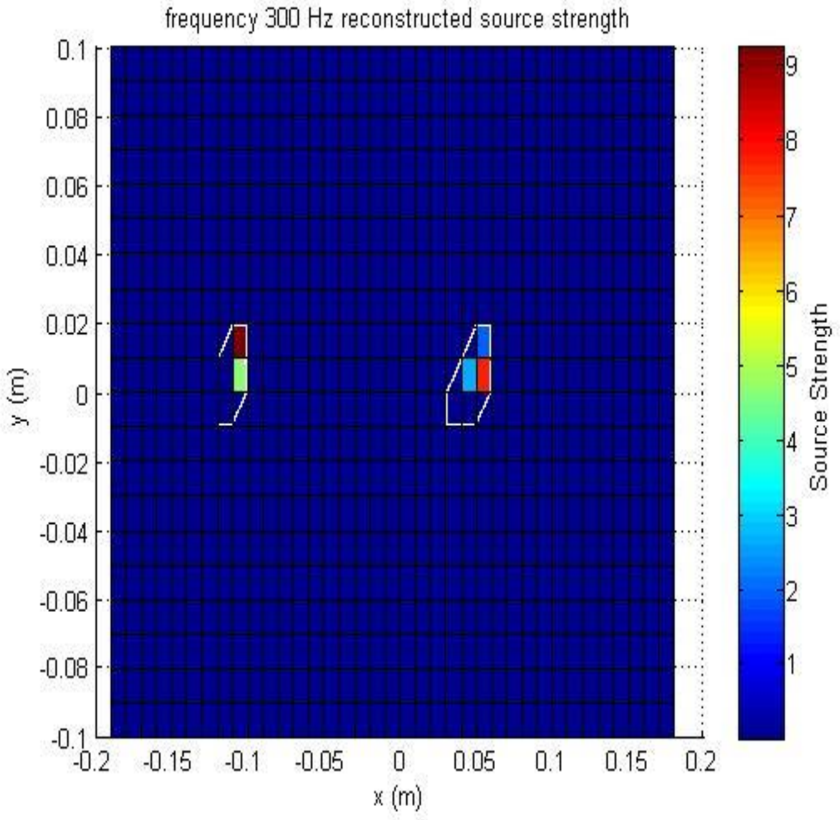
minimize $\|\vec{S}\|_1 + 2\left\|A\left(\vec{X}_S\right)\vec{S} - \overrightarrow{\hat{P}_m}\right\|_2$

Error < 10%, D_max = 60



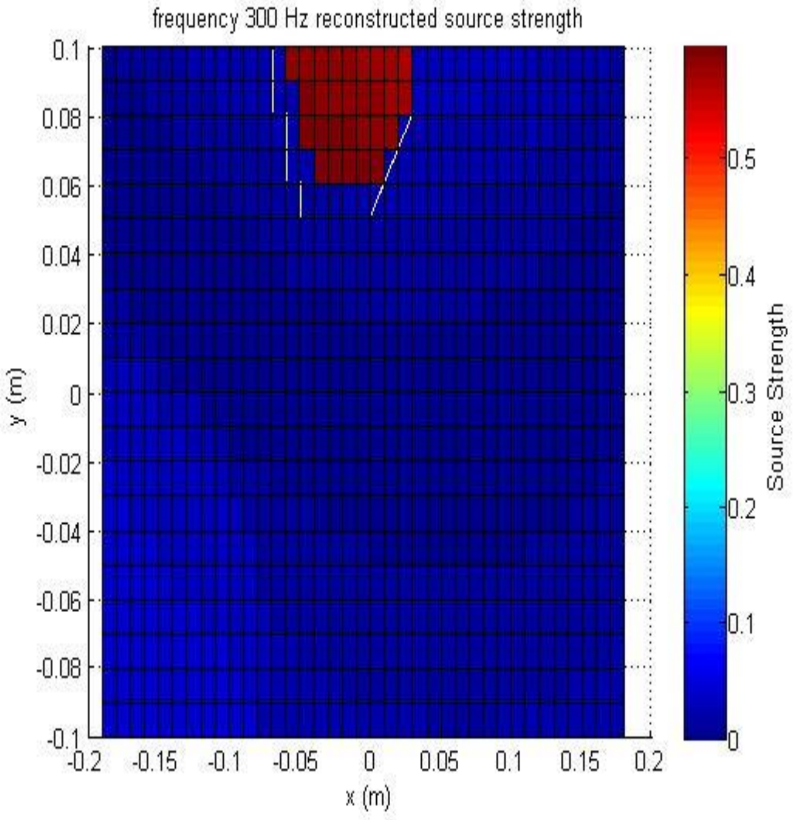
True sources distribution

Total source strength 27



Convex Optimization reconstructed sources

Total source strength 25.73



WBH reconstructed sources

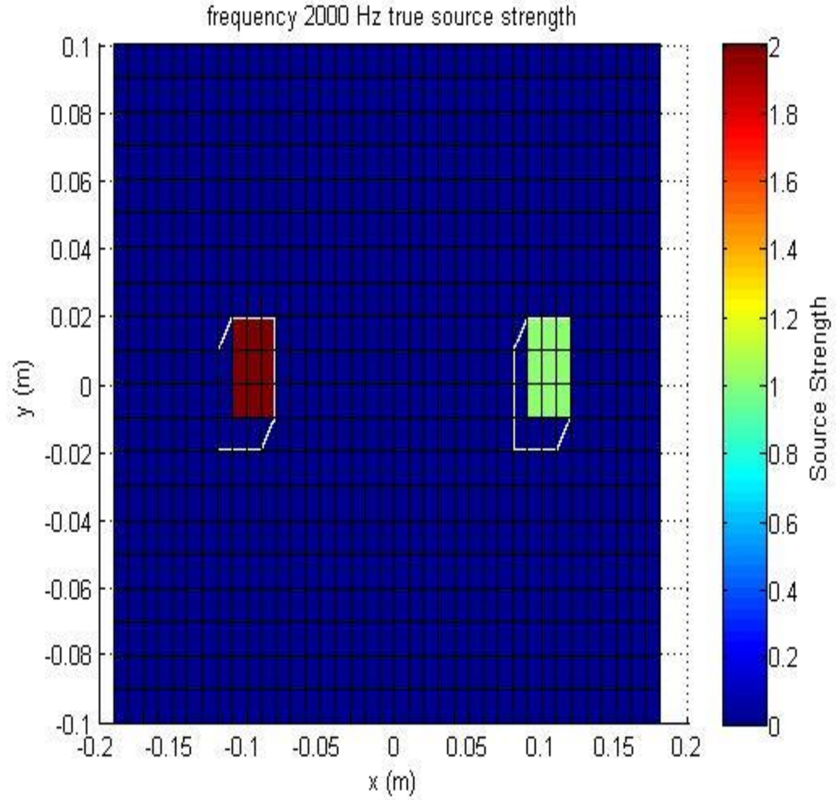
Total source strength 46.53

- Two separated sources were identified by Convex Optimization near true source location with nearly-correct source strength, but WBH failed to find either correct source location or source strength.

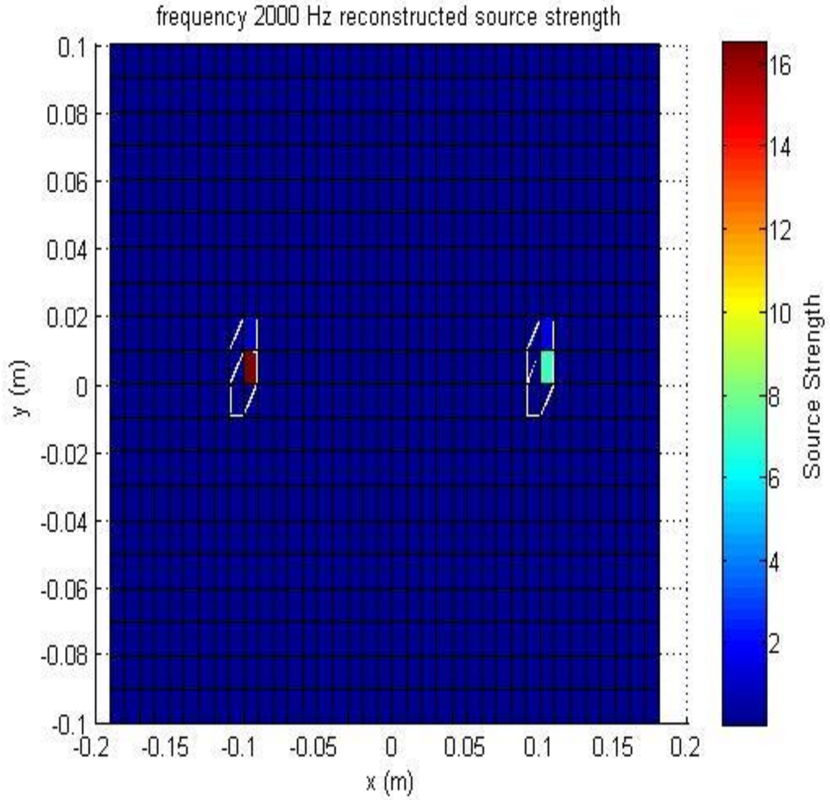
Reconstruction on Equivalent Source Plane at 2000 Hz

minimize $\left\|\vec{S}\right\|_1 + 2\left\|A\left(\vec{X}_S\right)\vec{S} - \vec{\hat{P}}_m\right\|_2$

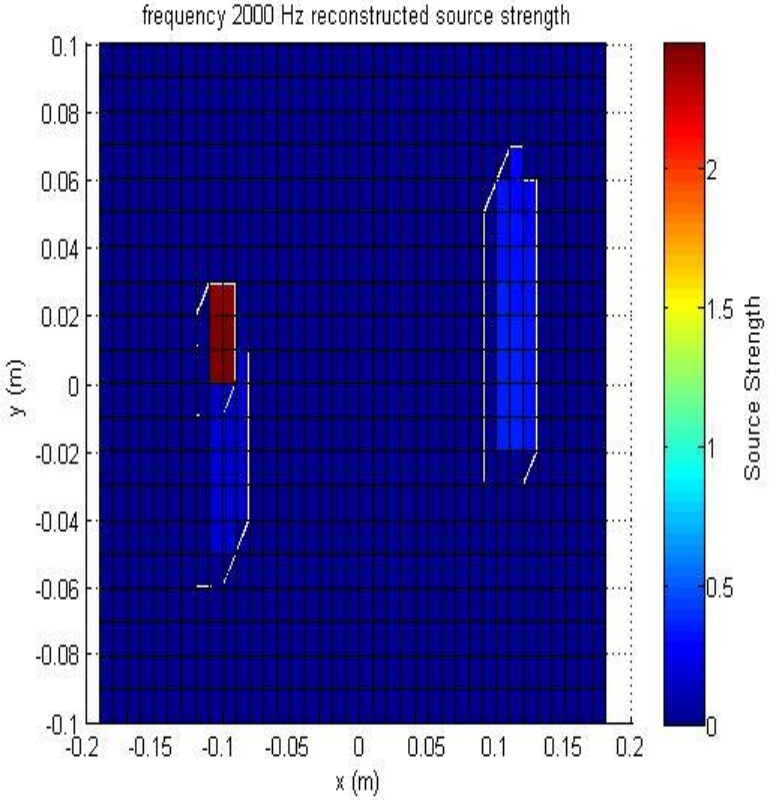
Error < 10%, D_max = 60



True sources distribution
Total source strength 27



Convex Optimization reconstructed sources
Total source strength 26.58



WBH reconstructed sources
Total source strength 27.28

- Convex Optimization reconstruction result is more concentrated and location is more accurate than WBH, and WBH underestimates the weaker source.

Loudspeaker Test

RAY W. HERRICK LABORATORIES

INTER-NOISE 2018 Impact of Noise Control Engineering 26-29 AUGUST CHICAGO, ILLINOIS

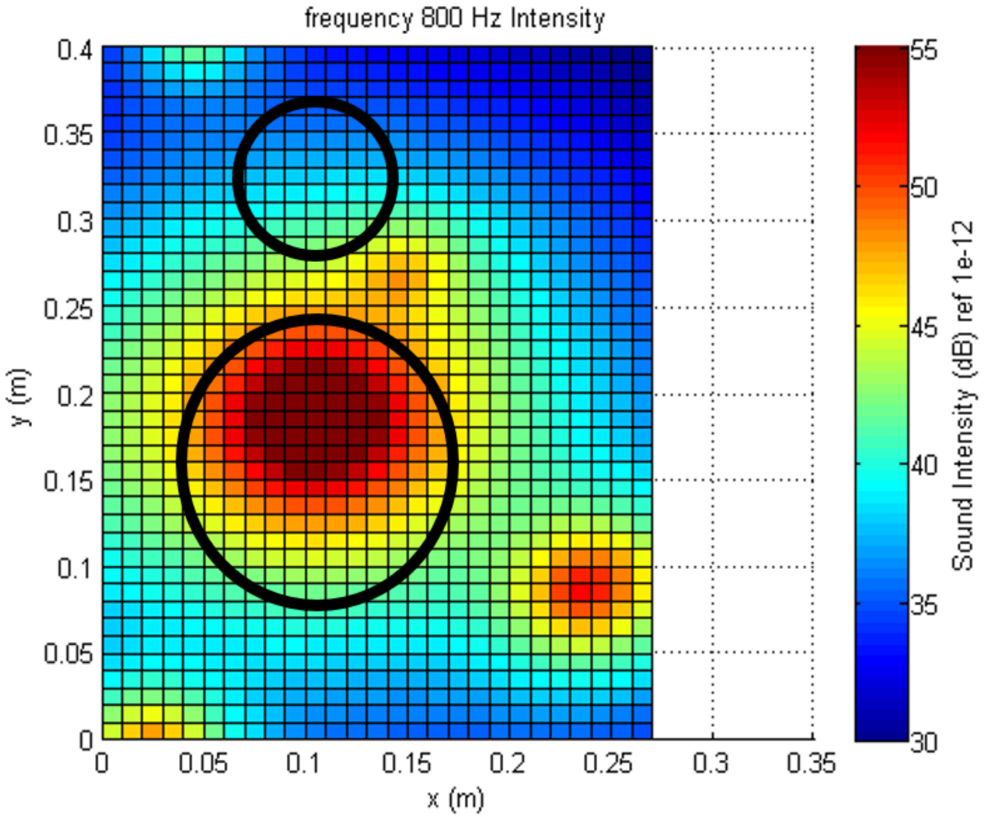
- Experimental set up
  - Test with loudspeaker (Infinity Primus P163) as a noise source
  - White noise as input
  - Brule and Kjaer 18 channel irregular array
  - 54 microphones measurement 0.23 m in front of loudspeaker, 10 second measurement duration

- Equivalent source plane
  - -0.2 - 0.4 m, in *x*-direction
  - -0.2 - 0.4 m, in *y*-direction
  - 0.01 m spacing in both *x*- and *y*-direction, 3721 monopoles.
  - 0.02 m in *z*-direction
  - Sound intensity was reconstructed on loudspeaker front face

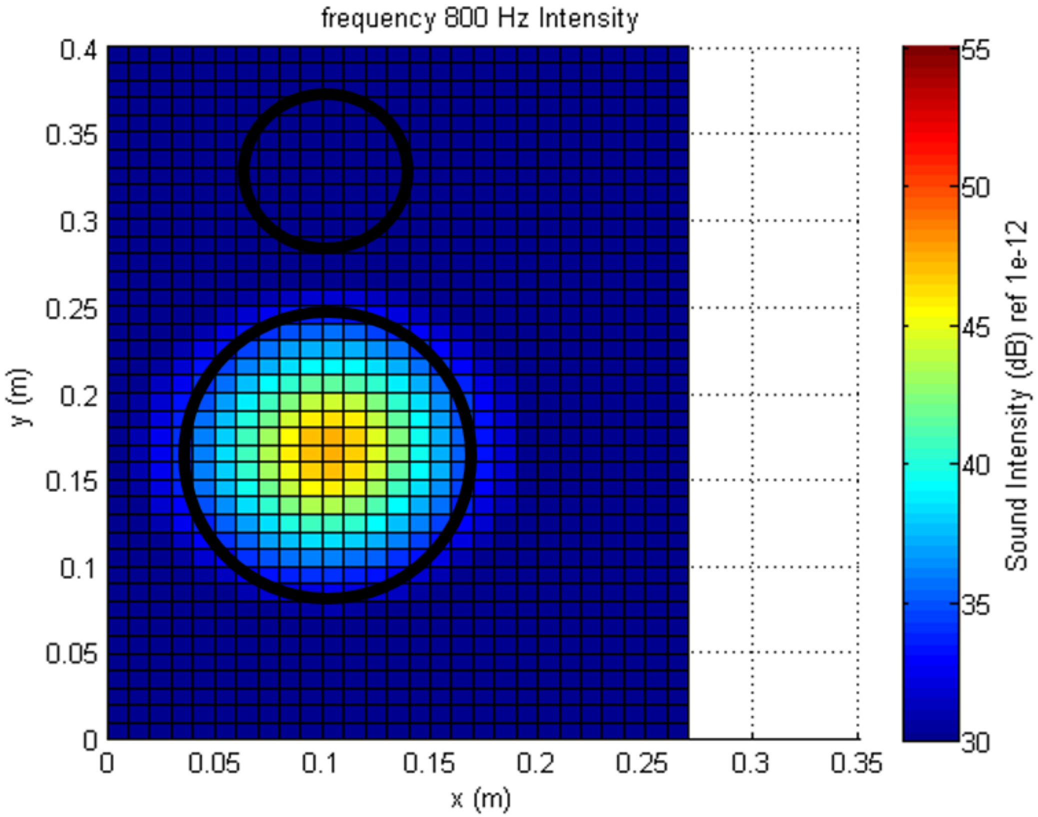
Sound Intensity Reconstruction on Loudspeaker Front Face at 800 Hz

minimize $\left\|\vec{S}\right\|_1 + 10\left\|A\left(\vec{X}_S\right)\vec{S} - \vec{\hat{P}_m}\right\|_2$

Error < 10%, D_max = 100



Convex Optimization



WBH

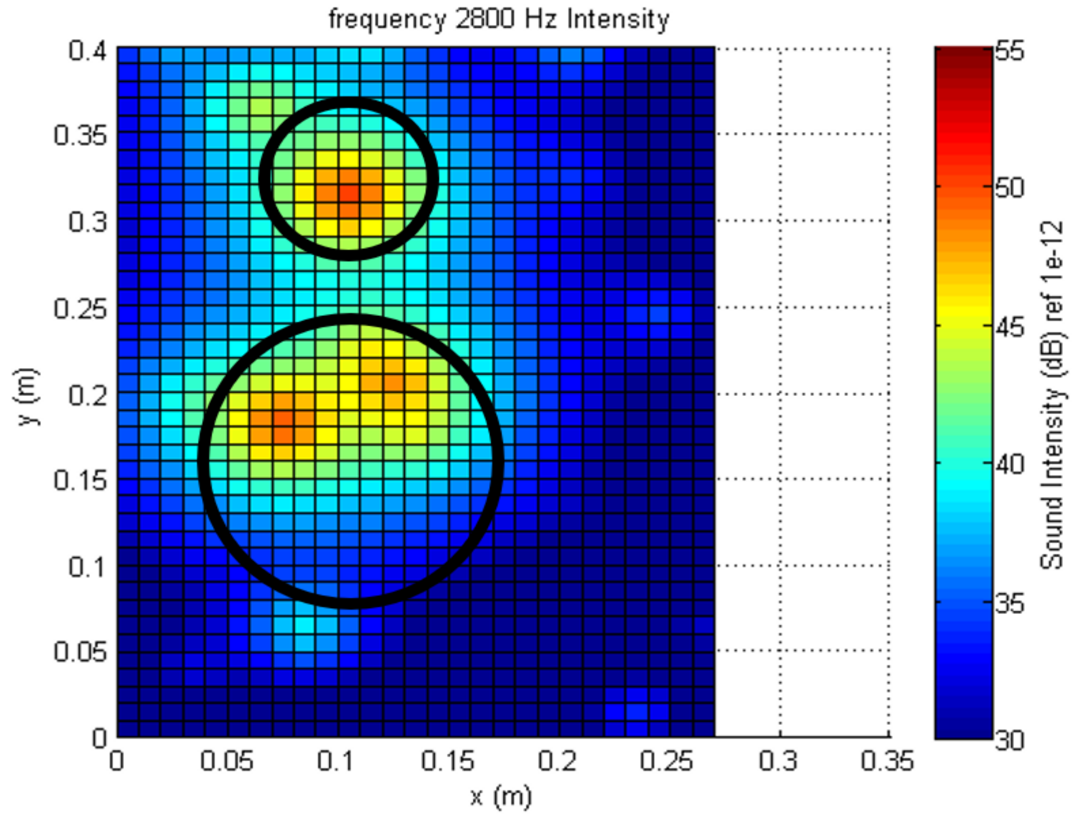
- WBH works well when only a single significant source
- Ghost sources appear in convex optimization case

Sound Intensity Reconstruction on Loudspeaker Front Face at 2800 Hz

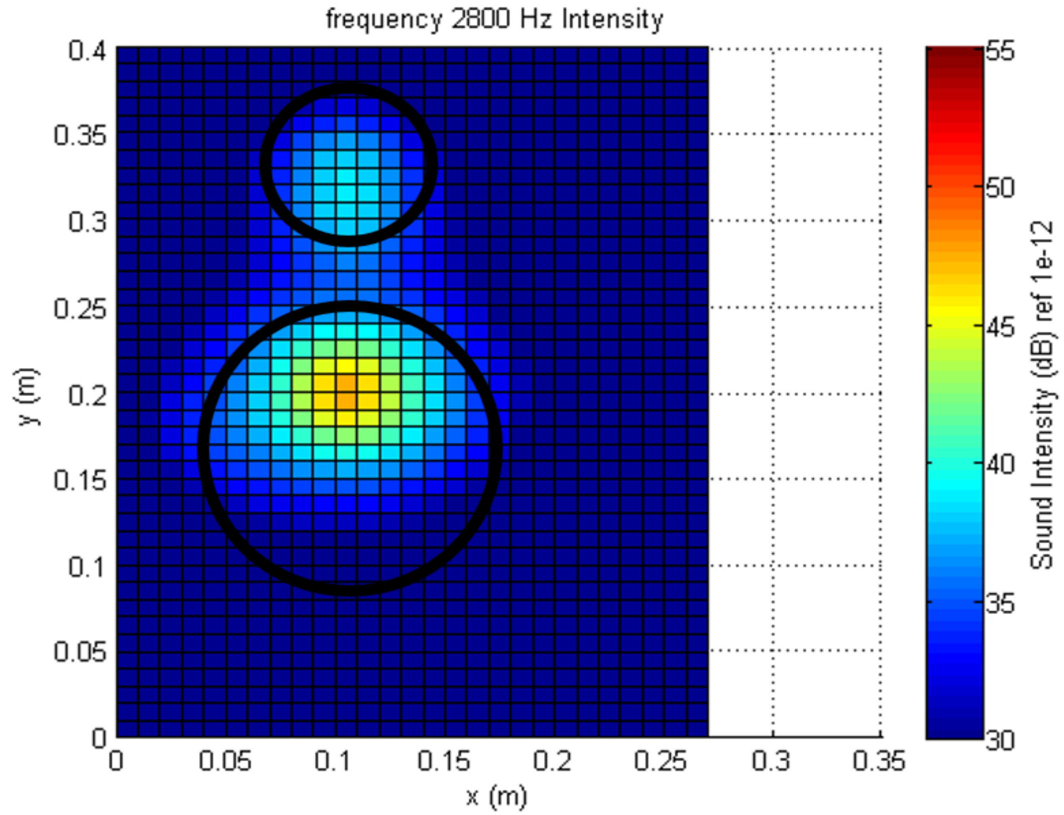
RAY W. HERRICK LABORATORIES
INTER-NOISE 2018 Impact of Noise Control Engineering 26-29 AUGUST CHICAGO, ILLINOIS

minimize $\left\|\vec{S}\right\|_1 + 10\left\|A\left(\vec{X}_S\right)\vec{S} - \overrightarrow{\hat{P}_m}\right\|_2$

Error < 10%, D_max = 100



Convex Optimization



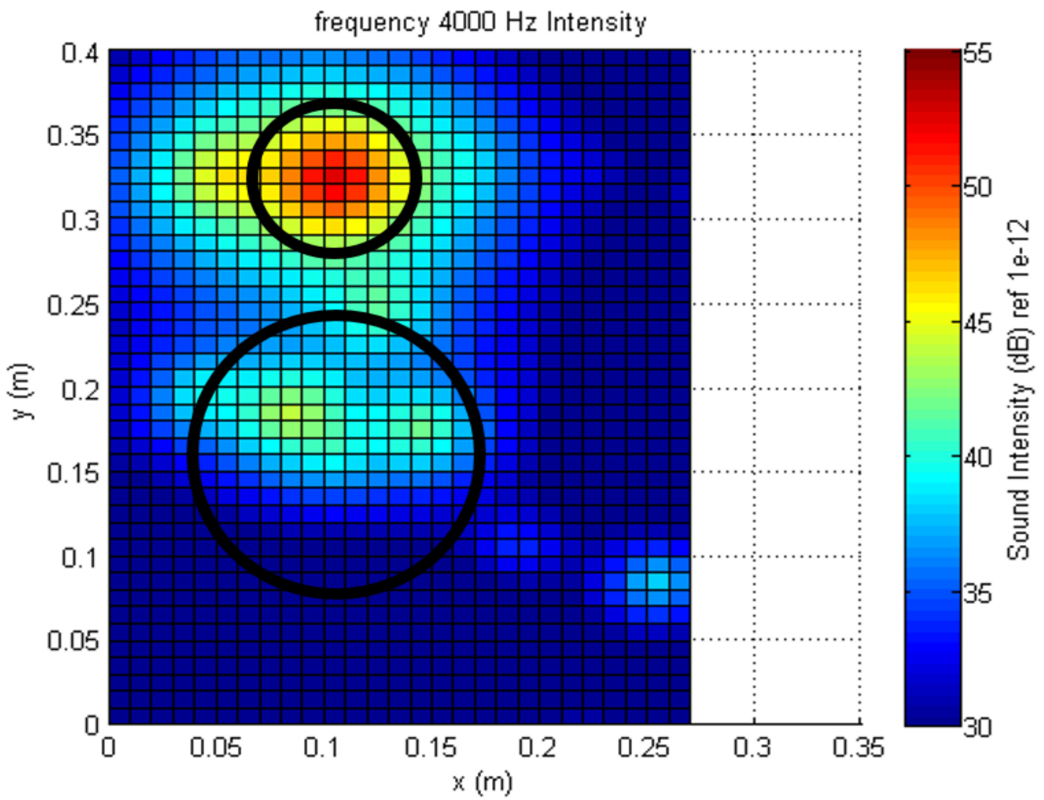
WBH

- WBH underestimates contribution from tweeter
- Convex optimization gives more balanced result

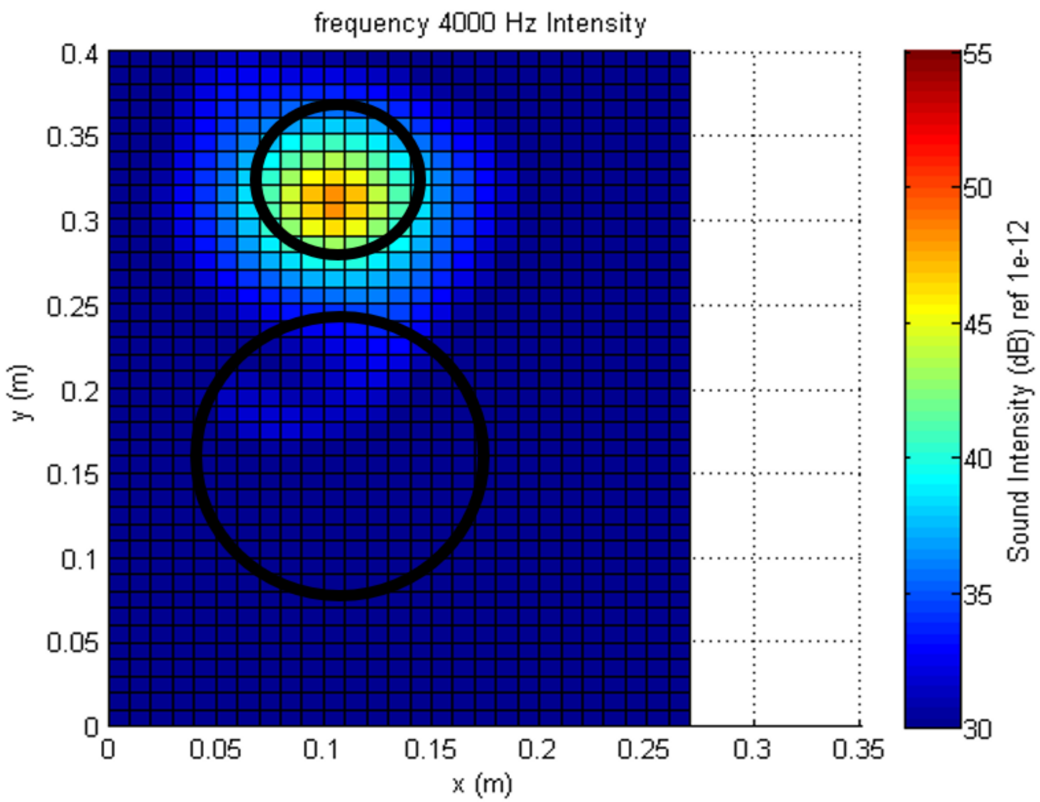
Sound Intensity Reconstruction on Loudspeaker Front Face at 4000 Hz

minimize $\left\|\vec{S}\right\|_1 + 10\left\|A\left(\vec{X}_S\right)\vec{S} - \overrightarrow{\hat{P}_m}\right\|_2$

Error < 10%, D_max = 100



Convex Optimization



WBH

- WBH underestimates contribution from diaphragm

Difference between WBH and Convex Optimization

- Wideband holography method roughly equivalent to

minimize $\left\| A\left(\vec{X}_S\right)\vec{S} - \overrightarrow{\hat{P}_m} \right\|_2$

subject to $\mathbf{card}(\vec{S}) \leq \tilde{t}_k$

  - Steepest gradient method
  - Adjust $\tilde{t}_k$ at each iteration through T_k

- Convex formulation

minimize $\left\| \vec{S} \right\|_1 + \lambda \left\| A\left(\vec{X}_S\right)\vec{S} - \overrightarrow{\hat{P}_m} \right\|_2$

- Different formulation to create the solution sparsity

Conclusions

RAY W. HERRICK LABORATORIES

INTER-NOISE 2018
Impact of Noise Control Engineering
26-29 AUGUST
CHICAGO, ILLINOIS

Noise source identification → Near-field Acoustical Holography (NAH) → Fewer microphone measurements → Under-determined System

Under-determined System →

- With Compressive Sensing, the least-square solution was formulated as a convex function which balances the solution sparsity and accuracy, then this function was solved by an open source Convex Optimization algorithm.
- From simulation and loudspeaker experiment, it was found that with the proposed solution, the reconstructed source is more accurate than WBH. Closely-positioned sources can be separated in space and recovered with appropriate source strength even at low frequency; at mid and high frequency, convex optimization gives more source details, and a better contour for the weaker source.
- Ghost sources are found in the reconstruction result, due to noise is included in the measurement.
- Choosing an appropriate weighting parameter is important, since it controls the model tolerance for noise.

Acknowledgement

- The authors are grateful for the funding provided by Cummins Inc., and for helpful discussions with the contract monitor, Frank Eberhardt